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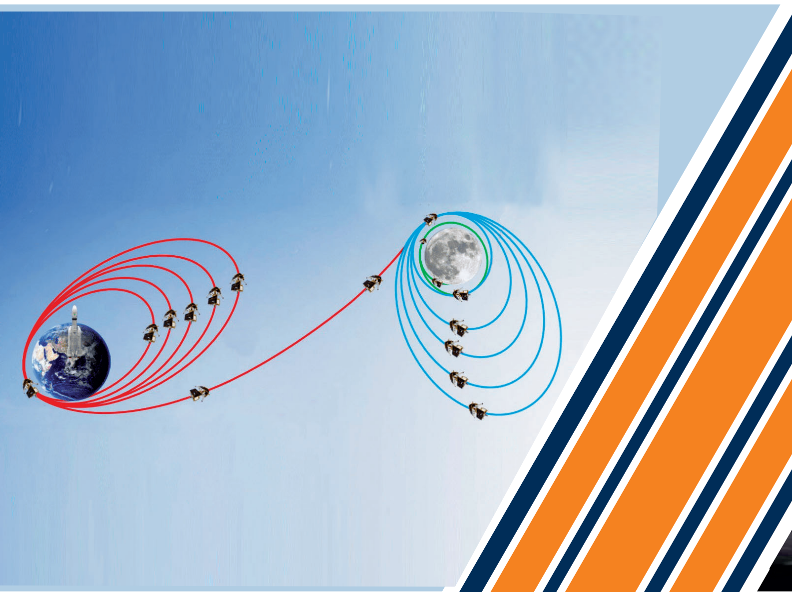

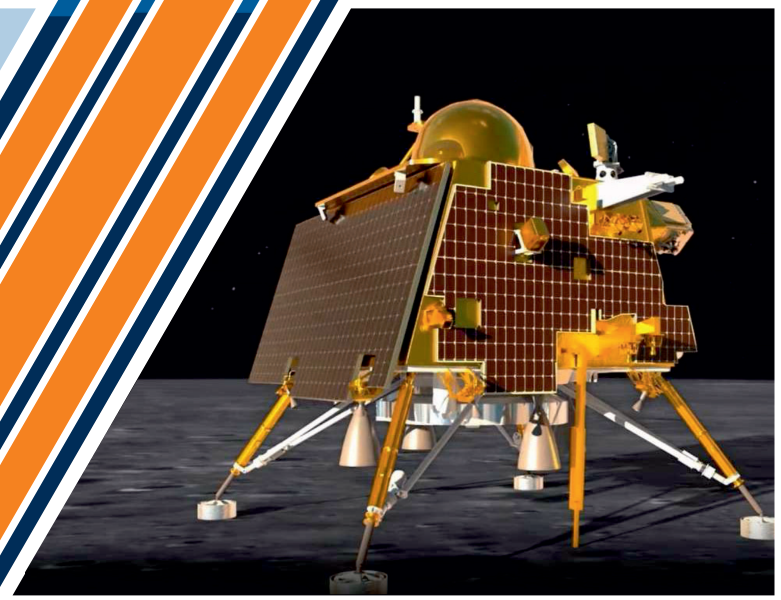

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312

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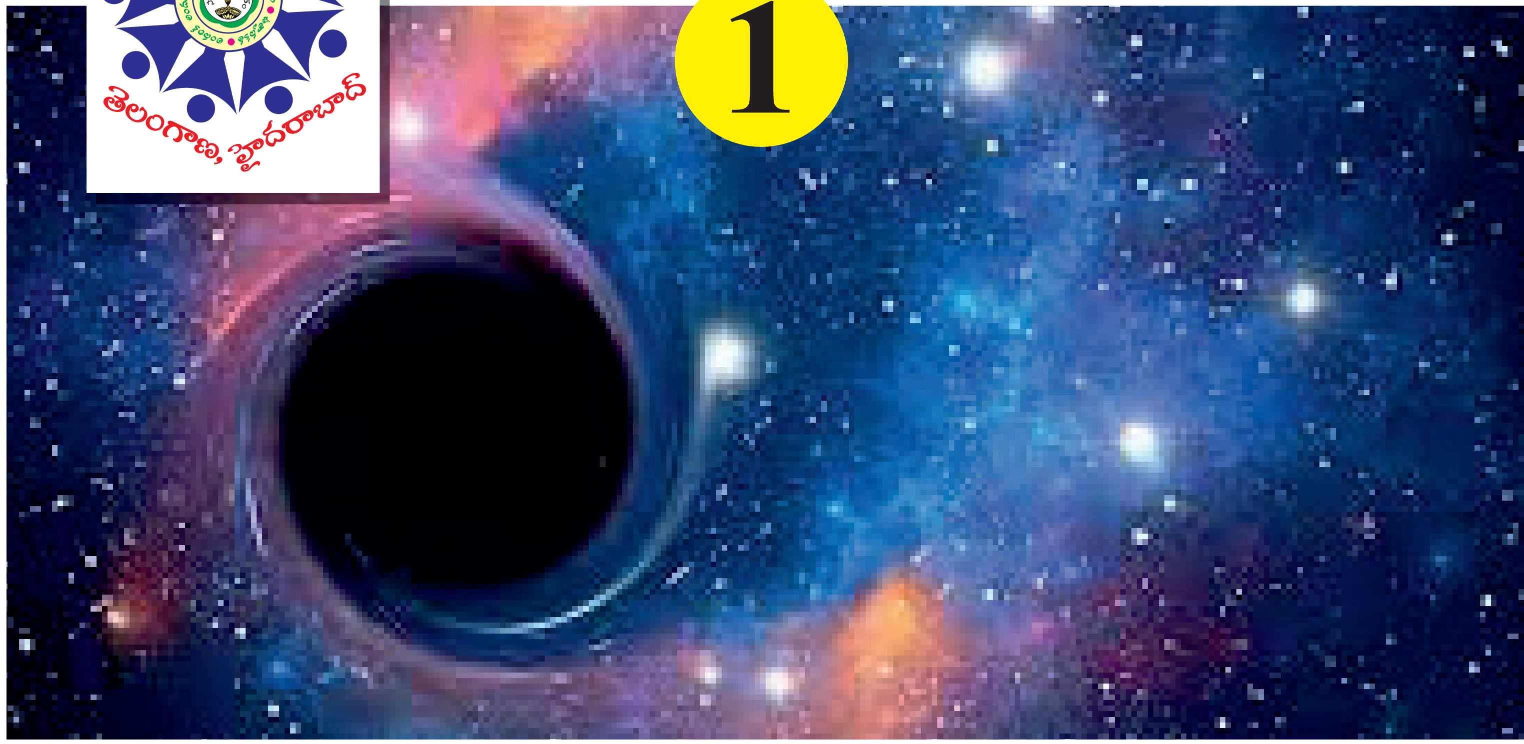

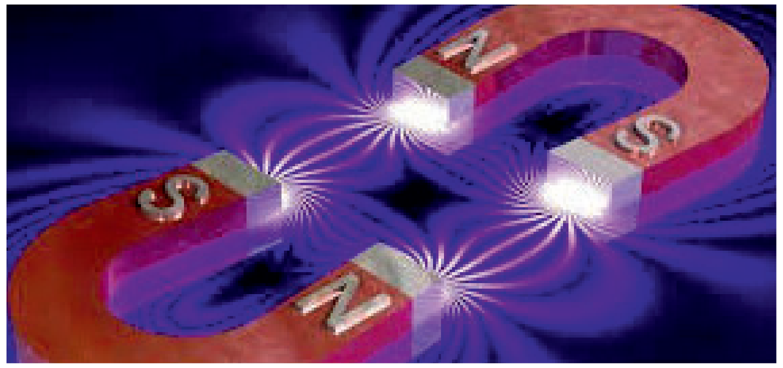

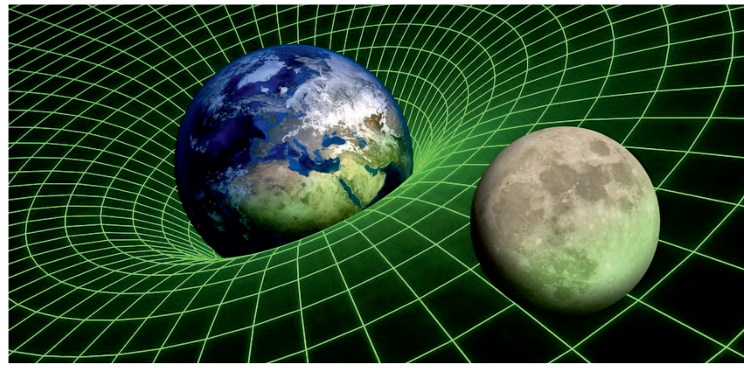

TELANGANA OPEN SCHOOL SOCIETY, HYDERABAD

312

PHYSICS

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FOREWORD

Providing education to children is a fundamental right and is essential for the overall development of society. The Government of Telangana plays a crucial role in ensuring that education is accessible to all, often establishing institutions like the Telangana Open School Society (TOSS) to serve children who may face challenges in accessing formal education due to various reasons.

In order to deliver quality education to students studying Intermediate Education in the Telangana Open School Society, starting from the 2023 academic year, we have undertaken the task of revising our textbooks to align them with the changing social landscape and to incorporate the fundamental principles of the National Education Policy 2020. The guidelines outlined in this policy aim to enhance the overall learning experience and cater to the diverse needs of our students. Unlike the earlier textbooks that primarily contained questions and answers, TOSS has taken a student-centric approach in designing these textbooks, taking into consideration the various learning styles and needs of our students. This approach encourages active engagement and participation in the learning process.

This Physics textbook is divided into two volumes, encompassing a total of 29 chapters. It has been thoughtfully crafted to foster the understanding and appreciation of this remarkable discipline. Each volume serves as a gateway to the intriguing and multifaceted realm of physics.

Volume-1 lays the essential foundation for students' journey into the world of physics. Mechanics, with its principles of motion, forces, and energy, provides the framework for comprehending how objects interact with one another. Heat and thermodynamics delve into the fascinating world of temperature, heat transfer, and the laws governing energy transformations.

Volume-2 will guide the learner through the enigmatic universe of light, exploring topics like reflection, refraction, and the formation of images, and delve deeper into the enigmas of the physical world. Electricity and magnetism will ignite the imagination of learners as they explore the principles of electric circuits, magnetic fields, and electromagnetic waves. Atoms and nuclei will unveil the intricacies of the atomic realm, from quantum mechanics to nuclear physics. Semiconductors and communication systems will introduce students to the technology that drives our modern world, from transistors to telecommunications.

We are indeed very grateful to the Government of Telangana and the Telangana State Board of Intermediate Education. Special thanks go to the editors, authors, co-coordinator, teachers, lecturers, and DTP operator who tirelessly contributed their services to create this textbook.

Date : 2023
Place : Hyderabad.

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A Word With You

Dear Learner

We are delighted to welcome you to the world of open and distance learning through TOSS. Your decision to embark on this educational journey as an Open and Distance learner is commendable, and we are thrilled that you have chosen physics as one of your subjects of study.

With great pleasure and enthusiasm, we present to you this comprehensive physics textbook, meticulously designed for senior secondary course in open and distance learning. Physics is a subject that unveils the mysteries of the universe, offering profound insights into the fundamental laws governing the cosmos. Our textbook, spanning two volumes and encompassing 29 chapters, has been thoughtfully crafted to foster your understanding and appreciation of this remarkable discipline.

As you set out on this educational voyage, you will explore the captivating domains of mechanics, heat and thermodynamics, optics, electricity and magnetism, atoms and nuclei, semiconductors, and communication systems. Each volume serves as a gateway to the intriguing and multifaceted realm of physics.

Volume-1, consisting of 14 chapters, lays the essential foundation for your journey into the world of physics. Mechanics, with its principles of motion, forces, and energy, provides the framework for comprehending how objects interact with one another. Heat and thermodynamics delve into the fascinating world of temperature, heat transfer, and the laws governing energy transformations.

Volume-2, with 15 chapters, will lead you through the enigmatic universe of light, exploring topics like reflection, refraction, and the formation of images, delves deeper into the enigmas of the physical world. Electricity and magnetism will electrify your imagination as you explore the principles of electric circuits, magnetic fields, and electromagnetic waves. Atoms and nuclei will unveil the intricacies of the atomic realm, from quantum mechanics to nuclear physics. Semiconductors and communication systems will introduce you to the technology that drives our modern world, from transistors to telecommunications.

Our textbook is tailored with your unique learning journey in mind, recognizing the distinctive challenges and opportunities that open and distance learning offers. Clear explanations, illustrative diagrams, and practical examples have been incorporated to make physics accessible and engaging, even when you are learning independently. Each chapter acts as a stepping stone, building upon previously introduced concepts, ensuring a coherent and structured learning experience.

Physics transcends being a mere subject; it is a path to discovery. It empowers you to explore the universe, from the vast expanses of space to the tiniest particles that constitute matter. Physics nurtures critical thinking, problem-solving abilities, and a profound appreciation for the natural world's beauty.

As you progress through these pages, remember that physics is not just a collection of equations and theories; it is a tool that equips you to understand and shape the world around you. Embrace the challenges, ask questions, and never cease to wonder. Your journey through these volumes will not only equip you with the knowledge and skills to excel academically but also inspire a lifelong passion for the marvels of physics.

We extend our heartfelt best wishes as you embark on this educational odyssey. May this textbook be your trusted companion and guide on your quest for knowledge. If you encounter any difficulties or have suggestions, please do not hesitate to reach out to us.

Sincerely,

The Curriculum Design and Course Development Team

(312)

PHYSICS

VOLUME - 1

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(312)

PHYSICS

VOLUME - 2

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CONTENTS

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1

UNITS AND MEASUREMENTS

INTRODUCTION

In science, particularly in physics, we try to make measurements as precisely as possible. Several times in the history of science, precise measurements have led to new discoveries or important developments. Obviously, every measurement must be expressed in some units. For example, if you measure the length of your room, it is expressed in suitable units. Similarly, if you measure the interval between two events, it is expressed in some other units. The unit of a physical quantity is derived, by expressing it in base units fixed by international agreement. The idea of base units leads us to the concept of **dimensions**, which as we shall see, has important applications in physics.

OBJECTIVES

After studying this lesson, you should be able to

- *distinguish between the fundamental and derived quantities;*
- *to study the international system of units;*
- *identify the number of significant figures in measurements and give their importance;*
- *write the dimensions of various physical quantities;*
- *apply dimensional analysis to check the correctness of an equation and determine the dimensional nature of 'unknown' quantities.*

1.1 NEED FOR MEASUREMENT

Physics is a branch of science which deals with nature and natural phenomena. For complete and proper study of any phenomenon, measurement of quantities involved is essential. For example, to study the motion of a particle, measurement of its displacement, velocity, and acceleration at any instant has to be made accurately. For this, measurement of time and distance has to be done. Similarly, measurement of volume, pressure and temperature is necessary to study the state of a gas fully. In case of liquids, mass, volume and temperature have to be measured to study the effect of heat. This explains the need for measurement.

1.2 UNIT OF MEASUREMENT

Any quantity that can be measured is called physical quantity. The laws of physics are expressed in terms of physical quantities such as distance, speed, time, force, volume, electric current and so on. A physical quantity which is independent of any other quantity is defined as a **fundamental** or **base quantity**. A physical quantity that can be derived from other physical quantities is called **derived quantity**. To measure any physical quantity, a standard of same quantity is essential and this is called a **unit.** For example, time could be measured in minutes, hours or days. But for the purpose of useful communication among different people, this unit must be compared with a standard unit acceptable to all. As another example, when we say that the distance between Mumbai and Kolkata is nearly 2000 kilometres, we have for comparison a basic unit in mind, called a kilometre. Some other units that you may be familiar with are a kilogram for mass, and a second, for time. It is essential that all agree on the standard units, so that when we say 100 kilometres, or 10 kilograms, or 10 hours, others not only understand but would have an intuitive feel of what it is meant. In science, international agreement on the basic units is absolutely essential. Otherwise scientists in one part of the world may experience difficulty in interpretation of the results of an investigation conducted in another part. Measurement of a physical quantity is expressed by a numerical followed by a unit. For example, length is 10 metres. 10 is numerical and metre is unit of length. A limited number of units are enough to express all physical quantities because, they are interrelated. The units for the fundamental or base quantities are called **fundamental** or **base units**. For example, length, mass and time are fundamental quantities and their respective units are metre, kilogram and second. The units of other physical quantities that can be expressed as a combination of base units are called **derived units**. For example, area, pressure and density are derived quantities and their respective units i.e., square metre, pascal, kgm^{-3} are derived from fundamental units.

1.3 SYSTEM OF UNITS

To measure the fundamental quantities viz., length, mass and time, there are three standardized system of units. They are: (1) FPS (British), (2) CGS, (3) MKS (Metric), as shown in Table 1.1.

Table - 1.1

		UNITS		
S.No.	**System**	**Length**	**Mass**	**Time**
1	FPS	Foot	Pound	Second
2	CGS	Centimeter	Gram	Second
3	MKS	Metre	Kilogram	Second

In due course of time, it was realized that only length, mass and time are not sufficient to derive all the physical quantities. Subsequently, four more physical quantities viz., thermodynamic temperature, illuminating power (Luminous intensity), strength of electric current and quantity of matter were added as fundamental quantities. Further, plane angle and solid angle were also included as supplementary quantities.

1.4 SI SYSTEM

A coherent, rational and comprehensive system of units was agreed upon at the 14^{th} General Conference on Weights and Measures held in 1971, which became known as SI which is the abbreviation for ***Système International d'Unités*** for the **International System of units**. The System adopted seven base or fundamental units. These units formed the SI system. SI system is also popularly known as the **metric** system. The SI units along with their symbols are given in Table 1.2.

Table - 1.2 : Base SI units

Fundamental quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Illuminous intensity	candela	cd
Quantity of matter	mole	mol
Plane angle	Radian	rad
Solid angle	steradian	sr

As may be noted, the SI system is a metric system. It is quite easy to handle because the smaller and larger units of the base units are always submultiples or multiples of ten. These multiples or submultiples are given special names. These are listed in Table 1.3.

Table - 1.3 : Prefixes for powers of 10

Power of ten	Prefix	Symbol	Example
10^{-18}	atto	a	Attometre (am)
10^{-15}	femto	f	Femtometre (fm)
10^{-12}	pico	p	Picofarad (pF)
10^{-9}	nano	n	Nanometer (nm)
10^{-6}	micro	m	Micron (mm)
10^{-3}	milli	m	Milligram (mg)
10^{-2}	centi	c	Centimeter (cm)
10^{-1}	deci	d	Decimeter (dm)
10^{1}	deca	Da	Decagram (Dag)
10^{2}	hecto	H	Hectometer (Hm)
10^{3}	kilo	K	Kilogram (Kg)
10^{6}	mega	M	Megawatt (MW)
10^{9}	giga	G	Gigahertz (GHz)
10^{12}	tera	T	Terahertz (THz)
10^{15}	peta	P	Petakilogram (Pkg)
10^{18}	exa	E	Exakilogram (Ekg)

Table - 1.4 : Order of magnitude of some masses

Object	Mass (Kg)
Electron	10^{-30}
Proton	10^{-27}
Amino acid	10^{-25}
Hemoglobin	10^{-22}
Flu virus	10^{-19}
Giant amoeba	10^{-8}
Raindrop	10^{-6}
Ant	10^{-2}
Human being	10^{2}
Saturn 5 rocket	10^{6}
Pyramid	10^{10}
Earth	10^{24}
Sun	10^{30}
Milky way galaxy	10^{41}
Universe	10^{55}

Table - 1.5 : Order of magnitude of some lengths

Length	Magnitude (m)
Radius of proton	10^{-15}
Radius of atom	10^{-10}
Radius of virus	10^{-7}
Radius of giant amoeba	10^{-4}
Radius of walnut	10^{-2}
Height of highest mountain	10^{4}
Radius of earth	10^{7}
Radius of sun	10^{9}
Earth-sun distance	10^{11}
Radius of solar system	10^{13}
Distance to nearest star	10^{16}
Radius of Milky Way Galaxy	10^{21}
Radius of visible universe	10^{26}

Table - 1.6 : Order of magnitude of a few time intervals

Interval	Seconds, s
Time for light to cross nucleus	10^{-23}
Period of visible light	10^{-15}
Period of microwaves	10^{-10}
Half-life of moon	10^{-6}
Period of highest audible sound	10^{-4}
Period of human heartbeat	10^{0}
Half-life of free neutron	10^{3}
Period of the Earth's rotation (day)	10^{5}
Period of revolution of the Earth (year)	10^{7}
Lifetime of human beings	10^{9}
Half-life of plutonium-239	10^{12}
Life-time of a mountain range	10^{15}
Age of Earth	10^{17}
Age of Universe	10^{18}

1.4.1. Measurements of Mass, Length and Time

Once we have chosen to use the SI system of units, we must decide on the set of standards against which these units will be measured. We define here standards of mass, length and time.

Mass : The SI unit of mass is kilogram. The standard kilogram was established in 1887. It is the mass of a particular cylinder made of platinum-iridium alloy, which is an unusually stable alloy. The standard is kept in the International Bureau of Weights and Measures in Paris, France. The prototype kilograms made of the same alloy have been distributed to all countries the world over. For India, the national prototype is the kilogram no. 57. This is maintained by the National Physical Laboratory, New Delhi (Fig. 1.1).

Fig. 1.1 : Prototype of Kilogram

Length : The SI unit of length is metre. It is defined in terms of a natural phenomenon: One metre is defined as the distance travelled by light in vacuum in a time interval of 1/299792458 second.

This definition of metre is based on the adoption of the speed of light in vacuum as 299792458 ms^{-1}

Time : One second is defined as the time required for a Cesium (Caesium-IUPAC) - 133 (133Cs) atom to undergo 9192631770 vibrations between two hyperfine levels of its ground state. This definition of a second has helped in the development of a device called atomic clock. The cesium clock maintained by the National Physical Laboratory (NPL) in India has an uncertainty of $\pm 1 \times 10^{-12}$ s, which corresponds to an accuracy of one picosecond in a time interval of one second.

As of now, clock with an uncertainty of 5 parts in 10^{15} have been developed. This means that if this clock runs for 10^{15} seconds, it will gain or lose less than 5 seconds. You can convert 10^{15} s to years and get the astonishing result that this clock could run for 6 million years and lose or gain less than a second. This is not all. Researches are being conducted today to improve upon this accuracy constantly. Ultimately, we expect to have a clock which would run for 10^{18} second before it could gain or lose a second. To give you an idea of this technological achievement, if this clock were started at the time of the birth of the universe, an event called the Big Bang, it would have lost or gained only two seconds till now.

1.5 ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENTS

Measurements are the numerical values of a physical quantity. Several instruments are used to measure the physical quantities. The measurement depends on the instruments, procedure, environment and the skill of the person using the instrument. Every measurement by any instrument contains some uncertainty which is called **error**. The reliability of measurements depends on the accuracy of the result. When the measured value is very close to the true value, then, it is called **accurate value**. When a physical quantity is measured repeatedly, the concept of closeness of measured values is called **precision**.

***Accuracy* :** Accuracy of a measurement is defined as the closeness of the measured value to the true value.

***Precision* :** Closeness of the measured values of repeated measurements of a physical quantity by an instrument is called precision.

***Error* :** The inaccuracy and lack of precision in a measurement without mistakes is called error.

1.6 TYPES OF ERRORS

Errors in measurements are broadly classified as **Systematic errors** and **Random errors**

1.6.1 Systematic errors

The errors due to a definite cause and which follow a particular rule are called systematic errors. Examples include, instrumental errors, theoretical errors and personal or parallax error.

Instrumental error : The error due to improper designing and calibration of the instrument for measurement is called instrumental error. These errors can be reduced by using more accurate instruments and also by applying zero-correction when required. For example, let zero error in a screw gauge be +0.04 mm. The correction for the observation becomes –0.04 mm. The diameter of a wire is measured as 10.04 mm using this screw gauge. The value after correction, becomes 10.04 – 0.04 = 10.00 mm. In every measurement with this instrument, this error +0.04 mm creeps in.

Theoretical error : The error due to approximations in deriving the formula is called theoretical error. For example, in measuring the time period of a simple pendulum, it is approximated as sin $\theta \approx \theta$ for smaller values of θ. When amplitude is 3°, the error is about 0.02%. If the amplitude is more, the error is more. This error can be minimized by keeping amplitude small.

Personal error : The errors due to the limitations of human senses are called personal error. An observer may consistently read the observed values either high or low, by keeping his view inclined. The error introduced in such a condition is called parallax error.

1.6.2 Random Errors

The errors which are not systematic and caused by uncontrolled disturbances which influence the physical quantity and instrument are called random errors. The error in the reading of electrical instruments due to change in line voltage and backlash error in screw gauge, spherometer and travelling microscope are some examples of random errors. Random errors can be minimized by repeating the observation a large number of times and taking a mean of the observations.

1.7 ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

1.7.1 Absolute error

Suppose the values obtained in several measurements are a_1, a_2, a_3 ... a_n. The ***arithmetic mean*** a_m of these values is known as *true value*.

$$a_m = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

The magnitude of the difference between the true value and the measured value of a quantity is called absolute error of the measurement, which may be positive or negative, denoted by Δa i.e.,

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

$$\Delta a_3 = a_m - a_3$$

....

....

$$\Delta a_n = a_m - a_n$$

1.7.2. Mean absolute error

Arithmetic mean of the magnitude of absolute errors in all the measurements is called mean absolute error denoted by Δa_{mean}.

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ...+ |\Delta a_n|}{n}$$

The final result of measurement can be written as $a = a_m \pm \Delta a_{mean}$

Relative error: The ratio of Mean absolute error Δa_{mean} to the mean value a_m of the quantity measured is called Relative Error or Fractional Error denoted by δa.

$$\delta a = \frac{\Delta a_{mean}}{a_m}$$

1.7.3 Percentage error

When the relative error is expressed in percent (%), it is called percentage error.

$$\text{Percentage error} = \frac{\Delta a_{mean}}{a_m} \times 100$$

Example 1.1

In successive measurements, the readings of the period of oscillation of a simple pendulum were found to be **2.63, 2.56, 2.42, 2.71 and 2.80** seconds in an experiment. Calculate absolute error, mean absolute error, relative error and percentage error of the period of oscillation.

Solution :

$$a_m = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$a_m = \frac{13.12}{5} = 2.624\,\text{s}$$

Absolute error :

$$\Delta a_1 = 2.62 - 2.63 = -0.01 \text{ s}$$
$$\Delta a_2 = 2.62 - 2.56 = 0.06 \text{ s}$$
$$\Delta a_3 = 2.62 - 2.42 = 0.20 \text{ s}$$
$$\Delta a_4 = 2.62 - 2.71 = -0.09 \text{ s}$$
$$\Delta a_5 = 2.62 - 2.80 = 0.18 \text{ s}$$

Mean Absolute error : $\Delta a_{mean} = \dfrac{|-0.01| + |0.06| + |0.20| + |-0.09| + |0.18|}{5}$

$$\Delta a_{mean} = \frac{0.54}{5} = 0.108\,\text{s} = 0.11 \text{ (rounded off)}$$

Relative error : $\delta a = \dfrac{\Delta a_{mean}}{a_m} = \dfrac{0.11}{2.62} = 0.04198 = 0.04$

Percentage error : $= \dfrac{\Delta a_{mean}}{a_m} \times 100 = 0.04 \times 100 = 4\%$

1.8 COMBINATION OF ERRORS

If we do an experiment involving several measurements, we must know how to combine the errors in all measurements. For example, density is obtained by dividing mass by the volume of the substance. If we have errors in the measurement of mass and volume, then we must know what the error will be in the density of the substance. To make such estimates, we should learn how errors combine in various mathematical operations.

Error in sum

Suppose two physical quantities have measured values a and b. Their sum x is given by x = a + b.

Further, let

Δa be absolute error in the measurement of a

Δb be absolute error in the measurement of b

Δx be absolute error in the measurement of x

Then,

$$x + \Delta x = (a \pm \Delta a) + (b \pm \Delta b)$$

$$x + \Delta x = a + b \pm \Delta a \pm \Delta b$$

$$\Delta x = \pm (\Delta a + \Delta b)$$

The maximum possible error in x is $\Delta x = \Delta a + \Delta b$

Error in Difference

Let $x = a - b$

$$x \pm \Delta x = (a \pm \Delta a) - (b \pm \Delta b)$$

$$x \pm \Delta x = a \pm \Delta a - b \pm \Delta b$$

$$x \pm \Delta x = (a - b) \pm (\Delta a \pm \Delta b)$$

The maximum possible error in x is $\Delta x = \Delta a + \Delta b$

Hence the rule is, when two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Error in product

Let $x = ab$ then,

$$x \pm \Delta x = (a \pm \Delta a)(b \pm \Delta b)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

$$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} + \frac{\Delta a}{a}\frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a}\frac{\Delta b}{b}$ is very small quantity. So, it can be neglected.

$$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Hence, the maximum relative error $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

Similarly, we can easily verify that this is true for division also.

Hence the rule is, when two quantities are multiplied or divided, the relative error in the result is the sum of relative errors in the measured quantities.

1.9 SIGNIFICANT FIGURES

As mentioned previously, errors are inherent in every measurement. Consequently, it is crucial to report measurement results in a manner that reflects their precision. Typically, the reported value includes all digits that are reliably known, as well as the first digit that introduces uncertainty. These reliable digits, along with the first uncertain digit, are referred to as **significant digits** or **significant figures**. For instance, if we state that the length of a line is 6.82 cm, the digits 6 and 8 are reliable and certain, whereas the digit 2 is uncertain. Thus, the measured value has three significant figures.

Similarly, the length of an object reported after measurement to be 647.5 cm has four significant figures, the digits 6, 4, 7 are certain while the digit 5 is uncertain.

1.9.1 Rules for counting significant figures

i. All non-zero digits are significant. For example, 315.58 has five significant figures.

ii. All zeros between two non-zero digits are significant. For example, 5300405.003 has ten significant figures.

iii. Trailing zero (s) after decimal places are significant. For example, 50.00 has four significant figures. 7.40 has three significant figures.

iv. The terminal or trailing zero (s) in a number without a decimal point are not significant. For example, 5000 has only one significant figure. The trailing zero (s) in a number with a decimal point are significant. For example, 3.500 has four significant figures.

v. If a measurement is less than one, then all zero (s) occurring to the left of last non-zero digit are not significant. Example, 0.0072 has two significant figures. 0.000072 also has two significant figures.

vi. The power of ten are not counted as significant figures. Example, 1.4×10^7 has two significant figures.

vii. The number of significant figures does not vary with the change in unit. For example, if the length of an object is 348.6 cm, it has 4 significant figures. If the length is expressed in metre, then it is equal to 3.486 m. It still has 4 significant figures.

1.9.2 Importance of significant figures in measurement

As stated earlier, the accuracy of the measurement determines the number of significant figures in the quantity. Suppose the diameter of a coin is 2 cm. If a student measures the diameter with a metre scale which can read up to .1 cm only (i.e. cannot read less than 0.1 cm) the student will report the diameter to be 2.0 cm i.e. up to 2 significant figures only. If the diameter is measured by an instrument which can read up to .01 cm only (or which cannot measure less than .01cm), viz., a Vernier Callipers, he will report the diameter as 2.00 cm i.e. up to 3 significant figure. Similarly, if the measurement is made by an instrument like a screw gauge which can measure up to .001 cm only (i.e. cannot measure less than .001 cm), the diameter will be recorded as 2.000 cm i.e. up to 4 significant figures.

1.9.3 Rules for rounding off the uncertain digits

i. If a digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, x = 7.82 is rounded off to 7.8

ii. If the digit to be dropped is more than 5, the preceding digit is raised by one. For example, x = 6.87 is rounded off to 6.9

iii. If the digit to be dropped is 5, followed by digits other than zero, then the preceding digit is raised by one. For example, x = 16.351 is rounded off to 16.4

iv. If the digit to be dropped is 5, or 5 followed by zero, then then the preceding digit is left unchanged if it is even. For example, x = 3.250 is rounded off to 3.2

v. If the digit to be dropped is 5, or 5 followed by zero, then then the preceding digit is raised by one if it is odd. For example, x = 3.750 is rounded off to 3.8

1.9.4 Rules for arithmetic operations with significant figures

Addition and subtraction – Suppose we have to add three quantities, 2.7 m, 3.68 m and 0.486 m. In these quantities, the first measurement is known up to one decimal place only, hence the sum of these numbers will be definite up to one decimal place only. Therefore, the correct sum of these numbers should not be written as 6.848 m but 6.8 m.

Similarly, to find the sum of quantities like 2.65×10^3 cm and 2.63×10^2 cm, all quantities should be converted to the same power of 10. These quantities will then be, 2.65×10^3 cm and $.263 \times 10^3$ cm. Since, the first number is known up to 2 decimal places, their sum will also be up to 2 decimal places. Hence 2.65×10^3cm + $.263 \times 10^3$ cm = 2.91×10^3 cm.

The same is done with subtraction. For example, the result of subtracting 2.38 cm from 4.6 cm will be 2.2 cm, not 2.22 cm.

Multiplication and division – Suppose the length of a plate is measured as 3.003 m and its width as 2.26 m. According to mathematical **c**alculation, the area of the plate will be 6.78678 m^2. But, it is not correct in scientific measurement. There are six significant figures in this result. But, the least number of significant figures (in the width) are only 3. Hence, the multiplication should also be written up to 3 significant figures. Therefore, the correct area would be 6.79 m^2.

The same method is applied for division also. For example, dividing 248.57 by 56.9 gives 4.3685413. But, the result should be recorded up to 3 significant figures only as the least number of significant figures in the given numbers is only 3. Hence, the result will be 4.37.

Intext Questions 1.1

1. Find the number of significant figures in the following quantity, quoting the relevant laws:

   (i) 426.69 (ii) 4200304.002 (iii) 0.3040 (iv) 4050 m (v) 5000

2. The length of an object is 3.486 m, if it is expressed in centimetre (i.e. 348.6 cm) will there be any change in number of significant figures in the two cases.
3. The mass of the sun is 2×10^{30} kg. The mass of a proton is 2×10^{-27} kg. If the sun was made only of protons, calculate the number of protons in the sun?
4. Earlier the wavelength of light was expressed in angstroms. One angstrom equals 10^{-8} cm. Now the wavelength is expressed in nanometers. How many angstroms make one nanometre?
5. A radio station operates at a frequency of 1370 kHz. Express this frequency in GHz.
6. How many decimetres are there in a decametre? How many MW are there in one GW?

Albert Abraham Michelson (1852-1931)

German-American Physicst, inventor and experimenter devised Michelson's interferometer with the help of which, in association with Morley, he tried to detect the motion of earth with respect to ether but failed. However, the failed experiment stirred the scientific world to reconsider all old theories and led to a new world of physics.

He developed a technique for increasing the resolving power of telescopes by adding external mirrors. Through his stellar interferometer along with 100" Hookes telescope, he made some precise measurements about stars.

1.10 DIMENSIONS OF PHYSICAL QUANTITIES

Most physical quantities you would come across in this course can be expressed in terms of five basic dimensions: mass (M), length (L), time (T), electrical current (I) and temperature (θ). Since all quantities in mechanics can be expressed in terms of mass, length and time, it is sufficient for our present purpose to deal with only these three dimensions. Following examples show how dimensions of the physical quantities are combinations of the powers of M, L and T :

i. Volume requires 3 measurements in length.

ii. Density is mass divided by volume. So it has 3 dimensions in length (L^3). Its dimensional formula is ML^{-3}.

iii. Speed is distance travelled in unit time or length divided by time. Its dimensional formula is LT^{-1}.

iv. Acceleration is change in velocity per unit time, i.e., length per unit time per unit time. Its dimensional formula is LT^{-2}.

v. Force is mass multiplied by acceleration. Its dimensions are given by the formula MLT^{-2}.

Similar considerations enable us to write dimensions of other physical quantities.

Note that numbers associated with physical quantities have no significance in dimensional considerations. Thus, if dimension of x is L, then dimension of 3x will also be L.

Write down the dimensions of momentum, which is product of mass and velocity and work which is product of force and displacement. Remember that dimensions are not the same as the units. For example, speed can be measured in m s^{-1} or kilometre per hour, but its dimensions are always given by length divided by time, or simply LT^{-1}.

Dimensional analysis is the process of checking the dimensions of a quantity, or a combination of quantities. One of the important principles of dimensional analysis is that **each physical quantity on the two sides of an equation must have the same dimensions**. Thus if x = p + q, then p and q will have the same dimensions as x. This helps us in checking the accuracy of equations, or getting the dimensions of a quantity using an equation. The following examples illustrate the use of dimensional analysis.

Example 1.2

You know that the kinetic energy of a particle of mass m is $\frac{1}{2}mv^2$ while its potential energy is mgh, where v is the velocity of the particle, h is its height from the ground and g is the acceleration due to gravity. Since the two expressions represent the same physical quantity i.e, energy, their dimensions must be the same. Let us prove this by actually writing the dimensions of the two expressions.

Solution :

The dimensions of $\frac{1}{2}mv^2$ are $M (LT^{-1})^2$ or ML^2T^{-2} (Remember that the numerical factors have no dimensions.) The dimensions of mgh are MLT^{-2}. L or ML^2T^{-2}. Clearly, the two expressions are the same and hence represent the same physical quantity.

Let us take another example to find an expression for a physical quantity in terms of other quantities.

Example 1.3

Experience tells us that the distance covered by a car, say x, starting from rest and having uniform acceleration depends on time t and acceleration a. Let us use dimensional analysis to find expression for the distance covered.

Solution :

Suppose x depends on the mth power of t and nth power of a. Then we may write $x \propto t^m a^n$

Expressing the two sides now in terms of dimensions, we get

$$L^1 \propto T^m \left(LT^{-2}\right)^n$$

$$L^1 \propto T^{m-2n} L^n$$

Comparing the powers of L and T on both sides, you will easily get n = 1, and m = 2. Hence, we have

$$x \propto t^2 a^1, \text{ or } x \propto at^2$$

This is as far as we can go with dimensional analysis. It does not help us in getting the numerical factors, since they have no dimensions. To get the numerical factors, we have to get input from experiment or theory. In this particular case, of course, we know that the complete relation is

$$x = \frac{1}{2}at^2$$

Intext Questions 1.2

1. Experiments with a simple pendulum show that its time period depends on its length (l) and the acceleration due to gravity (g). Use dimensional analysis to obtain the dependence of the time period on l and g.
2. Consider a particle moving in a circular orbit of radius r with velocity v and acceleration a towards the center of the orbit. Using dimensional analysis, show that $a \propto v^2/r$.
3. You are given an equation: mv = Ft, where m is mass, v is speed, F is force and t is time. Check the equation for dimensional correctness.

WHAT YOU HAVE LEARNT

- The number of significant figures determines the accuracy of a measurement.
- Every physical quantity must be measured in some unit and also expressed in this unit. The SI system has been accepted and followed universally for scientific reporting.
- Base SI units for mass, length and time are respectively kg, m and s. In addition to base units, there are derived units.

- Every physical quantity has dimensions. Dimensional analysis is a useful tool for checking correctness of equations.

TERMINAL EXERCISE

1. A unit used for measuring very large distances is called a light year. It is the distance covered by light in one year. Express light year in metres. Take speed of light as 3×10^8 m s^{-1}.
2. State the number of significant figures in the following.
   (a) 5769
   (b) 0.042
   (c) 0.08420
   (d) 6.033
   (e) 3.56×10^8
3. A stick has a length of 12.132 cm and another has a length of 12.4 cm. (a) If the two sticks are placed end-to-end what is the total length? (b) If the two sticks are placed side-by-side, what is the difference in their lengths?
4. The distance covered by a particle in time t while starting with the initial velocity u and moving with a uniform acceleration a is given by $s = ut + \frac{1}{2}at^2$. Check the correctness of the expression using dimensional analysis.
5. Newton's law of gravitation states that the magnitude of force between two particles of mass m_1 and m_2 separated by a distance r is given by $F = G\frac{m_1 m_2}{r^2}$ where G is the universal constant of gravitation. Find the dimensions of G.
6. Explain the different types of errors that can occur in measurements.
7. Distinguish between fundamental and derived units.
8. What is meant by significant figures in measurement?

ANSWERS TO INTEXT QUESTIONS

1.1

1. (i) 5 (ii) 10 (iii) 4 (iv) 4 (v) 1
2. No, in both cases, the number of significant figures will be 4.
3. Mass of the sun = 2×10^{30} kg; Mass of the proton: 2×10^{-27} kg;

   No. of protons in the sun : $\frac{2 \times 10^{30}}{2 \times 10^{-27}} = 10^{57}$

4. 1 angstrom = 10^{-8} cm = 10^{-10} m

   1 nanometer (nm) = 10^{-9} m

   $\therefore$ 1 nm/1 angstrom = 10^{-9} m/10^{-10}; m = 10. So, 1 nm = 10 Å

5. 1370 kHz = 1370 × 10^3 Hz = (1370 × 10^3) /10^9 GHz = 1.370 × 10^{-3} GHz

6. 1 decametre (dam) = 10 m, 1 decimetre (dm) = 10^{-1} m;

   $\therefore$ 1 dam =100 dm

   1 MW= 10^6 W

   1 GW = 10^9 W

   1 GW = 10^3 MW

1.2

1. Dimension of length = L; Dimension of time = T; Dimensions of g = LT^{-2}

   Let time period t be proportional to l^α and g^β,

   Then, writing dimensions on both sides $T = L^a (LT^{-2})^\beta = L^{\alpha+\beta} T^{-2\beta}$

   Equating powers of L and T,

   $\alpha + \beta = 0,\ 2\beta = -1 \Rightarrow \beta = -1/2$ and $\alpha = ½$

   So, $t \propto \sqrt{\frac{l}{g}}$

2. Dimension of a = LT^{-2}; Dimension of v = LT^{-1}; Dimension of r = L

   Let a be proportional to v^α and r^β

   Then dimensionally,

   $LT^{-2} = (LT^{-1})^\alpha L^\beta = L^{\alpha+\beta} T^{-\alpha}$

   Equating powers of L and T,

   $\alpha + \beta = 1,\ \alpha = 2,\ \Rightarrow \beta = -1$

   So, $a \propto v^2/r$

3. Dimensions of mv = MLT^{-1}; Dimensions of Ft = $MLT^{-2}\,T^1 = MLT^{-1}$;

   Dimensions of both the sides are the same, therefore, the equation is dimensionally correct.

ANSWERS TO TERMINAL EXERCISE

1. 1 ly = 9.4673 × 10^{15} m
2. (a) 4 (b) 2 (c) 4 (d) 4 (e) 3
3. (a) 24.5 cm (b) 0.3 cm

2

MOTION IN A STRAIGHT LINE

INTRODUCTION

In the universe, we observe number of objects moving though some objects appear to be stationary. For example humans, animals, vehicles are seen moving on land. Fish, frogs and other aquatic animals move in water. Birds and aeroplanes move in air. The earth on which we live also revolves around the sun once in a year and rotates about its own axis. It is quite apparent that we live in a world that is very much in constant motion. Therefore, the study of motion is essential. Motion is change in position of an object with time. Motion can be in a straight line (1D), in a plane (2D) or in space (3D). If the motion of the object is only in one direction, it is said to be the motion in a straight line. For example, motion of a bus on a straight road, motion of a train on straight rails, motion of a freely falling body, motion of a lift etc.

In this lesson, you will learn about motion in a straight line. Next you will study the motion in plane, laws of motion and other types of motion.

OBJECTIVES

After studying this lesson, you should be able to

- *distinguish between distance and displacement and speed and velocity;*
- *explain the terms instantaneous velocity, relative velocity and average velocity;*
- *define acceleration and instantaneous acceleration;*
- *interpret position - time and velocity - time graphs for uniform as well as non-uniform motion;*
- *derive equations of motion with constant acceleration;*
- *describe motion under gravity;*
- *solve numericals based on equations of motion.*

2.1 DISTANCE AND DISPLACEMENT

When an object changes its position with time, the object is said to be in motion. We locate an object by its position with respect to some reference point on a line or an axis. The reference point is called origin. Positions to the right of origin are taken as positive and to the left of origin as negative.

Distance

Consider the motion of object along a straight line. Now choose the x-axis such that it coincides with the path of the object's motion and origin of the axis as the point from where the object started moving i.e; the object was at x = 0 at t = 0. (Fig. 2.1)

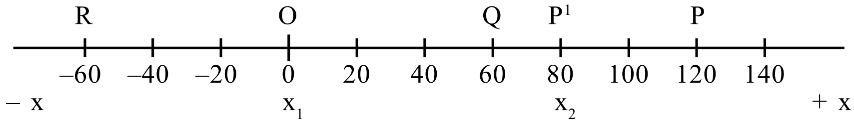


Fig. 2.1 : Motion along a straight line

Let P, Q and R represent the positions of the object at different instants of time. First consider that the object moves from O to P. Then the path covered by the object is OP = +120 m. Now consider that the object moves from O to P and then moves back from P to Q. During this course of motion, the path traversed by the object is OP + PQ = (+120 +60) m = +180 m.

Hence the total length of the path covered by the object is the distance travelled by it. Distance is a scalar - which has only magnitude but no direction.

Displacement

Let the positions of an object at time t_1 be x_1 and at time t_2 be x_2 (Fig. 2.1). Then the object is said to be displaced and the displacement is given by the difference between the final and initial positions of the object. Displacement is denoted by Δx.

$\Delta x = x_2 - x_1$

If $x_2 > x_1$, Δx is positive and if $x_2 < x_1$, Δx is negative.

Basically, displacement is the shortest distance between the two positions and has a certain direction. Thus displacement is a vector - which has both magnitude and direction. For example, displacement of the object in moving from O to P' is

$\Delta x = x_2 - x_1 = (+80 \text{ m}) - 0 \text{ m} = +80 \text{ m}$

The displacement has a magnitude of 80 m and is directed in the positive x-direction as shown by the +sign. Similarly, the displacement of the object from P to Q is 60 m – 120 m = –60 m. The negative sign indicates the direction of displacement.

We have seen that the distance travelled by the object when it moves from O to P and back P to Q is +180 m, whereas, the displacement in this case is +60 m only. This is illustrated in below example 2.1.

2.1.1 Speed and Velocity

The rate of change of distance with time is called speed. It is a scalar.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

The rate of change of displacement is known as velocity. Velocity is a vector. The SI unit for velocity is ms^{-1}.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

For 1-D motion, the directional aspect of the vector is taken care of by putting + and – signs and we do not have to use vector notation for displacement, velocity and acceleration for motion in one dimension.

Average Velocity and Average Speed

When an object is in motion and travels certain distance with different velocities, its motion is specified by its average velocity. Average velocity of an object is defined as the displacement per unit time. Let x_1 and x_2 be the positions of object at time t_1 and t_2 respectively. Then average velocity can be expressed as

$$\bar{v} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad (2.1)$$

where $x_2 - x_1$ denotes change in position (Δx) and $t_2 - t_1$ is the corresponding change in time (Δt). Here the bar over the symbol for velocity (v) is standard notation used to indicate an average quantity. Average velocity can be represented as v_{av} also. Like displacement, average velocity is also a vector.

The average speed of an object is obtained by dividing the total distance travelled by the total time taken.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} \qquad (2.2)$$

If the motion of an object is in the same direction along a straight line, the average speed is the same as the magnitude of the average velocity. However, this is always not the case.

To understand the difference between average speed and average velocity, consider the examples given below.

Example 2.1

A car is moving along a straight line from point O to P and covers a distance of 120 m in 3 seconds Fig (2.1). It returns back to point Q after travelling 60 m in 2 seconds. What are the average velocity and average speed of the car (a) when it moves from O to P (b) when moves from O to P and back to Q.

Solution :

(a) When car moves from O to P,

Displacement of the car = +120 m

Distance or path length travelled by car = 120 m

Time taken = 3 s

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{+120}{3} = +40\,ms^{-1}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{120}{3} = 40\,ms^{-1}$$

Thus in this case the magnitude of average velocity is equal to the average speed.

(b) In second case, when car moves from O to P and back to point Q,

Displacement of the car = +120 m – 60 m = +60 m

Distance or path length of car = 120 m + 60 m = 180 m

Time taken = 3 s + 2 s = 5 s

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}} = \frac{+60}{5} = +12\,ms^{-1}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time taken}} = \frac{180}{5} = 36\,ms^{-1}$$

Thus in this case the average speed is not equal to the magnitude of average velocity. This is because the motion involves change in direction so that the distance travelled is greater than the displacement.

Example 2.2

A person runs on a 300 m circular track and comes back to the starting point in 200 seconds. Calculate the average speed and average velocity.

Solution :

Total length of the track = 300 m

Time taken to cover the track = 200 s

$$\text{Average speed} = \frac{\text{distance}}{\text{time taken}} = \frac{300}{200} = 1.5\,ms^{-1}$$

As the person comes back to the same point, the displacement is zero. Therefore, the average velocity is also zero.

2.1.2 Relative Velocity

Consider a car moving at 20 kmh^{-1} due north. It means that the car travels a distance of 20 km in 1 h in northward direction from its starting position. Thus it is implied that the referred velocity is with respect to some reference point. In fact, the velocity of a body is always specified with respect to some other body. Since all bodies are in motion, we can say that every velocity is relative in nature.

The relative velocity of an object with respect to another object is the rate at which it changes its position relative to the object / point taken as reference. For example, if v_A and v_B are the velocities of the two objects along a straight line, the relative velocity of B with respect to A will be $v_B - v_A$.

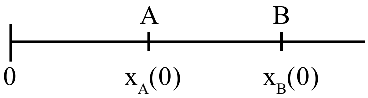
The rate of change of the relative position of an object with respect to the other object is known as the ***relative velocity*** *of that object with respect to the other.*

If the reference body is at rest, the motion of the body can be described easily. But, if the reference body is also moving, then the motion is seen to be of the two body system by a stationary observer. However, it can be simplified by invoking the concept of relative motion.

Consider two bodies A and B moving along positive x-direction with velocities v_A and v_B respectively.

If $x_A(0)$ and $x_B(0)$ are the initial positions of two bodies A and B at time t = 0, then the positions of bodies A and B after 't' seconds will be given by

$$x_A(t) = x_A(0) + v_A t$$
$$x_B(t) = x_B(0) + v_B t$$



Therefore, the relative separation of B from A will be

$$\begin{aligned} x_{BA}(t) &= x_B(t) - x_A(t) = x_B(0) - x_A(0) + (v_B - v_A)\, t \\ &= x_{BA}(0) + v_{BA} t \end{aligned}$$

where $v_{BA} = (v_B - v_A)$ is called the relative velocity of B with respect to A. Thus by applying the concept of relative velocity, a two body problem can be reduced to a single body problem.

Example 2.3

A train A is moving on a straight track (or railway line) from North to South with a speed of 60 km h^{-1}. Another train B is moving from South to North with a speed of 70 km h^{-1}. What is the (a) velocity of B relative to the train A

(b) velocity of ground with respect to B.

Solution :

Considering the direction from South to North as positive, we have

(a) velocity (v_B) of train B $= + 70$ km h^{-1}

and, velocity (v_A) of train A $= - 60$ km h^{-1}

Hence, the velocity of train B relative to train A

$$= v_B - v_A$$
$$= 70 - (-60) = 130 \text{ km h}^{-1}.$$

It is seen that the relative velocity of one train with respect to the other is equal to the sum of their respective velocities. This is why a train moving in a direction opposite to that of the train in which you are travelling appears to be travelling very fast. But, if the other train were moving in the same direction as your train, it would appear to be very slow.

(b) Relative velocity of ground with respect to B $= 0 - v_B = -70$ km h^{-1}

2.1.3 Acceleration

When you are travelling in a bus or a car, you might have noticed that sometimes it moves fast and sometimes it slows down. That is, its velocity changes with time and the bus or car is said to be accelerated. ***The acceleration is defined as time rate of change of velocity.*** Acceleration is a vector quantity and its SI unit is ms^{-2}. In one dimension, like velocity, there is no need to use vector notation for acceleration. The average acceleration of an object is given by,

$$\text{Average acceleration } (\overline{a}) = \frac{\text{Final velocity - Initial velocity}}{\text{Time taken for change in velocity}}$$

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad (2.3)$$

where v_1 and v_2 are the velocities of object at time t_1 and t_2.

The acceleration of a moving object may be positive or negative. If the acceleration is in the same direction as the motion or velocity, the acceleration is positive and if acceleration is in the opposite direction of motion, then the acceleration is taken as negative and is called as deceleration or retardation. So we can say that positive acceleration means the velocity is increasing with time. Similarly negative acceleration means the velocity is decreasing with time.

Example 2.4

The velocity of a car moving towards the East increases from 0 to 12 ms^{-1} in 3.0 s. Calculate its average acceleration.

Solution :

Initial velocity, v_1 $= 0 \ ms^{-1}$

Final velocity, v_2 $= 12 \ ms^{-1}$

Time, t $= 3$ s

Acceleration , a $= \frac{12-0}{3} = 4 \ ms^{-2}$

Intext Questions 2.1

1. Is it possible for a moving body to have non-zero average speed but zero average velocity during any given interval of time? If so, explain.
2. Can a moving body have zero relative velocity with respect to another body? Give an example.
3. A person strolls inside a train with a velocity of 1.0 m s^{-1} in the direction of motion of the train. If the train is moving with a velocity of 3.0 ms^{-1}, calculate his (a) velocity as seen by passengers in the compartment, and (b) velocity with respect to a person sitting on the platform.

2.2 POSITION – TIME GRAPH

Motion of an object can be represented by position - time graph. A moving body is found to be at different positions at different times. The different positions and corresponding times can be plotted on a graph taking time along x-axis and position of the body along y-axis which gives us a curve. Such a curve is known as position-time curve.

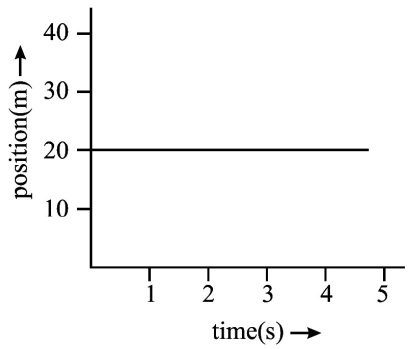


Fig. 2.2 : Position-time graph for a body at rest

If we plot the position - time graph for a body at rest, we get a straight line parallel to the time axis. For example consider a body at rest and at a distance of 20 m from the origin. The position-time graph is a straight line parallel to the time axis as shown in Fig 2.2.

2.2.1 Position-time graph for uniform motion

Consider an object moving along the straight line and which covers equal distances in equal intervals of time. For example, suppose that the object covers a distance of 10 m in each second for 5 seconds. The positions of the object at different times will be as shown below.

Time (t) in s	1	2	3	4	5
Position (x) in m	10	20	30	40	50

The position-time graph for the above moving object will be as shown in Fig. 2.3. The graph is a straight line inclined with the x-axis.

A motion in which the velocity of the moving object is constant is known as uniform motion. Its position-time graph is a straight line inclined to the time axis. In other words, we can say that if a moving object covers equal distances in equal intervals of time, then it is said to be in uniform motion.

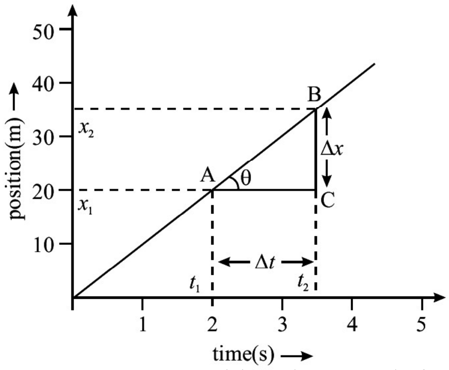


Fig. 2.3 : Position-time graph for uniform motion

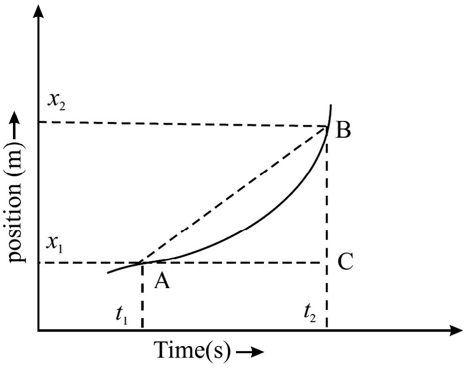


Fig. 2.4 : Position-time graph of accelerated motion as a continuous curve.

2.2.2 Position-Time Graph for Non-Uniform Motion

Consider the motion of a train which starts from a station. The train speeds up and moves with uniform velocity for certain duration and then slows down before steaming in the next station. In this case, it is observed that the distances covered by train in equal intervals of time are not equal. Such a motion is said to be non-uniform motion. ***If the distances covered by a moving body are not equal in equal intervals***

of time, then the motion is said to be non-uniform motion. If the distances covered in successive intervals are increasing, the motion said to be accelerated motion. The position-time graph for such an object is shown in Fig. 2.4.

Note that the position-time graph of accelerated motion is a continuous curve. Hence, the velocity of body changes continuously. In such a situation, it is more appropriate to define average velocity of the body over an extremely small interval of time or instantaneous velocity.

2.2.3 Interpretation of Position-time Graph

It is seen that, the position-time graphs of different moving objects have different shapes. If it is a straight line parallel to the time axis, we can say that the body is at rest (Fig. 2.2), the straight line inclined to the time axis shows that the motion is uniform (Fig.2.3) and a continuous curve implies continuously changing velocity.

(a) Velocity from Position-time graph : The slope of the straight line of position-time graph gives the average velocity of the object in motion. To determine the slope, consider two widely separated points (say A and B) on the straight line (Fig 2.3). Now construct a triangle by drawing lines parallel to y-axis and x-axis. The average velocity of the object is

$$\overline{\mathrm{v}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{BC}{AC} \qquad (2.4)$$

Hence, average velocity of object is equal to the slope of the straight line AB. Note that greater the value of the slope (Δx / Δt) of the straight line position-time graph, more will be the average velocity. Also we know that the slope is equal to the tangent of the angle that the straight line makes with a horizontal line, i.e., tan θ = Δx / Δt.

Any two corresponding Δx and Δt intervals can be used to determine the slope and thus the average velocity during that time interval.

(b) Instantaneous velocity : We have seen that, a body having uniform motion along a straight line has the same velocity at every instant. But in the case of non-uniform motion, the position-time graph is a curved line as shown in Fig.2.5. As a result the slope or the average velocity varies, depending on the size of the time intervals selected. The velocity of particle at any instant of time or at some point of its path is called its instantaneous velocity. We know that the average velocity over a time interval 'Δt' is given by

$$\overline{\mathrm{v}} = \frac{\Delta x}{\Delta t}$$

As Δt is made smaller and smaller the average velocity approaches instantaneous velocity. i.e., in the limit $\Delta t \to 0$, the slope (Δx / Δt) of a line tangent

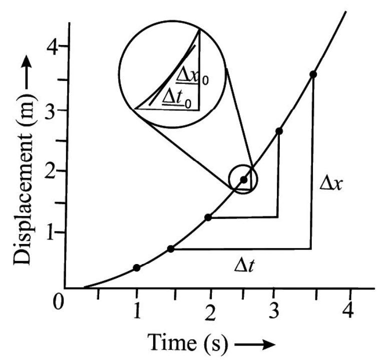


Fig. 2.5 : Displacement-time graph for non-uniform motion

to the curve at that point gives the instantaneous velocity. Note that for uniform motion, the average velocity is equal to the instantaneous velocity.

Example 2.5

The position - time graph for the motion of an object for 20 seconds is shown in Fig. 2.6. What distances and with what speeds does it travel in time intervals (i) 0 s to 5 s, (ii) 5 s to 10 s, (iii) 10 s to 15 s. Calculate the average speed for this total journey.

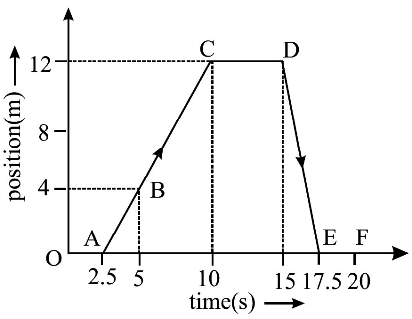


Fig. 2.6 : Position-time graph

Solution :

(i) During 0 s to 5 s, distance travelled = 4 m

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{4\,\text{m}}{(5-0)\,\text{s}} = \frac{4\,\text{m}}{5\,\text{s}} = 0.8\,\text{ms}^{-1}$$

(ii) During 5 s to 10 s, distance travelled = 12 – 4 = 8 m

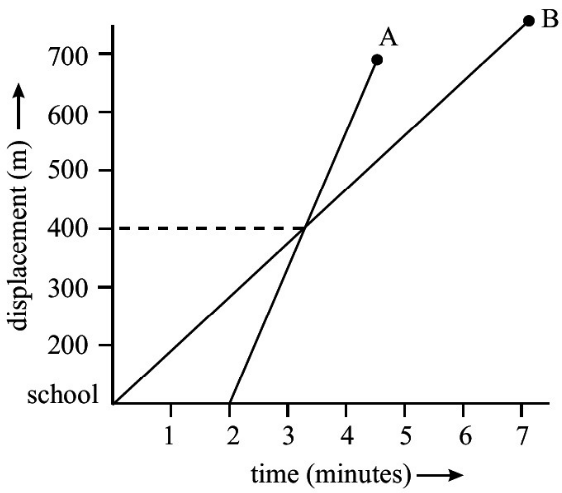
$$\text{speed} = \frac{(12-4)\,\text{m}}{(10-5)\,\text{s}} = \frac{8\,\text{m}}{5\,\text{s}} = 1.6\,ms^{-1}$$

(iii) During 10 s to 15 s, distance travelled = 12 – 12 = 0 m

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{0}{5} = 0$$

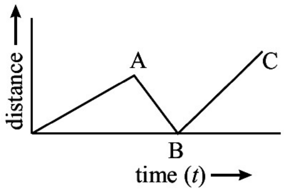
Intext Questions 2.2

1. Draw the position-time graph for a motion with zero acceleration.
2. The following figure shows the displacement - time graph for two students A and B who start from their school and reach their homes. Look at the graphs carefully and answer the following questions.
   (i) Do they both leave school at the same time?
   (ii) Who stays farther from the school?
   (iii) Do they both reach their respective houses at the same time?
   (iv) Who moves faster?
   (v) At what distance from the school do they cross each other?

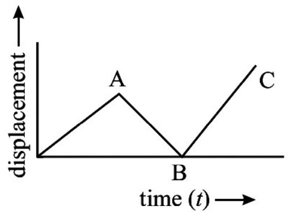


3. Under what conditions is average velocity of a body equal to its instantaneous velocity?

4. Which of the following graphs is not possible? Give reason for your answer?



(a)



(b)

2.3 VELOCITY-TIME GRAPH

We can also plot velocity-time graph by taking time along the x-axis and the velocity along the y-axis.

2.3.1 Velocity-Time Graph for Uniform Motion

We know that in uniform motion the velocity of the body remains constant, i.e., there is no change in the velocity with time. The velocity-time graph for such a uniform motion, is a straight line parallel to the time axis, as shown in Fig. 2.7.

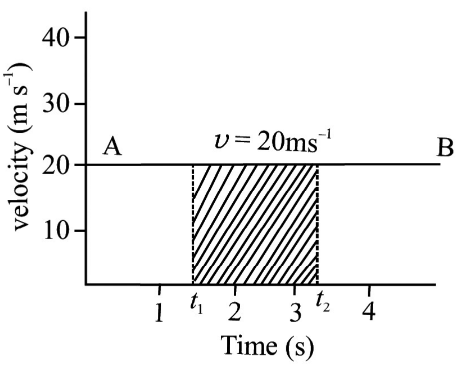


Fig. 2.7 : Velocity-time graph for uniform motion

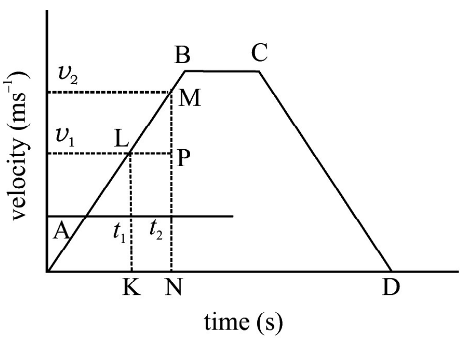


Fig. 2.8 : Velocity-time graph for motion with three different stages of constant acceleration

2.3.2 Velocity-Time Graph for Non-Uniform Motion

If the velocity of a body changes uniformly with time, its acceleration is constant. The velocity-time graph for such a motion is a straight line inclined to the time axis. This is shown in Fig. 2.8 by the straight line AB. It is clear from the graph that the velocity increases by equal amounts in equal intervals of time. The average acceleration of the body is given by

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} = \frac{MP}{LP}$$

$$= \text{slope of the straight line}$$

Since the slope of the straight line is constant, the average acceleration of the body is constant. However, it is also possible that the rate of variation in the velocity is not constant.

Such a motion is called non-uniformly accelerated motion. In such a situation, the slope of the velocity-time graph will vary at every instant, as shown in Fig.2.9. It can be seen that θ_A, θ_B and θ_C are different at points A, B and C.

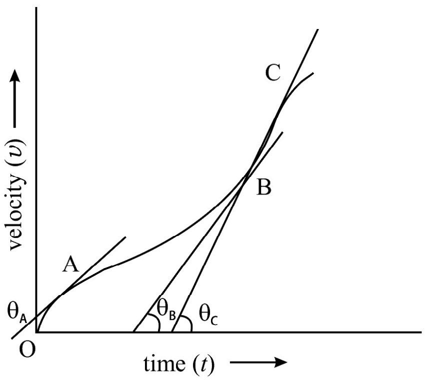


Fig. 2.9 : Velocity-time graph for a motion with varying acceleration

2.3.3 Interpretation of Velocity-Time Graph

Using v – t graph of the moving body, we can determine the distance travelled by it and the acceleration of the body at different instants.

(a) Determination of the distance travelled by the body : Let us consider the velocity-time graph shown in Fig.2.8. The portion AB shows the motion with constant acceleration, where as the portion CD shows the constantly retarded motion. The portion BC represents uniform motion (i.e., motion with zero acceleration).

For uniform motion, the distance travelled by the body from time t_1 to t_2 is given by $s = v(t_2 - t_1)$ = area under the curve between t_1 and t_2. Generalising this result for Fig.2.8, we find that the distance travelled by the body between time t_1 and t_2

$$s = \text{area of trapezium KLMN}$$
$$= (½) \times (KL + MN) \times KN$$
$$= (½) \times (v_1 + v_2) \times (t_2 - t_1)$$

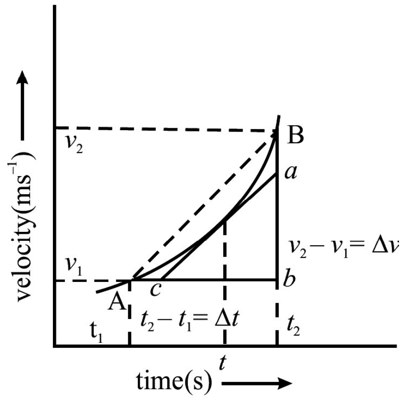


Fig : 2.10 : velocity time graph of non uniformly accelerated motion

(b) Determination of the acceleration of the body : We know that acceleration of a body is the rate of change of its velocity with time. If we look at the velocity- time graph given in the Fig.2.10, we will note that the average acceleration is represented by the slope of the chord AB, which is given by

$$\text{average acceleration } (\bar{a}) = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

If the time interval Δt is made smaller and smaller, the average acceleration becomes instantaneous acceleration. Thus, instantaneous acceleration

$$a = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \text{slope of the tangent at}(t = t) = \frac{ab}{bc}$$

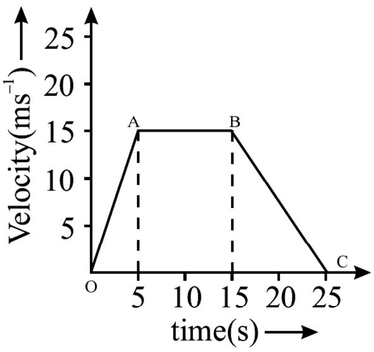
Thus, the slope of the tangent at a point on the velocity-time graph gives the acceleration at that instant.

Intext Questions 2.3

1. The motion of a particle moving in a straight line is depicted in the adjoining v – t graph.

   (i) Describe the motion in terms of velocity, acceleration and distance travelled

   (ii) Find the average speed.



2.4 EQUATIONS OF MOTION

In order to describe the motion of an object,we use physical quantities like distance, velocity and acceleration. In the case of constant acceleration, the velocity acquired and the distance travelled in a given time can be calculated by using one or more of three equations. These equations are known as equations of motion for constant acceleration or kinematical equations. They are very easy to use and find many applications.

2.4.1 Equation of Uniform Motion

To derive these equations, let us consider the initial time to be zero i.e. $t_1 = 0$. We can then assume $t_2 = t$ to be the elapsed time. Let the initial position of object be x_0 and initial velocity u. After time t, the final position of object be x and final velocity v. The average velocity during the time t will be

$$\overline{v} = \frac{x - x_0}{t} \qquad (2.5)$$

2.4.2 First Equation of Uniformly Accelerated Motion

The first equation of uniformly accelerated motion helps in determining the velocity of an object after a certain time when the acceleration is given. We know that

$$\text{Acceleration}(a) = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$$

for convenience let $t_1 = 0$, $v_1 = u$ and $t_2 = t$, $v_2 = v$. Then

$$a = \frac{v - u}{t} \qquad (2.6)$$

$$\Rightarrow \qquad v = u + at \qquad (2.7)$$

Example 2.6

A car starting from rest has an acceleration of 20 ms^{-2}. How fast will it be going after 4 s?

Solution :

Given,

Initial velocity $u = 0$

Acceleration $a = 20\ ms^{-2}$

Time $t = 4\ s$

Using first equation of motion

$$v = u + at$$

The velocity after time t = 4 s is given by

$$v = 0 + (20\ ms) \times (4\ s)$$

$$= 80\ ms^{-1}$$

2.4.3 Second Equation of Uniformly Accelerated Motion

Second equation of motion is used to calculate the position of an object after time t when it is moving with constant acceleration a.

Suppose that at $t = 0$, $x_1 = x_0$; $v_1 = u$ and at $t = t$, $x_2 = x$; $v_2 = v$.

The distance travelled = area under v – t graph

= Area of trapezium OABC

$$= \frac{1}{2}(CB + OA) \times OC$$

$$x - x_0 = \frac{1}{2}(v + u)t$$

Since $v = u + at$, we can write

$$x - x_0 = \frac{1}{2}(u + at + u)t$$

$$= ut + \frac{1}{2}at^2$$

or $$x = x_0 + ut + \frac{1}{2}at^2 \qquad (2.8)$$

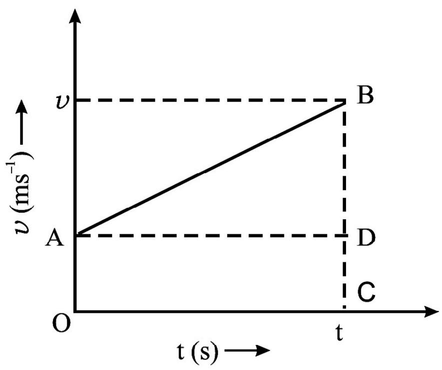


Fig. 2.11 : v–t graph for uniformly accelerated motion

Example 2.7

A car A is travelling on a straight road with a uniform speed of 60 km h^{-1}. Car B is following it with uniform velocity of 70 km h^{-1}. When the distance between them is 2.5 km, the car B is given a decceleration of 20 km h^{-1}. At what distance and time will the car B catch up with car A?

Solution :

Suppose that car B catches up with car A at a distance x after time t. For car A, the distance travelled in t time, $x = 60\ t$.

For car B, the distance travelled in t time is given by

$$x' = x_0 + ut + 1/2\, at^2$$

$$= 0 + 70 \times t + ½\,(–20) \times t^2$$

$$x' = 70\,t - 10\,t^2$$

But the distance between two cars is

$$x' - x = 2.5$$

$\therefore$ $(70\,t - 10\,t^2) - (60\,t) = 2.5$

or $10\,t^2 - 10\,t + 2.5 = 0$

It gives t = ½ hour

$\therefore$ $x = 70\,t - 10\,t^2$

$$= 70 \times ½ - 10 \times (½)^2$$

$$= 35 - 2.5 = 32.5 \text{ km.}$$

2.4.4 Third Equation of Uniformly Accelerated Motion

The third equation is used to calculate the final velocity when the acceleration, position and initial velocity of moving body are known.

From Eqn. (2.8) we can write

$$x - x_0 = ½\,(v + u)\,t.$$

Also from Eqn. (2.7), we recall that

$$t = \frac{v-u}{a}$$

Substituting this in above expression we get

$$x - x_0 = \frac{1}{2}(v + u)\left(\frac{v-u}{a}\right)$$

$$\Rightarrow \quad 2a(x - x_0) = v^2 - u^2$$

$$\Rightarrow \quad v^2 = u^2 + 2a\,(x - x_0) \qquad (2.9)$$

Thus the three equations for constant acceleration

$$v = u + at$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

and $v^2 = u^2 + 2a(x - x_0)$

Example 2.8

A body moves along a straight line with a constant acceleration of 4 ms^{-2}. If initially the body was at a position of 5 m and had a velocity of 3 ms^{-1}, calculate

(i) the position and velocity at time t = 2 s, and

(ii) the position of the body when its velocity is 5 ms^{-1}.

Solution :

We are given

$$x_0 = 5 \text{ m}, u = 3 \text{ ms}^{-1}, a = 4 \text{ ms}^{-2}.$$

(i) Using Eqn. (2.8)

$$x = x_0 + ut + 1/2\ at^2$$

$$= 5 + 3 \times 2 + ½ \times 4\ (2)^2 = 19 \text{ m}$$

From Eqn. (2.7)

$$v = u + at$$

$$= 3 + 4 \times 2 = 11 \text{ ms}^{-1}$$

velocity, $v = 11 \text{ ms}^{-1}$.

(ii) Using equation

$$v^2 = u^2 + 2a\ (x - x_0)$$

$$(5)^2 = (3)^2 + 2 \times 4 \times (x - 5)$$

$$\Rightarrow \quad x = 7 \text{ m}$$

Hence position of the body (x) = 7 m.

Intext Questions 2.4

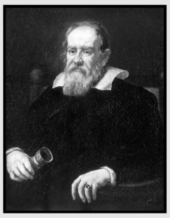
1. A body starting from rest covers a distance of 40 m in 4 s with constant acceleration along a straight line. Compute its final velocity and the time required to cover half of the total distance.
2. A car moves along a straight road with constant acceleration of 5 ms^{-2}. Initially at 5 m, its velocity was 3 ms^{-1}. Compute its position and velocity at t = 2 s.
3. With what velocity should a body be thrown vertically upward so that it reaches a height of 25 m? For how long will it be in the air?
4. A ball is thrown upward in the air. Is its acceleration greater while it is being thrown or after it is thrown?

2.5 MOTION UNDER GRAVITY

We know that all objects when dropped fall towards the earth. This is because of the gravitational force of earth. The gravitational force acts in vertical direction, therefore, motion under gravity is along a straight line. It is a one dimensional motion. ***The free fall of a body towards the earth is one of the most common examples of motion with constant acceleration.*** In the absence of air resistance, it is found that all bodies, irrespective of their size or weight, fall with the same acceleration. Though the acceleration due to gravity varies with altitude, for small distances compared to the earth's radius, it may be taken constant throughout the fall. For our practical use, the effect of air resistance is neglected.

The acceleration acquired by a freely falling body due to gravity is called acceleration due to gravity and is denoted by *g*. At or near the earth's surface, its magnitude is approximately 9.8 ms^{-2}.

Galileo Galilei (1564 – 1642)



He was born at Pisa in Italy in 1564. He enunciated the laws of falling bodies. He devised a telescope and used it for astronomical observations. His major works are : Dialogues about the Two great Systems of the World and Conversations concerning Two New Sciences. He supported the idea that the earth revolves around the sun.

Example 2.9

A stone is dropped from a height of 50 m and it falls freely. Calculate the (i) distance travelled in 2 s, (ii) velocity of the stone when it reaches the ground, and (iii) velocity at 3 s i.e., 3 s after the start.

Solution :

Given

Height h = 50 m and Initial velocity u = 0

Consider, initial position (y_0) to be zero and the origin at the starting point. Thus, the y-axis (vertical axis) below it will be negative. Since acceleration is downward in the negative y-direction, the value of $a = -g = -9.8\ ms^{-2.}$

(i) From Eqn. (2.8), we recall that

$$y = y_0 + ut + ½\ at^2$$

For the given data, we get

$$y = 0 + 0 - ½\ gt^2 = -½ \times 9.8 \times (2)^2$$
$$= -19.6\ m.$$

The negative sign shows that the distance is below the starting point in downward direction.

(ii) At the ground $y = -50$ m,

Using equation (2.9),

$$v^2 = u^2 + 2a\ (y - y_0)$$
$$= 0 + 2\ (-9.8)\ (-50 - 0)$$
$$v = 9.9\ ms^{-1}$$

(iii) Using $v = u + at$, at $t = 3$ s, we get

$$\therefore \quad v = 0 + (-9.8) \times 3$$

$$v = -29.4 \text{ ms}^{-1}$$

This shows that the velocity of the stone at $t = 3$ s is 29.4 ms^{-1} and it is in downward direction.

Note : It is important to note that in kinematic equations, we use the sign convention according to which quantities directed upwards and rightwards are taken as positive and those directed downwards and leftwards are taken as negative.

2.6 CONCEPT OF DIFFERENTIATION AND INTEGRATION

All branches of Mathematics have been very useful tools in understanding and explaining the laws of Physics and finding the relations between different Physical quantities. You are already familiar with the use of Algebra and Trigonometry in this connection. In the further study of Physics, you will come across the use of Differentiation (or Differential Calculus) and Integration (or Integral Calculus). A brief and simple description of the concept of Differentiation and Integration is, therefore, being given below. You may consult books on Mathematics for more information on these topics.

We will often come across the following terms in this topic. Let us define these terms:

Constant: It is a quantity whose value does not change during mathematical operations, e.g. integers like 1, 2, 3, …., fractions, π, e, etc.

Variable: It is a quantity which can take different values during mathematical operations. A variable is generally denoted by x, y, z etc.

Function: 'y' is said to be a function of 'x', if for every value of 'x' there is definite value of 'y'.

Mathematically, it is represented by

$$y = f(x)$$

i.e. 'y' is a function of 'x'

Differential Coefficient: Of any variable 'y' with respect to any other variable 'x', is the instantaneous rate of change of 'y' with respect to 'x'.

Let 'y' be a function of 'x' i.e. $y = f(x)$. Suppose 'x' is increased by a very small amount δx or say there is a very small increament 'δx' in 'x'. Let there be a corresponding increament 'δy' in 'y'. Then, $y + \delta y$ is a function of $(x + \delta x)$

or $$y + \delta y = f(x + \delta x)$$

or $$\delta y = f(x + \delta x) - y$$

or $$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

The quantity $\frac{\delta y}{\delta x}$ is called increment ratio and represents the average rate of change of ‘y’ with respect to ‘x’ in the range between the time interval x and (x + δx).

To find the instantaneous rate of change of ‘y’ with respect to ‘x’, we will have to calculate the limit of $\frac{\delta y}{\delta x}$ as δx tends to zero ($\delta x \to 0$).

i.e. $$\underset{\delta x \to 0}{\mathrm{Lt}} \frac{\delta y}{\delta x} = \underset{\delta x \to 0}{\mathrm{Lt}} \frac{f(x+\delta x)-f(x)}{\delta x}$$

Thus, the instantaneous rate of change of ‘*y*’ with respect to ‘*x*’ is given by $\underset{\delta x \to 0}{\mathrm{Lt}} \frac{\delta y}{\delta x}$.

This is called the **differential coefficient** of ‘y’ with respect to ‘*x*’ and is denoted by $\frac{dy}{dx}$.

Integration

Integration is a mathematical process which is reverse of differentiation. In order to understand this concept, let a constant force **F** act on a body moving it through a distance **S**. Then, the work done by the force is calculated by the product W = **F.S**.

But, if the force is variable, ordinary algebra does not give any method to find the work done.

For example when a body is to be moved to a long distance up above the surface of the earth, the force of gravity on the body goes on changing as the body moves up. In such cases a method called integration is used to calculate the work done.

The work done by a variable force can be calculated as (see for details section 5.2 work done by a variable force)

$$W = \Sigma F(x)\, \Delta x$$

For infinitesimally small values of Δx,

$$W = \sum_{\lim \Delta x \to 0} F(x)dx$$

This may be written as

$$W = \int F(x)dx$$

This expression is called integral of function *F*(*x*) with respect to *x*, where the symbol ‘$\int$’ denotes integration.

Some often used formulae of Integration and Differentiation

(i) $\int x^n dx = \frac{x^{n+1}}{n+1}(\text{for n} \neq -1)$	(i) $\frac{d}{dx}x^n = nx^{n-1}$
(ii) $\int x^{-1}dx = \int \frac{1}{x}dx = \log x$	(ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$
(iii) $\int dx = \int x^0 dx = \frac{x^1}{1} = x$	(iii) $\frac{d}{dx}(x) = 1$
(iv) $\int cxdx = c\int xdx(\text{c is a constant})$	(iv) $\frac{d}{dx}(cu) = c\frac{d}{dx}(u)$
(v) $\int (u \pm v \pm w)dx = \int udx \pm \int vdx \pm \int wdx$	(v) $\frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$
(vi) $\int e^x dx = e^x$	(vi) $\frac{d}{dx}(e^x) = e^x$
(vii) $\int \sin xdx = -\cos x$	(vii) $\frac{d}{dx}\{\sin(x)\} = +\cos x$
(viii) $\int \cos xdx = \sin x$	(viii) $\frac{d}{dx}(\cos x) = -\sin x$
(ix) $\int \sec^2 xdx = \tan x$	(ix) $\frac{d}{dx}(\tan x) = \sec x$
(x) $\int \text{cosec}^2 xdx = -\cot x$	(x) $\frac{d}{dx}(\cot x) = -\cos ec^2 x$

A close look at the table shows that Integration and differentiation are converse mathematical operations.

WHAT YOU HAVE LEARNT

- The total path covered by a body during its motion is called distance.
- Displacement is the shortest distance between the initial and final positions of the body and has a direction.
- The ratio of the displacement of an object to the time interval is known as average velocity.
- The total distance travelled divided by the time taken is average speed.

- The rate of change of the relative position of an object with respect to another object is known as the relative velocity of that object with respect to the other.
- The change in the velocity in unit time is called acceleration.
- The position-time graph for a body at rest is a straight line parallel to the time axis.
- The position-time graph for a uniform motion is a straight line inclined to the time axis.
- A body covering equal distance in equal intervals of time, however small, is said to be in uniform motion.
- The velocity of a particle at any one instant of time or at any one point of its path is called its instantaneous velocity.
- The slope of the position-time graph gives the average velocity.
- The velocity-time graph for a body moving with constant acceleration is a straight line inclined to the time axis.
- The area under the velocity-time graph gives the displacement of the body.
- The average acceleration of the body can be computed by the slope of velocity-time graph.
- The motion of a body can be described by following three equations :

  (i) $v = u + at$

  (ii) $x = x_0 + u\,t + \frac{1}{2}at^2$

  (iii) $v^2 = u^2 + 2a\,(x - x_0)$

TERMINAL EXERCISE

1. A car C moving with a speed of 65 km h^{-1} on a straight road is ahead of motorcycle M moving with the speed of 80 km h^{-1} in the same direction. What is the velocity of M relative to C?
2. How long does a car take to travel 30 m, if it accelerates from rest at a rate of 2.0 m s^{-2}?
3. A motorcyclist covers half of the distance between two places at a speed of 30 km h^{-1} and the second half at the speed of 60 km h^{-1}. Compute the average speed of the motorcycle.
4. A duck, flying directly south for the winter, flies with a constant velocity of 20 km h^{-1} to a distance of 25 km. How long does it take for the duck to fly this distance?
5. Bangalore is 1200 km from New Delhi by air (straight line distance) and 1500 km by train. If it takes 2 h by air and 20 h by train, calculate the ratio of the average speeds.

6. A car accelerates along a straight road from rest to 50 kmh^{-1} in 5.0 s. What is the magnitude of its average acceleration?
7. A body with an initial velocity of 2.0 ms^{-1} is accelerated at 8.0 ms^{-2} for 3 seconds.

   (i) How far does the body travel during the period of acceleration?

   (ii) How far would the body travel if it were initially at rest?
8. A body is thrown vertically upward, with a velocity of 10 m/s. What will be the value of the velocity and acceleration of the body at the highest point?
9. A lady drove to the market at a speed of 8 km h^{-1}. Finding market closed, she came back home at a speed of 10 km h^{-1}. If the market is 2 km away from her home, calculate the average velocity and average speed.
10. A body starting from rest covers a distance of 40 m in 4 s with constant acceleration along a straight line. Compute its final velocity and the time required to cover half of the total distance.

ANSWERS TO INTEXT QUESTIONS

2.1

1. Yes. When body returns to its initial postion, its velocity is zero but speed is non-zero.
2. Yes,two cars moving with same velocity in the same direction, will have zero relative velocity with respect to each other.
3. (a) 1 ms^{-1} (b) 2 ms^{-1}

2.2

1. See Fig. 2.2.
2. (i) No, (ii) B (iii) No (iv) A (v) 400 m
3. In the uniform motion.
4. (a) is wrong, because the distance covered cannot decrease with time or become zero.

2.3

1. (i) The body starts with a zero velocity.

   Motion of the body between start and 5th seconds is uniformly accelerated. It has been represented by the line OA.

$$a = \frac{15-0}{5-0} = 3ms^{-2}$$

Motion of the body between 5th and 10th second is a uniform motion

(represented by AB). $a = \frac{15-15}{15-5} = \frac{0}{10} = 0\,ms^{-2}$.

Motion between 15th and 25th second is uniformly retarded.

(represented by the line BC). $a = \frac{0-15}{25-15} = -1.5\,ms^{-2}$

(ii) $\text{Average speed} = \frac{\text{Distance covered}}{\text{time taken}} = \frac{\text{Area of OA BC}}{(25-0)}$

$$= \frac{\left(\frac{1}{2}\times 15\times 5\right) + (15\times 10) + \left(\frac{1}{2}\times 15\times 10\right)}{25} = \frac{525}{50} = 10.5\,ms^{-1}.$$

2.4

1. Using $x = x_0 + ut + \frac{1}{2}at^2$

   $40 = \frac{1}{2} \times a \times 16 \quad \Rightarrow \quad a = 5\,ms^{-2}$

   Next using $v^2 = u^2 + 2a(x - x_0)$

   $v = 20\ ms^{-1}$,

   $20 = 0 + \frac{1}{2}\times 5 \times t^2 \quad \Rightarrow \quad t = 2\sqrt{2}\,s.$

2. Using eqn. $x = x_0 + ut + \frac{1}{2}at^2, \quad x = 21\,m$

   and using eqn. $v = u + at$, $v = 13\ ms^{-1}$.

3. At maximum height, $v = 0$

   $u = 7\sqrt{10}\,ms^{-1} = 22.6\,ms^{-1}$.

   The body will be in the air for the twice of the time it takes to reach the maximum height.

4. The acceleration of the ball is greater while it is thrown.

ANSWERS TO TERMINAL EXERCISE

1. 15 km h^{-1}
2. 5.47 s
3. 40 ms^{-1}
4. 1.25 h
5. 8 : 1
6. 2.8 ms^{-2} (or 3000 km h^{-2})
7. (i) 42 m (ii) 36 m
8. 0 and 9.8 ms^{-2}.
9. Average velocity = 0, Average speed = 8.89 km h^{-1}
10. v = 20 ms^{-1}, $t = 2\sqrt{2}$ s.

3

MOTION IN A PLANE

INTRODUCTION

In the last chapter, we studied the concepts of distance, displacements, velocity and acceleration related to the motion of an object along a straight line. It is seen that the directional aspect of displacement, velocity, acceleration is taken care of by positive and negative signs, since in 1D only two directions are possible. But to describe the motion of objects moving in a plane i.e., in 2D, we have to introduce certain new concepts. A simple example of motion in two dimensions is the motion of a ball thrown at an angle to the horizontal. This motion is called a *projectile motion*. We will discuss in detail the projectile motion and also circular motion. Generally, circular motion refers to motion in a horizontal circle. We will introduce the concepts of angular speed, centripetal acceleration, and centripetal force to explain this kind of motion.

OBJECTIVES

After studying this lesson, you should be able to

- *differentiate between scalar and vector quantities and give examples of each;*
- *add and subtract two vectors and resolve a vector into its components;*
- *calculate the product of two vectors;*
- *explain projectile motion and circular motion;*
- *derive the equation of the trajectory of a projectile;*
- *derive expressions for the maximum height, time of flight and range of a projectile;*
- *define velocity and acceleration of a particle in circular motion.*

3.1 VECTORS AND SCALARS

3.1.1 Scalar and Vector Quantities

In physics we classify physical quantities in two categories. In one case, we need only to state their magnitude with proper units and that gives their complete description. Take, for example, mass. If we say that the mass of a ball is 50 g, we do not have to add anything to the description of mass. Similarly, the statement that the density of water is 1000 kg m^{-3} is a complete description of density. Such quantities are called scalars. **A scalar quantity has only magnitude but no direction.**

On the other hand, there are quantities which require both magnitude and direction for their complete description. A simple example is velocity. The statement that the velocity of a train is 100 km h^{-1} does not make much sense unless we also tell the direction in which the train is moving. Force is another such quantity. We must specify not only the magnitude of the force but also the direction in which the force is applied. Such quantities are called vectors. **A vector quantity has both magnitude and direction.**

Some examples of vector quantities which you come across in mechanics are: displacement Fig. 3.1, acceleration, momentum, angular momentum and torque etc.

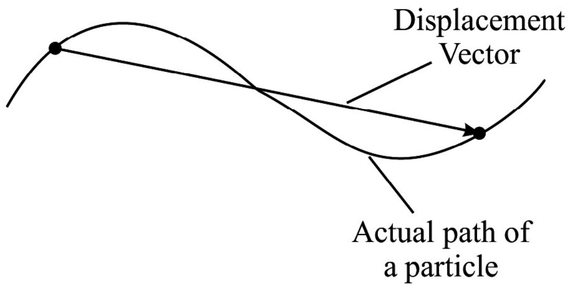


Fig. 3.1 : Displacement Vector

3.1.2 Representation of Vectors

A vector is represented by a line with an arrow indicating its direction. Take vector AB in Fig. 3.2. The length of the line represents its magnitude on some scale. The arrow indicates its direction. Vector CD is a vector in the same direction but its magnitude is smaller. Vector EF is a vector whose magnitude is the same as that of vector CD, but its direction is different. In any vector, the initial point, (point A in AB), is called the tail of the vector and the final point, (point B in AB) with the arrow mark is called its tip (or head). A vector is written with an arrow over the letter representing the vector, for example, $\vec{A}$. The magnitude of vector $\vec{A}$ is simply A or $|\vec{A}|$. In print, a vector is indicated by a bold letter as **A**.

Two vectors are said to be equal if their magnitudes are equal and they point in the same direction. This means that all vectors which are parallel to each other have the same magnitude and point in the same direction are equal. Three vectors **A**, **B** and **C** shown in Fig. 3.3 are equal. We say **A** = **B** = **C**. But **D** is not equal to **A**.

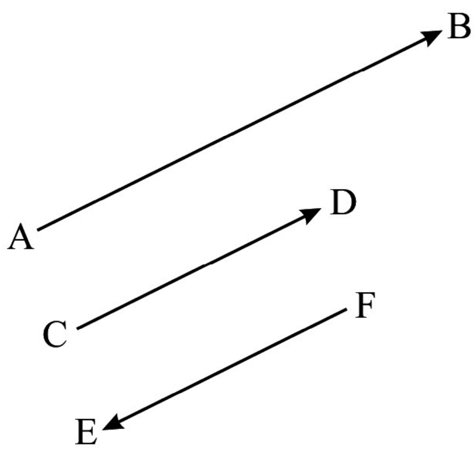


Fig. 3.2 : Directions and magnitudes of vectors

A vector (here **D**) which has the same magnitude as **A** but has opposite direction, is called negative of **A**, or –**A**. Thus, **D** = –**A**.

For representing a physical vector quantitatively, we have to invariably choose a proportionality scale. For instance, the vector displacement between Delhi and Agra, which is about 300 km, is represented by choosing a scale 100 km = 1 cm, say. Similarly, we can represent a force of 30 N by a vector of length 3cm by choosing a scale 10 N = 1cm.

From the above we can say that if we translate a vector parallel to itself, it remains unchanged. This important result is used in addition of vectors.

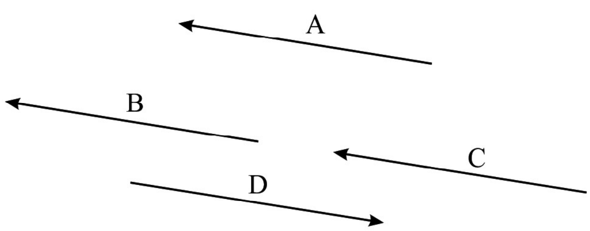


Fig. 3.3 : Three vectors are equal but fourth vector D is not equal.

3.1.3 Addition of Vectors

You should remember that only vectors of the same kind can be added. For example, two forces or two velocities can be added. But a force and a velocity cannot be added.

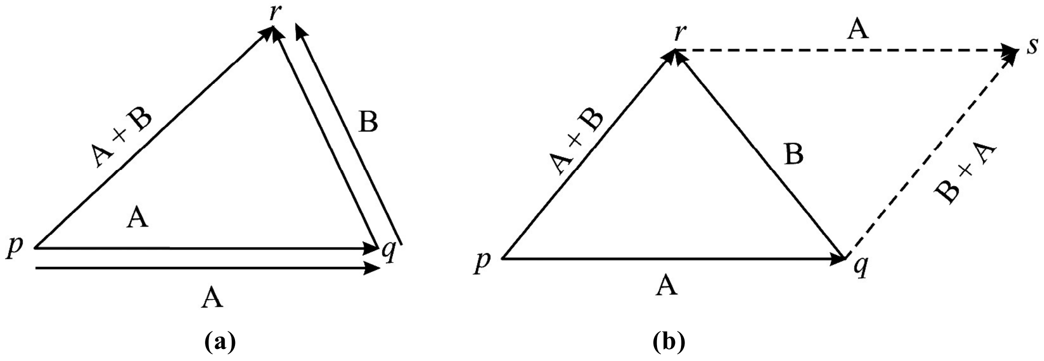


Fig. 3.4 : Addition of vectors A and B

Suppose we wish to add vectors **A** and **B**. First redraw vector **A** [Fig. 3.4 (a)]. For this draw a line (say pq) parallel to vector **A**. The length of the line i.e. pq should be equal to the magnitude of the vector. Next draw vector B such that its tail coincides with the tip of vector **A**. For this, draw a line qr from the tip of **A** (i.e., from the point q) parallel to the direction of vector **B**. The sum of two vectors then is the vector from the tail of A to the tip of B, i.e. the resultant will be represented in magnitude and direction by line pr. You can now easily prove that vector addition is commutative. That is, **A** + **B** = **B** + **A**, as shown in Fig. 3.4 (b). In Fig. 3.4 (b) we observe that pqr is a triangle and its two sides pq and qr respectively represent the vectors **A** and **B** in magnitude and direction, and the third side pr, of the triangle represents the resultant vector with its direction from p to r. This gives us a rule for finding the resultant of two vectors:

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, the resultant is represented by the third side of the triangle taken in the opposite order. This is called triangle law of vectors.

The sum of two or more vectors is called the **resultant** vector. In Fig. 3.4 (b), **pr** is the resultant of **A** and **B**. What will be the resultant of three forces acting along the three sides of a triangle in the same order? If you think that it is zero, you are right.

Let us now learn to calculate resultant of more than two vectors. The resultant of more than two vectors, say **A**, **B** and **C**, can be found in the same manner as the sum of two vectors. First we obtain the sum of **A** and **B**, and then add the resultant of the two vectors, (**A** + **B**), to **C**. Alternatively, you could add **B** and **C**, and then add **A** to (**B** + **C**) Fig. 3.5. In both cases you get the same vector. Thus, vector addition is associative. That is, **A + (B + C) = (A + B) + C.**

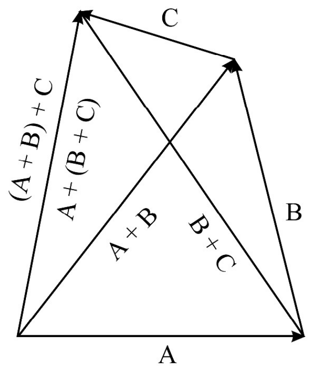


Fig. 3.5 : Addition of three vectors in two different orders

If you add more than three vectors, you will discover that **the resultant vector is the vector from the tail of the first vector to the tip of the last vector**.

Many a time, the point of application of vectors is the same. In such situations, it is more convenient to use parallelogram law of vector addition. Let us now learn about it.

3.1.4 Parallelogram Law of Vector Addition

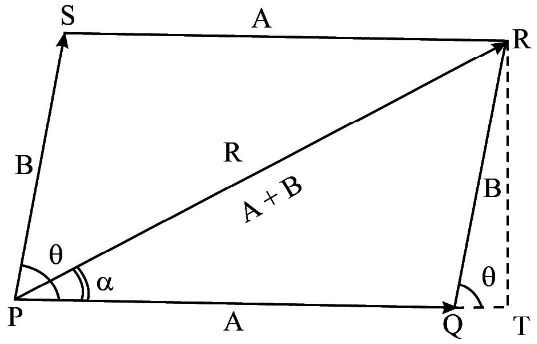


Fig. 3.6 : Parallelogram law of addition of vectors

Let **A** and **B** be the two vectors and let θ be the angle between them as shown in Fig. 3.6. To calculate the vector sum, we complete the parallelogram. Here side PQ represents vector **A**, side PS represents **B** and the diagonal PR represents the resultant vector **R**. Can you recognize that the diagonal PR is the sum vector **A + B**? It is called the **resultant** of vectors **A** and **B**. The resultant makes an angle α with the direction of vector **A**. Remember that vectors **PQ** and **SR** are equal to **A**, and vectors **PS** and **QR** are equal, to **B**. To get the magnitude of the resultant vector **R**, drop a perpendicular RT as shown. Then in terms of magnitudes

$$
\begin{aligned}
(PR)^2 &= (PT)^2 + (RT)^2 \\
&= (PQ + QT)^2 + (RT)^2 \\
&= (PQ)^2 + (QT)^2 + 2PQ.QT + (RT)^2 \\
&= (PQ)^2 + [(QT)^2 + (RT)^2] + 2PQ.QT \qquad (3.1) \\
&= (PQ)^2 + (QR)^2 + 2PQ.QT \\
&= (PQ)^2 + (QR)^2 + 2PQ.QR\ (QT / QR)
\end{aligned}
$$

$$R^2 = A^2 + B^2 + 2\ AB.\cos\theta$$

Therefore, the magnitude of **R** is

$$|\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB.\cos\theta} \qquad (3.2)$$

For the direction of the vector **R**, we observe that

$$\tan\alpha = \frac{RT}{PT} = \frac{RT}{PQ+QT} = \frac{B\sin\theta}{A+B\cos\theta} \qquad (3.3)$$

So, the direction of the resultant can be expressed in terms of the angle it makes with base vector.

Special cases

When the two vectors **A** and **B** are in the same direction (parallel), i.e., θ = 0° and cos θ = 1. Then, the resultant vector **R** is equal to the sum of their magnitudes, i.e., **R** = **A + B** and acts in the direction of these vectors.

When the two vectors are perpendicular to each other, i.e., θ = 90° and cos θ = 0, then the resultant vector **R** is the square root of the sum of squares of the vectors.

$$R = \sqrt{A^2 + B^2}$$

When the two vectors are in opposite direction (anti-parallel), i.e., θ = 180° and cos θ = –1, then the resultant vector **R** is equal to the difference of their magnitudes and direction will be along **A** or **B**, depending upon which of the vectors has larger magnitude.

$$\mathbf{R} = \mathbf{A} - \mathbf{B}$$

3.1.5 Subtraction of Vectors

How do we subtract one vector from another? If you recall that the difference of two vectors, **A – B**, is actually equal to **A + (–B)**, then you can adopt the same method as for addition of two vectors. It is explained in **Fig. 3.7**. Draw vector **–B** from the tip of **A**. Join the tail of **A** with the tip of **–B**. The resulting vector is the difference **(A – B)**.

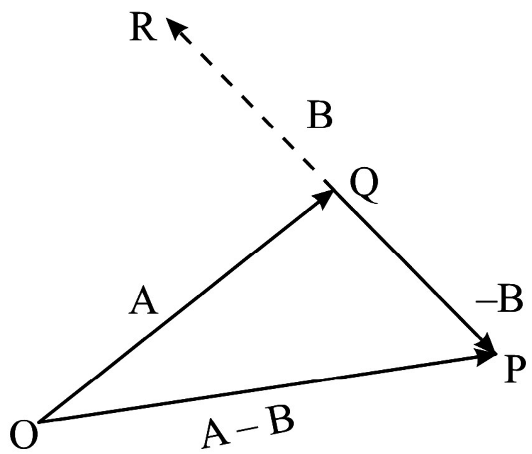
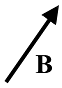


Fig. 3.7 : Subtraction of vector B from vector A

Intext Questions 3.1

Given vectors 

and 

1. Make diagrams to show how to find the following vectors:

   (a) **B – A**, (b) **A + B**, (c) **A – 2B** and (d) **B – 2A**.

2. Two vectors **A** and **B** of magnitudes 10 units and 12 units are anti-parallel. Determine **A + B** and **A – B**.

3. Two vectors A and B of magnitudes A = 30 units and B = 60 units respectively are inclined to each other at angle of 60 degrees. Find the resultant vector.

3.1.6 Multiplication of Vector by a scalar

If we multiply a vector **A** by a scalar k, the product is a vector whose magnitude is the absolute value of k times the magnitude of **A**. This means that the magnitude of the resultant vector is k |**A**|. The direction of the new vector remains unchanged if k is positive. If k is negative, the direction of the new vector is opposite to its original direction. For example, vector 3**A** is thrice the magnitude of vector **A**, and it is in the same direction as **A**. But vector –3**A** is in a direction opposite to vector **A**, although its magnitude is thrice that of vector **A**.

3.1.7 Scalar product of Vectors

The **scalar product** of two vectors **A** and **B** is written as **A.B** and is equal to AB cosθ where θ is the angle between the vectors. If you look carefully at Fig. 3.8, you would notice that B cos θ is the projection of vector **B** along vector **A**. Therefore, the scalar product of **A** and **B** is the product of magnitude of **A** with the length of the projection of **B** along **A.**

Another thing to note is that even if we take the angle between the two vectors as 360 – θ, it does not matter because the cosine of both angles is the same. Since a dot between the two vectors indicates the scalar product, it is also called the **dot product**.

Remember that the scalar product of two vectors is a scalar quantity.

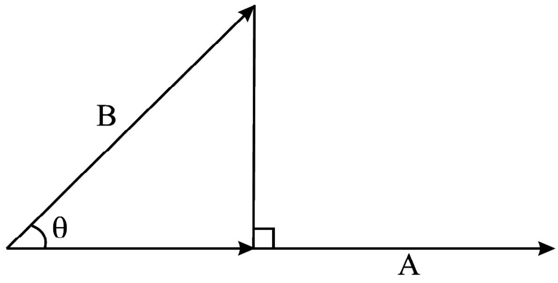


Fig. 3.8 : Projection of **B** on **A**

A familiar example of the scalar product is the work done when a force **F** acts on a body moving at an angle to the direction of the force. If **d** is the displacement of the body and θ is the angle between **F** and **d**, then the work done by the force is Fdcosθ.

Since dot product is a scalar, it is commutative: **A.B** = **B.A** = ABcosθ. It is also distributive: **A. (B + C) = A.B + A.C**.

3.1.8 Vector product of Vectors

Suppose we have two vectors **A** and **B** inclined at an angle θ. We can draw a plane which contains these two vectors. Let that plane be called Ω Fig. 3.9 (a).

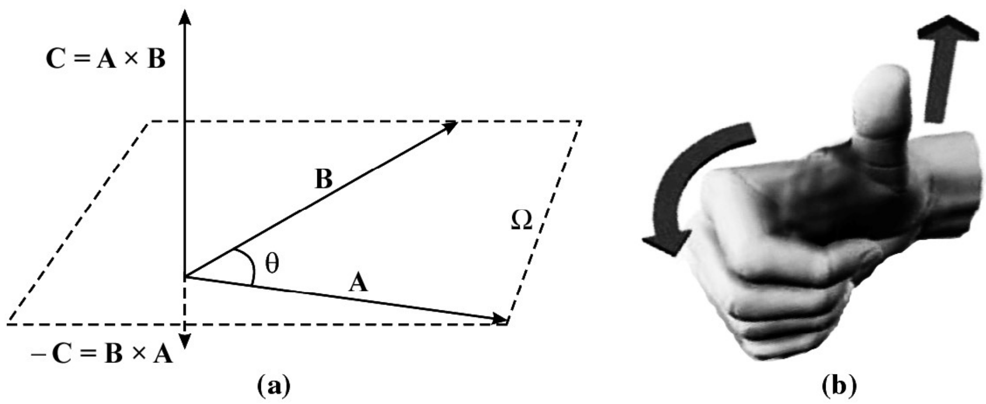


Fig. 3.9 : (a) Product of vectors
(b) Direction of the product vector **C = A × B** is given by the right hand rule. If the right hand is held so that the curling fingers point from **A** to **B** through the smaller angle between the two, then the thumb stretched at right angles to fingers will point in the direction of **C**.

which is perpendicular to the plane of paper here. Then the vector product of these vectors, written as **A × B**, is a vector, say **C**, whose magnitude is AB sinθ and whose direction is perpendicular to the plane **Ω**. The direction of the vector **C** can be found by **right-hand rule** Fig. 3.9 (b). Imagine the fingers of your right hand curling from **A** to **B** along the smaller angle between them. Then the direction of the thumb gives the direction of the product vector C. If you follow this rule, you can easily see that direction of vector **B × A** is opposite to that of the vector **A × B**. This means that **the vector product is not commutative**. Since a cross is inserted between the two vectors to indicate their vector product, the vector product is also called the cross product.

A familiar example of vector product is the angular momentum possessed by a rotating body.

Intext Questions 3.2

1. Suppose vector **A** is parallel to vector **B.** What is their vector product**?** What will be the vector product if **B** is anti-parallel to **A?**

2. Suppose we have a vector **A** and a vector $\mathbf{C} = \frac{1}{2}\mathbf{B}$. How is the direction of vector **A** × **B** related to the direction of vector **A** × **C.**

3. Suppose vectors **A** and **B** are rotated in the plane which contains them. What happens to the direction of vector **C = A** × **B**.

4. Suppose you were free to rotate vectors **A** and **B** through arbitrary amounts keeping them confined to the same plane. Can you make vector **C = A** × **B** to point in exactly opposite direction?

5. If vector **A** is along the x-axis and vector **B** is along the **y-**axis**,** what is the direction of vector **C = A** × **B?** What happens to **C** if **A** is along the **y-**axis and **B** is along the **x-**axis**?**

6. **A** and **B** are two mutually perpendicular vectors. Calculate (a) **A . B** and (b) **A** × **B.**

3.1.9 Resolution of Vectors

Resolution of vectors is converse of addition of vectors. Here we calculate components of a given vector along any set of coordinate axes. Suppose we have vector A as shown in Fig. 3.10 and we need to find its components along x and y-axes. Let these components be called A_x and A_y respectively. Simple trigonometry shows that

$$A_x = A\cos\theta \qquad (3.4)$$

and
$$A_y = A\sin\theta \qquad (3.5)$$

where θ is the angle that **A** makes with the x-axis. What about the components of vector **A** along X and Y-axes Fig. 3.10? If the angle between the X-axis and **A** is ϕ,

$$A_X = A\cos\phi$$

and
$$A_Y = A\sin\phi.$$

It must now be clear that the components of a vector are not fixed quantities; they depend on the particular set of axes along which components are required. Note also that the magnitude of vector **A** and its direction in terms of its components are given by

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{A_X^2 + A_Y^2} \qquad (3.6)$$

and
$$\tan\theta = A_y/A_x,\ \tan\phi = A_Y/A_X \qquad (3.7)$$

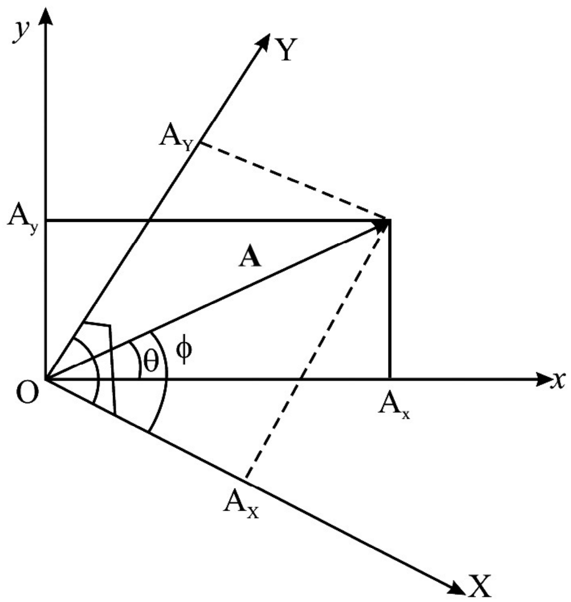


Fig. 3.10 : Resolution of vector **A** along two sets of coordinates (x, y) and (X, Y)

3.1.10 Unit Vector

At this stage we introduce the concept of a **unit vector**. As the name suggests, a unit vector has unitary magnitude and has a specified direction. It has no units and no dimensions. As an example, we can write vector **A** as $A\hat{\mathbf{n}}$ where a cap on **n** (i.e. $\hat{\mathbf{n}}$) denotes a unit vector in the direction of **A**. Notice that a unit vector has been introduced to take care of the direction of the vector; the magnitude has been taken care of by A. Unit vector along x-axis is denoted by $\hat{\mathbf{i}}$, along y-axis by $\hat{\mathbf{j}}$ and along z-axis by $\hat{\mathbf{k}}$. Using this notation, vector **A**, whose components along x and y axes are respectively A_x and A_y, can be written as

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{3.8}$$

Another vector B can similarly be written as

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} \tag{3.9}$$

The sum of these two vectors can now be written as

$$\mathbf{A}+\mathbf{B} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \tag{3.10}$$

By the rules of scalar product, you can show that

$$\hat{\mathbf{i}}\bullet\hat{\mathbf{i}} = 1,\ \hat{\mathbf{j}}\bullet\hat{\mathbf{j}} = 1,\ \hat{\mathbf{k}}\bullet\hat{\mathbf{k}} = 1,\ \hat{\mathbf{i}}\bullet\hat{\mathbf{j}} = 0,\ \hat{\mathbf{i}}\bullet\hat{\mathbf{k}} = 0, \text{ and } \hat{\mathbf{j}}\bullet\hat{\mathbf{k}} = 0 \tag{3.11}$$

The dot product between two vectors A and B can now be written as

$$\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}).(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

$$= A_xB_x(\hat{\mathbf{i}}\bullet\hat{\mathbf{i}}) + A_xB_y(\hat{\mathbf{i}}\bullet\hat{\mathbf{j}}) + A_yB_x(\hat{\mathbf{j}}\bullet\hat{\mathbf{i}}) + A_yB_y(\hat{\mathbf{j}}\bullet\hat{\mathbf{j}})$$

Using Eqn. (3.11),

$$= A_xB_x + A_yB_y \tag{3.12}$$

The cross product of two vectors can also be written in terms of the unit vectors. For this we first need the cross product of unit vectors. For this remember that the angle between the unit vectors is a right angle. Consider, for example, $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$. Sine of the angle between them is one. The magnitude of the product vector is also 1. Its direction is perpendicular to the xy-plane containing $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, which is the z-axis. By the right hand rule, we also find that this must be the positive z-axis. And what is the unit vector in the positive z-direction. The unit vector $\hat{\mathbf{k}}$. Therefore,

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \tag{3.13}$$

Using similar arguments, we can show,

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}},\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}},\ \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}},\ \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},\ \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}} \tag{3.14}$$

And

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0 \qquad (3.15)$$

Example 3.1

Calculate the dot product and cross product of vectors $\mathbf{C} = 4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}$ and $\mathbf{D} = 6\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$

Solution :

The dot product of **C** and **D:**

$$\mathbf{C} \bullet \mathbf{D} = (4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}) \bullet (6\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$$

$$= 24\,(\hat{\mathbf{i}} \bullet \hat{\mathbf{i}}) - 16\,(\hat{\mathbf{i}} \bullet \hat{\mathbf{j}}) + 30\,(\hat{\mathbf{j}} \bullet \hat{\mathbf{i}}) - 20(\hat{\mathbf{j}} \bullet \hat{\mathbf{j}})$$

Using equation **3.11,**

$$= 24 - 20$$

$$= 4$$

The cross-product of **C** and **D:**

$$\mathbf{C} \times \mathbf{D} = (4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}) \times (6\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$$

$$= 24\,(\hat{\mathbf{i}} \times \hat{\mathbf{i}}) - 16\,(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) + 30\,(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) - 20(\hat{\mathbf{j}} \times \hat{\mathbf{j}})$$

Using Eqns. (3.13 to 3.15), we can write

$$\mathbf{C} \times \mathbf{D} = -16\,\hat{\mathbf{k}} - 30\,\hat{\mathbf{k}} = -46\,\hat{\mathbf{k}}$$

So, the cross product of C and D is a vector of magnitude 46 and in the negative z direction. Since C and D are in the xy-plane, it is obvious that the cross product must be perpendicular to this plane, that is, it must be in the z-direction.

3.2 POSITION VECTOR AND DISPLACEMENT

Consider a body located at point P in xy plane Fig. 3.11. Then the position of the body with reference to the origin 'O' in xy plane is given by its position vector **r**, which can be written as

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

where x, y are the components of **r** along x and y-axis.

Now consider a body moving along the curve. Let the body is at P at time t and P' at time t' whose coordinates are (x, y) and (x', y') as shown in Fig. 3.12. Then the displacement is

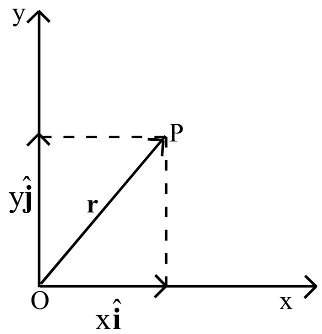


Fig. 3.11 : Position vector **r**

$\Delta\mathbf{r} = \mathbf{r}^1 - \mathbf{r}$ and is directed from P to P'.

$$\Delta\mathbf{r} = (x'\hat{\mathbf{i}} + y'\hat{\mathbf{j}}) - (x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$$

$$= \Delta x\,\hat{\mathbf{i}} + \Delta y\,\hat{\mathbf{j}} \text{ where}$$

$$\Delta x = x' - x, \quad \Delta y = y' - y.$$

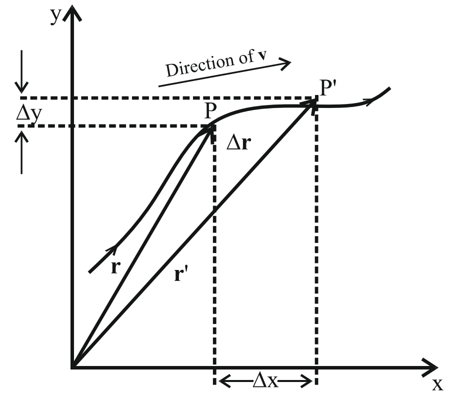


Fig. 3.12 : Displacement $\Delta\mathbf{r}$ and average velocity $\mathbf{v}$

3.2.1 Velocity and Acceleration

The average velocity ($\mathbf{v}$) of an object is the ratio of displacement and the corresponding time interval.

$$\mathbf{v} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{\Delta x\,\hat{\mathbf{i}} + \Delta y\,\hat{\mathbf{j}}}{\Delta t} = \hat{\mathbf{i}}\frac{\Delta x}{\Delta t} + \hat{\mathbf{j}}\frac{\Delta y}{\Delta t}$$

$= v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$ where v_x, v_y are the scalar components of velocity $\mathbf{v}$ and represent speeds of the body along x- and y-axis respectively. The direction of average velocity is same as that of displacement $\Delta\mathbf{r}$ as shown in Fig. 3.12.

Similarly the acceleration of body is given by

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta(v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}})}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta v_y}{\Delta t}\hat{\mathbf{j}}$$

$= a_x\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ where a_x, a_y are the accelerations of the body along x and y directions respectively.

3.2.2 Projectile Motion

Projectile is defined as a body projected into the air at an angle other than 90° with the horizontal near the surface of the earth. The first breakthrough in the description of projectile motion was made by Galileo. He showed that the horizontal and vertical motions of projectile are independent. This can be understood by doing the following activity.

Take two cricket balls. Project one of them horizontally from the top of building. At the same time drop the other ball downward from the same height. It is observed that both the balls hit the ground at the same time. This shows that the downward acceleration of a projectile is same as that of freely falling body. Moreover, this takes place

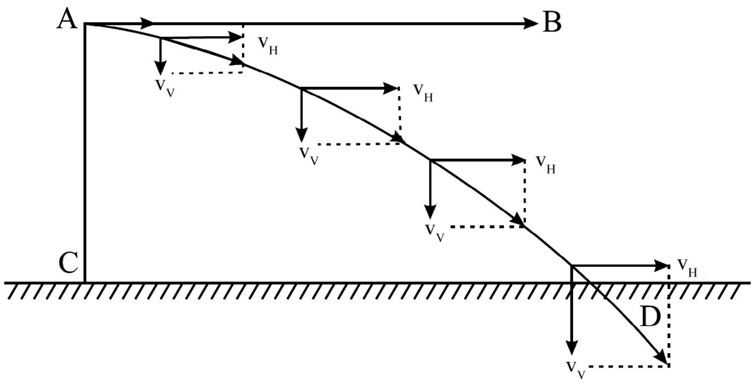


Fig. 3.13 : Curved path of projectile

independent of its horizontal motion. Further, measurement of time and distance will show that the horizontal velocity continues unchanged and takes place independent of the vertical motion.

In other words, the two important properties of projectile motion are

(i) a constant horizontal velocity component

(ii) a constant vertically downward acceleration component.

The combination of these two motions results in the curved path of the projectile.

3.2.3 Trajectory or path of a projectile

The path followed by a projectile is called its trajectory. Now we will derive the equation for the path of a projectile. Let us assume that the air resistance is neglected. Suppose that a body is projected with velocity **u**, such that it makes an angle θ with the x-axis as shown in Fig. 3.14. The components of velocity **u** along x- and y- directions are

$$u_x = u\cos\theta \qquad (3.16\ [a])$$

$$u_y = u\sin\theta \qquad (3.16\ [b])$$

We know that the projectile motion can be thought of as the result of two independent simultaneously occurring straight line motions. They are (i) horizontal motion with zero acceleration and (ii) vertical motion with constant downward acceleration.

First let us consider the horizontal motion. As the horizontal motion has zero acceleration, the horizontal component of velocity u_x remains constant throughout the motion. The horizontal displacement of the projectile after time 't' from initial position is given by

$$x = u_x\,t = (u\cos\theta)\,t \qquad (3.17)$$

and $a_x = 0$

Now consider the vertical motion.

In vertical direction, the acceleration of the projectile, **a** is equal to the acceleration of freely falling body which is constant and is always acting downwards.

i.e., $a_y = -g$

The component of velocity at any time 't' is obtained by the equation

$$v_y = u_y - gt$$

using equation (3.16 [b]), we get

$$v_y = u\sin\theta - gt \qquad (3.18)$$

The equation for vertical displacement of the projectile after time 't' is given by

$$y = u_y\,t + \frac{1}{2}a_y\,t^2$$

Using equation (3.16 [b]), we get

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \qquad (\because\ a_y = g) \qquad (3.19)$$

substituting the value of 't' from equation (3.17) in equation (3.19), we get

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$y = (\tan\theta)\,x - \frac{gx^2}{2u^2\cos^2\theta} \qquad (3.20)$$

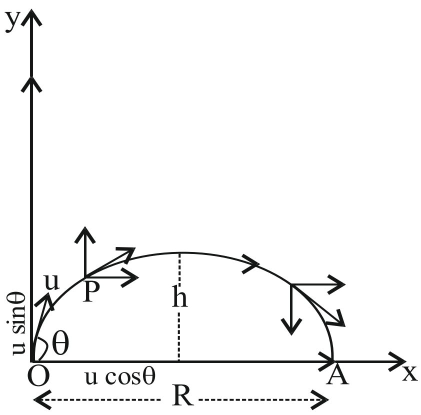


Fig. 3.14 : Path of projectile

As θ, g, u are constants, we can say that the equation (3.20) is of the form $y = ax + bx^2$ which is the equation of a parabola. Thus if air resistance is negligible, the path of projectile is parabola.

Now we would like to determine how high and how far does a projectile go and for how long does it remain in air.

3.2.4 Maximum height of a projectile

As projectile travels through air, it climbs up to some maximum height (h_{max}) and then begins to come down. The height of the projectile at which the vertical component of its velocity becomes zero is called maximum height of projectile. At this instant, the projectile stops to move upward. Thus when the projectile is at the maximum height, we have $v_y = 0$, $u_y = u \sin \theta$ and $a = -g$.

So from equation (3.18) we get

$$0 = u \sin\theta - gt$$

$$t = \frac{u\sin\theta}{g} \qquad (3.21)$$

The above equation gives the time taken by the projectile to reach maximum height.

using the equation $v^2 - u^2 = 2as$ we get,

$$0 - (u \sin \theta)^2 = -2gh_{max}$$

$$h_{max} = \frac{u^2\sin^2\theta}{2g} \qquad (3.22)$$

3.2.5 Time of flight

The time of flight of a projectile is the time interval between the point of its launch and the instant when it hits the ground.. This can be obtained by substituting $y = 0$ and $t = T$ in equation (3.19).

i.e., $$0 = (u \sin\theta)\,T - \frac{1}{2}gT^2$$

$$\frac{1}{2}gT^2 = u\sin\theta\, T$$

$$T = \frac{2u\,\sin\theta}{g} \qquad (3.23)$$

comparing equations (3.21) and (3.23), we get

$$T = 2t$$

i.e., the time 't' given by equation (3.21) is the time for half the flight of the projectile.

3.2.6 Range

Let us calculate the horizontal distance travelled by the projectile. The horizontal distance travelled by the projectile during the time of flight T is called range of projectile. It is denoted by R.

$$R = \text{horizontal velocity} \times \text{time of flight}$$

$$= u_x \times T$$

Using equations (3.17) and (3.23), we get

$$R = \frac{u\cos\theta\,(2u\sin\theta)}{g}$$

$$= \frac{2u^2\cos\theta\sin\theta}{g}$$

But $\quad 2\sin\theta\cos\theta = \sin 2\theta$

Hence range, $\quad R = \dfrac{u^2\sin 2\theta}{g} \qquad (3.24)$

From the above equation we see that the range of projectile depends on initial velocity u and its direction given by θ. For a given value of projection velocity u, range R is maximum when $\sin 2\theta = 1$ or $2\theta = 90°$ i.e., $\theta = 45°$.

Thus for R to be maximum at a given speed u, θ should be equal to 45°.

Example 3.2

A ball is thrown with the velocity $u = 10\ ms^{-1}$ at an angle of 30° with the horizontal. Find (i) the maximum height reached by the ball, (ii) the horizontal distance from the point of projection to the point where it hits the ground and (iii) the time during which the ball will be in motion. (neglect the air resistance and take $g = 10\ ms^{-2}$)

Solution :

Given

$u = 10\ ms^{-1}$, $\theta = 30°$ and $g = 10\ ms^{-2}$

(i) $$h_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(10)^2 \sin^2 30}{2\times10} = \frac{10}{2} \times \frac{3}{4} = 3.75\,\text{m}$$

(ii) $$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(10)^2 \sin(2 \times 30)}{10}$$

$$= 10 \times \sin 60 \;\; = 10 \times \frac{1}{2} = 5\,\text{m}$$

(iii) $$T = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \sin 30}{10}$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} = 1.7\,\text{s}$$

Intext Questions 3.3

1. Identify examples of projectile motion from among the following situations:
   (a) An archer shoots an arrow at a target
   (b) Rocks are ejected from an exploding volcano
   (c) A truck moves on a mountainous road
   (d) A bomb is released from a bomber plane. [**Hint:** Remember that at the time of release the bomb shares the horizontal motion of the plane.]
   (e) A boat sails in a river.
2. An athlete set the record for the long jump with a jump of 8.90 m. Assume his initial speed on take-off to be 9.5 ms^{-1}. How close did he come to the maximum possible range in the absence of air resistance? Take g = 9.78 ms^{-2}
3. A body is projected vertically upwards. Can its velocity at any point on the trajectory be zero?

So far we have studied motion of objects in a plane, which can be placed in the category of projectile motion. In projectile motion, the acceleration is constant both in magnitude and direction. There is another kind of two-dimensional motion in which acceleration is constant in magnitude but not in direction. Uniform circular motion is one such kind of motion and let us discuss in the following section.

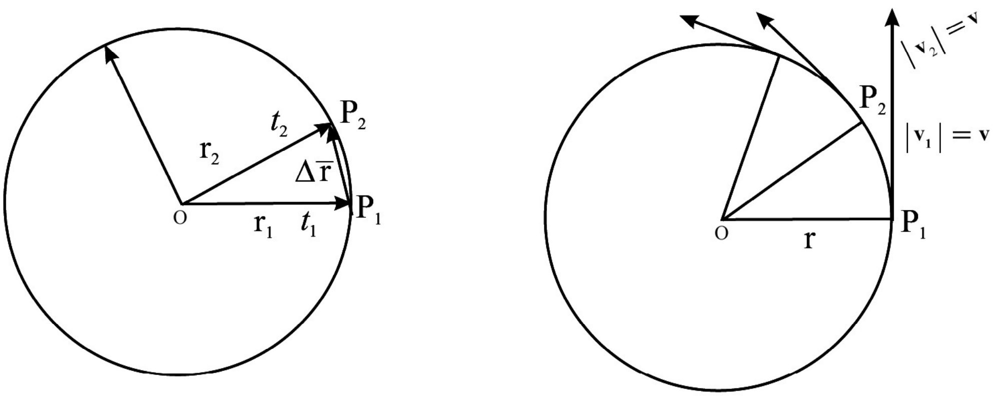
Evangelista Torricelli (1608 – 1647)

Italian mathematician and a student of Galelio Galili, he invented mercury barometer, investigated theory of projectiles, improved telescope and invented a primitive microscope. Disproved that nature abhors vacuum, presented torricellis theorem.

3.3 CIRCULAR MOTION

Consider the Fig. 3.15 (a) shown below. It shows the position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ of a particle in uniform circular motion at two different instants of time t_1 and t_2 respectively. The word uniform refers to constant speed. We have said that the speed of the particle is constant. Let us find the velocity. Recalling the definition of average velocity and applying it to points P_1 and P_2

$$\mathbf{v}_{avg} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} = \frac{\Delta \mathbf{r}}{\Delta t} \tag{3.25}$$



(a) (b)

Fig. 3.15 : (a) Positions of a particle in uniform circular motion
(b) Uniform circular motion

The motion of a grinding wheel at constant speed, the moving hands of an ordinary clock, a vehicle turning around a corner are examples of circular motion.

The simplest kind of circular motion is uniform circular motion. A point on a rotating fan blade, a satellite revolving round the earth in a circular orbit performs uniform circular motion. Let us study about uniform circular motion.

3.3.1 Uniform Circular Motion

When a body moves along the circumference of circle with constant speed, the motion of body is called uniform circular motion. The vector $\Delta\mathbf{r}$ is shown in Fig. 3.15 (a). Now let us suppose that the time interval Δt is made smaller and smaller such that it approaches zero. What happens to $\Delta\mathbf{r}$. In particular, what is the direction of $\Delta\mathbf{r}$. It approaches the tangent to the circle at point P_1 as Δt tends to zero. We define the instantaneous velocity at point P_1 as

$$v = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

Thus in uniform circular motion, the velocity vector changes continuously. This is because the direction of velocity is not constant. It goes on changing continuously as the particle travels around the circle as shown in Fig. 3.15(b). Hence velocity changes and uniform circular motion is accelerated motion. The acceleration of a particle in uniform circular motion is termed as centripetal acceleration. Let us discuss about centripetal acceleration in detail.

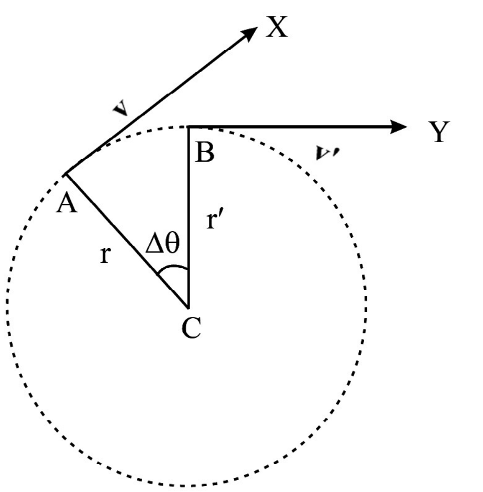
3.3.2 Centripetal acceleration

Consider a particle of mass m moving on the circumference of a circle with uniform speed v. At any instant of time let its position is at A and its motion is directed along AX. After a small time Δt, the position of particle is at B and its velocity is represented by the tangent at B directed along BY.

Let $\mathbf{r}$ and $\mathbf{r}'$ be the position vectors and $\mathbf{v}$ and $\mathbf{v}'$ be the velocities of the particle at A and B respectively as shown in Fig. 3.16 (a). The change in velocity $\Delta\mathbf{v}$ is obtained using the triangle law of vectors. As the path of the particle is circular and velocity is along its tangent, $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\Delta\mathbf{v}$ is perpendicular to $\Delta\mathbf{r}$. As the average acceleration $\mathbf{a}$ is along $\Delta\mathbf{v}$, we can say that average acceleration is perpendicular to $\Delta\mathbf{r}$.

Let $\Delta\theta$ be the angle between the position vectors $\mathbf{r}$ and $\mathbf{r}'$. Then the angle between velocity vectors $\mathbf{v}$ and $\mathbf{v}'$ will also be $\Delta\theta$ as velocity vectors are always perpendicular to the position vectors.

To determine change in velocity $\Delta\mathbf{v}$, consider a point O outside the circle. Draw a line OP parallel and equal to AX (or $\mathbf{v}$) and a line OQ parallel and equal to BY (or $\mathbf{v}'$). As magnitudes of $\mathbf{v}$ and $\mathbf{v}'$ are equal, we have OP = OQ. Join PQ. We get a triangle OPQ as shown in Fig. 3.16 (b).



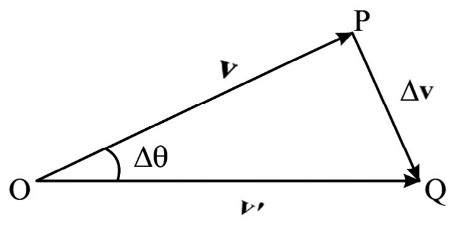


Fig. 3.16 : Uniform circular motion

The sides OP and OQ of triangle OPQ represent velocity vectors **v** and **v'** at A and B respectively. Hence their difference is represented by the side PQ in magnitude and direction. In other words the change in velocity equal to PQ in magnitude and direction takes place as the particle moves from A to B in time Δt.

Acceleration = Rate of change of velocity

$$\mathbf{a} = \frac{PQ}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t}$$

As Δt is very small AB is also very small and is nearly a straight line. Then Δ ACB and Δ POQ are isosceles triangles having their included angles equal. Therefore the triangles are similar and hence,

$$\frac{PQ}{AB} = \frac{OP}{CA}$$

$$\frac{\Delta v}{r \cdot \Delta\theta} = \frac{v}{r}$$

(as magnitudes of velocity vectors **v** and **v'** = v (say))

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

But $\frac{\Delta v}{\Delta t}$ is the acceleration of the particle. Hence

Centripetal acceleration, $a = \frac{v^2}{r}$ (3.26)

The resultant external force that causes centripetal acceleration is called centripetal force. It is given by

$$\mathbf{F} = m\mathbf{a} = \frac{mv^2}{r} = mr\omega^2 \qquad (v = r\omega)$$

The direction of the centripetal force is in the direction of centripetal acceleration i.e., it is directed along the radius towards the centre. From definition, it is seen that some minimum centripetal force has to be applied on a body to make it move in a circular path. In the absence of such a force, the body will move in a straight line path.

Some applications of centripetal force

(i) **Centrifuge :** This is a spinning device used for separating materials having different densities. When a mixture of two materials of different densities placed in a vessel is rotated at high speed, the centripetal force on the heavier material will be more. Therefore, it will move to outermost position in the vessel and hence can be separated.

These devices are being used for uranium enrichment.

(ii) Mud clings to an automobile tyre until the speed becomes too high and then it flies off tangentially.

(iii) Planetary motion : The earth and other planets revolving round the sun get necessary centripetal force from the gravitational force between them and the sun.

(iv) When a ball on the end of the string is rotated about an axis the centripetal force required is provided by the tension on the string.

Example 3.3

A stone of mass 2.0 kg is tied to the end of a string of length 2 m length. It is whirled in a horizontal circle with a velocity of 20 ms^{-1}. Calculate the centripetal force.

Solution :

Given

mass of stone, m = 2 kg

length of string = radius of circle, r = 2 m

velocity, v = 20 ms^{-1}

$$\text{centripetal force, } F = \frac{mv^2}{r}$$

$$= \frac{2 \times 20 \times 20}{2}$$

= 400 N

Example 3.4

Astronauts experience high acceleration in their flights in space. In the training centres for such situations, they are placed in a closed capsule, which is fixed at the end of a revolving arm of radius 15 m. The capsule is whirled around in a circular path, just like the way we whirl a stone tied to a string in a horizontal circle. If the arm revolves at a rate of 24 revolutions per minute, calculate the centripetal acceleration of the capsule.

Solution :

The circumference of the circular path is $2\pi \times$ (radius) = 2π x 15 m. Since the capsule makes 24 revolutions per minute or 60 s, the time taken by it to go once round this circumference is $\frac{60}{24}$ s.

$$\text{Speed of the capsule, } v = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 15 \times 24}{60} = 38\,ms^{-1}$$

The magnitude of centripetal acceleration

$$a = \frac{v^2}{r} = \frac{(38\,ms^{-1})^2}{15\,m} = 96\,ms^{-2}$$

Intext Questions 3.4

1. In uniform circular motion, (a) Is the speed constant? (b) Is the velocity constant? (c) Is the magnitude of the acceleration constant? (d) Is acceleration constant? Explain.
2. An athlete runs around a circular track with a speed of 9.0 ms^{-1} and a centripetal acceleration of 3 ms^{-2}. What is the radius of the track?

WHAT YOU HAVE LEARNT

- In physics, we deal with two kinds of quantities, scalars and vectors. A scalar has only magnitude. A vector has both direction and magnitude.
- Vectors are added according to the parallelogram rule.
- The scalar product of two vectors is a scalar.
- The vector product of two vectors is a vector perpendicular to the plane containing two vectors.
- Vectors can be resolved into components along a specified set of coordinates axes.
- Projectile motion is defined as the motion which has constant velocity in a certain direction and constant acceleration in a direction perpendicular to that velocity.

$$a_x = 0 \qquad a_y = -g$$

$$v_x = u_x = u\cos\theta \qquad v_y = u\sin\theta - gt$$

$$x = x_0 + (u\cos\theta)\,t \qquad y = y_0 + (u\sin\theta) - \frac{1}{2}gt^2$$

- The trajectory of a projectile is parabola.
- Height reached by projectile, $h = \dfrac{u^2\sin^2\theta}{2g}$.
- Time of flight, $T = \dfrac{2u\sin\theta}{g}$.
- Range of projectile, $R = \dfrac{u^2\sin 2\theta}{g}$.
- Circular motion is uniform when the speed of the particle is constant. A particle undergoing uniform circular motion in a circle of radius r at constant speed v has

a centripetal acceleration given by

$$a = -\frac{v^2}{r}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector directed from the centre of the circle to the particle. The speed v of the particle is related to its angular speed ω by v = rω.

- The centripetal force acting on the particle is given by

$$F = ma = \frac{mv^2}{r}\hat{\mathbf{r}} = m r \omega^2$$

TERMINAL EXERCISE

1. Define unit vector and position vector.
2. State and explain parallelogram law of vectors. Deduce expression for the magnitude and direction of the resultant vector.
3. State triangle law of forces.
4. Define scalar product and vector product of two vectors.
5. A force $6\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$ produces a displacement of $2\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. Find the work done.
6. Two vectors are given by $5\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$. Calculate their scalar and vector products.
7. What is a projectile. Show that the path of a projectile is parabola.
8. Obtain expressions for the time of flight and the maximum height for a projectile.
9. Derive an equation for the trajectory of a projectile
10. A string can sustain a maximum force of 100 N without breaking. A mass of 1 kg is tied to one end of the piece of string of 1 m long and it is rotated in a horizontal plane. Compute the maximum speed with which the body can be rotated without breaking the string?
11. A motorcyclist passes a curve of radius 50 m with a speed of 10 ms^{-1}. What will be the centripetal acceleration when turning the curve?
12. A bullet is fired with an initial velocity 300 ms^{-1} at an angle of 300 with the horizontal. At what distance from the gun will the bullet strike the ground?
13. A shell is fired at an angle of elevation of 30° with a velocity of 500 ms^{-1}. Calculate the vertical and horizontal components of the velocity, the maximum height that the shell reaches, and its range.
14. An aero plane drops a food packet from a height of 2000 m above the ground while in horizontal flight at a constant speed of 200 kmh^{-1}. How long does the packet take

to fall to the ground? How far ahead (horizontally) of the point of release does the packet land?

15. A car is rounding a curve of radius 200 m at a speed of 60 kmh^{-1}. What is the centripetal force on a passenger of mass m = 90 kg?

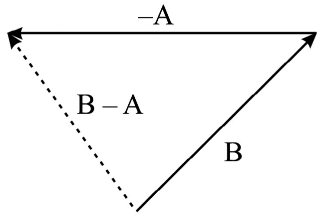
16. Define uniform circular motion. Give a few examples.

17. Derive an expression for centripetal acceleration.

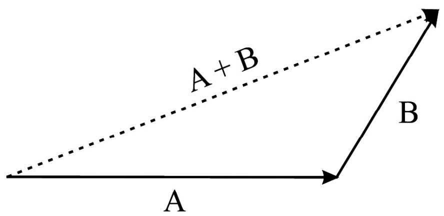
ANSWERS TO INTEXT QUESTIONS

3.1

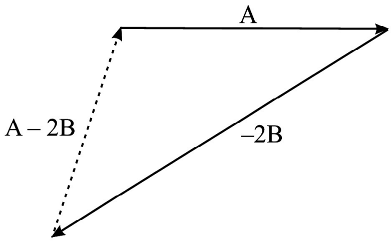
1. (a)



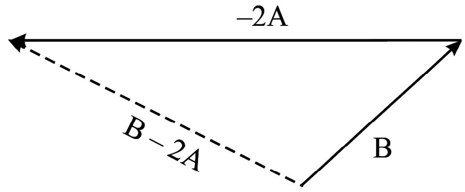
(b)



(c)



(d)

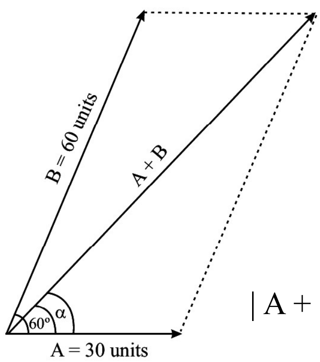


2.

A + **B** = 10 + (–12) = 2 units

A – **B** = 10 – (–12) = 22 units

3.



| A + B | = 79 units

3.2

1. If **A** and **B** are parallel, the angle θ between them is zero. So, their cross product **A** × **B** = AB sin θ = 0.

   If they are antiparallel then the angle between them is 180°. Therefore,

   A × **B** = AB sin θ = 0, because sin 180°= 0.

2. If magnitude of **B** is halved, but it remains in the same plane as before, then the direction of the vector product **C** = **A** × **B** remains unchanged. Its magnitude may change.

3. Since vectors **A** and **B** rotate without change in the plane containing them, the direction of **C** = **A** × **B** will not change.

4. Suppose initially the angle between **A** and **B** is between zero and 180°. Then **C** = **A** × **B** will be directed upward perpendicular to the plane. After rotation through arbitrary amounts, if the angle between them becomes > 180°, then **C** will drop underneath but perpendicular to the plane.

5. If **A** is along x-axis and **B** is along y-axis, then they are both in the xy plane. The vector product **C** = **A** × **B** will be along z-direction. If **A** is along y-axis and **B** is along x-axis, then **C** is along the negative z-axis.

6. (a) **A.B** = | **A** | | **B** | cos θ = 0 when θ = 90°

   (b) **A** × **B** = | **A** | | **B** | sin θ = | **A** | | **B** |, as sin θ = 1 when θ = 90°

3.3

1. (a), (b), (d)
2. Maximum range : 9.23 m, Difference is : 9.23 m – 8.90 m = 0.33 m
3. At the maximum height, the velocity is zero on its trajectory.

3.4

1. (a) Yes (b) No (c) Yes (d) No

   The velocity and acceleration are not constant because their directions are changing continuously.

2. 27 m

ANSWERS TO TERMINAL EXERCISE

5. 124
6. Scalar product : 30, Cross product: magnitude 16 along negative z-direction
10. 10 ms^{-1}
11. 2 ms^{-2}
12. 900 × 1.732 m
13. u_x = 250 × 1.732 ms^{-1} v_y = 250 ms^{-1} Max.height : 500 m Range : 3125 m
14. t = 20 s, 999.9 m
15. 125 N

4

NEWTON'S LAWS OF MOTION

INTRODUCTION

In the previous lesson you learnt to describe the motion of an object in terms of its distance, displacement, speed, velocity and acceleration. But an important questions is : What makes a ball to move? The general answer that you say is hit the ball with a bat. The other question: What causes a moving ball in air to come to a stop, while you catch it? In this case, you would say, hold it tightly. From our everyday experiences, we come to a conclusion that, we need to push or pull an object for a change in its state of motion or rest. Similarly, a football requires a hard kick so that it reaches the goal post. A cricket ball has to be hit hard by a batter to send it across the boundary for a four or six. In all these situations muscular activity is involved and its effect is quite visible.

There are, however, many situations where the cuase behind an action is not visible. For example (1) the cause behind the falling of rain drops (2) the cause behind the revolving of Earth around the Sun.

In this lesson you will learn the basic laws of motion and discover that the forces cause motion. Newton showed that force and motion are intimately connected. These laws are fundamental and enable us to understand everyday phenomena connected to rest and motion.

OBJECTIVES

After studying this lesson, you should be able to

- *explain the significance of Force in the motion of an object;*
- *state Newton's laws of motion and illustrate them with examples;*
- *explain the significance of Inertia;*
- *explain the significance of friction in our daily activities;*
- *define co-efficient of friction and be in a position to differentiate one type of co-efficient of friction form other type;*
- *suggest different methods of reducing friction;*
- *analyse a given situation and apply Newton's laws of motion using a free body diagram.*

4.1 CONCEPT OF FORCE AND INERTIA

We all know that stationary objects remain wherever they are placed. These objects can not move on their own from one place to another place unless forced to change their state of rest. Similarly, an object moving with constant velocity has to be forced to change its state of motion.

"The property of an object by virtue of which it opposes a change in its state of rest or of uniform motion along a straight line is called inertia." Inertia depends only on the mass of the object and it can only be measured interms of mass.

In a way, inertia is a fantastic property. If it were not present, your books or classnotes could mingle with those of your younger brother or sister. Your wardrobe could move to your friend's house creating chaos. You must however recall that the state of rest or of uniform motion of an object is not absolute. In the previous lesson you have learnt that an object at rest with respect to one observer may appear to be in motion with respect to some other observer. Observations show that the change in velocity of an object can only be brought, if a net force acts on it.

You are very familiar with the term force. We use it in so many situations in our daily life. We are exerting (applying) force when we are pulling, pushing, kicking, hitting etc., Though a force is not visible, its effect can be seen or experienced. Forces are known to have different kinds of effects.

(a) They may change the shape and size of an object. A ballon changes shape depending on the magnitude of force acting on it.

(b) Forces also influence the motion of an object. A force can set an object into motion or it can bring a moving object to rest. A force can also change the direction or speed of moving object.

(c) Forces acting at a distance from the fulcrum (or) axis of rotation of the object, can make the object to under go rotatory motion. We will learn about it in lesson seven.

4.1.1 Force and Motion

Force is a vector quantity. For this reason, when several forces act on a body simultaneously, a net equivalent force can be caluclated by vector addition, as discussed in previous lesson.

Motion of a body is charcterised by its displacement, velocity etc. We come across many situations where the velocity of an object is either continuously increasing or decreasing. For example, in the case of a body falling freely, the velocity of the body increases continuously, till it hits the ground. Similarly, in the case of a ball rolling on a horizontal surface, the velocity of the ball decreases continuously and ultimately becomes zero (Why?)

From experience we know that a net non zero force is required to change the state of a body. The resultant of many forces, acting on a body, is called net force. For a body (1) in motion, the velocity changes depending on the magnitude and the direction of net force. (2) in rest, gets into state of motion depending on the force and its direction.

Both magnitude and direction of force are equally responsible for the chagne Viz.. rest or motion. If the direction of the net force is opposite to the direction of motion, the velocity (speed) of the object decreases. However, if a net force acts on a body in a direction perpendicular to its velocity, the magnitude of velocity (speed) of the body remains constant (see 4.3). Such a force changes only the direction of velocity of the body. We may conclude that velocity of a body changes as long as a net force is acting on it.

4.1.2 First Law of Motion

When we roll a marble on a smooth floor, it stops after some time. It is obvious that its velocity decreases and ultimately it becomes zero. However, if we want it to move continuously with the same velocity, a force will have to be applied, constantly, on it.

We also see that in order to move a trolley at constant velocity, it has to be continuously pushed or pulled. Is there any net force acting on the marble or trolley in the situations mentioned here?

Motion and Inertia

Galileo carried out experiments to prove that in the absence of any external force, a body would continue to be in its state of rest or of uniform motion in a straight line. He observed that a body is accelerated while moving down an inclined plane (Fig. 4.1 [a]) and is retarded while moving up an inclined plane (Fig. 4.1 [b]). He argued that if the plane is neither inclined upwards nor downwards (i.e. if it is a horizontal plane surface), the motion of the body will neither be accelerated nor retarded (velocity neither increases nor decreases). That is on a horizontal plane surface, a body will move with a uniform speed/velocity (if there is no external force). (Fig. 4.1 [c])

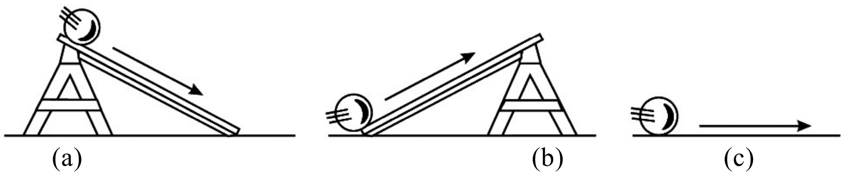


Fig. 4.1: Motion of a body on inclined and horizontal planes

In another thought experiment, he considered two inclined planes facing each other, as shown in Fig. 4.2. The inclination of the plane PQ is same in all the three cases, where the inclination of the plane RS in Fig. 4.2 (a) is more than that in (b) and (c). The planes PQ and RS one very smooth and the ball is of marble. When the ball is allowed to roll down the plane PQ, it rises to nearly the same height on the face RS. As the inclination of the plane RS decreases, the ball moves a longer distance to rise to the same height on the inclined plane (Fig. 4.2 [b]). When the plane RS becomes horizontal, the ball keeps moving to attain the same height as on the plane PQ, i.e., on a horizontal plane, the ball will keep moving if there is no friction between the plane and the ball. (Fig. 4.2 [c])

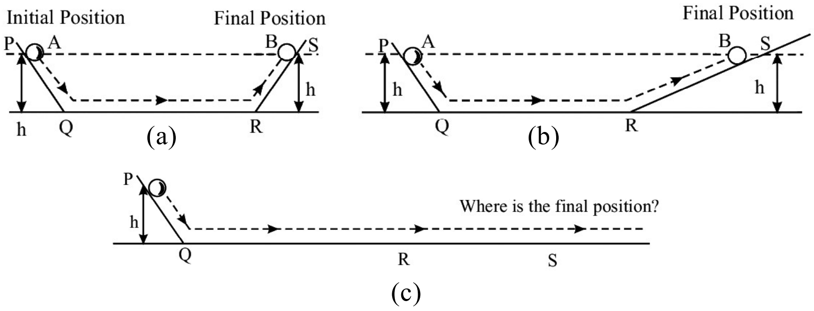
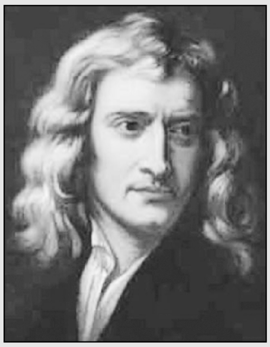


Fig 4.2 Motion of a ball along planes inclined to each other

Sir Isaac Newton (1642-1727)



Newton was born at Wollsthorpe in England in 1642. He studied at Trinity College, Cambridge and became the most profound scientist. The observation of an apple falling towards the ground helped him to formulate the basic law of gravitation. He enunciated the laws of motion and the law of gravitation. Newton was a genius and contributed significantly in all fields of science, including mathematics. His contributions are of a classical nature and form the basis of the modern science. He wrote his book **"Principia"** in Latin and his book on optics was written in English.

You may logically ask : Why is it necessary to apply a force continuously to the trolley to keep it moving uniformly? We know that a forward force on the cart is needed for balancing out the force of friction on the cart. That is, the force of friction on the trolley can be overcome by continuously pushing or pulling it.

Isaac Newton generalized Galileo's Conclusion in the form of a law known as Newton's first law of motion, which states that a body continues to be in a state of rest or in uniform motion in a straight line unless it is acted upon by a net external force.

As you know, the state of rest or motion of a body depends on its relative position with respect to an observer. A person in a running car is at rest with respect to another person in the same car. But the same person in motion with respect to a person standing on the road. For this reason, it is necessary to record measurements of changes in position, velocity, acceleration and force with respect to a chosen frame of reference.

"Reference frame is an imaginary co-ordinate system (X, Y & Z axes) attached to a rigid system."

A reference frame relative to which a body in translatory motion has constant velocity, if no net external force acts on it, is known as inertial frame of reference. The nomenclature follows from the property of "**Inertia**" of bodies due to which they tend to preserve their state (of rest or of uniform linear motion). A reference frame fixed to the Earth (for all practical purposes) is considered as an inertial frame of reference.

Accelerating and rotating objects are attached with reference frames called as "non-inertial frame of reference."

Now you may like to take a break and answer the following questions.

Intext Questions 4.1

1. Is it correct to state that a body always moves in the direction of the net external force acting on it?
2. What physical quantity is a measure of the Inertia of a body?

3. Can a force change only the direction of velocity of an object keeping its magnitude constant.
4. State the different types of changes which a force can bring in a body when applied on it.
5. Is rest or motion absolute or relative?
6. What is a frame of reference?

4.2 CONCEPT OF MOMENTUM

- You must have observed Sachin Tendulkar, a great batter of India liked to use a cricket bat of more mass, compared to the one others use, in general. You also observe in cricket, a fielder finds it difficult to stop a cricket ball moving with a large velocity although its mass is small. Similarly, it is difficult to stop a truck moving with a small velocity because its mass is larger. These examples suggest that both mass and velocity of a body, are important when we study the effect of force on the motion of the body.
- Here, we introduce a new physical quantity momentum ($\overline{p}$).
- The product of mass (m) and velocity (v) of a body is defined as (linear) momentum.

$$\overline{p} = m\overline{v}$$

- It is a vector quantity. Its units are kg m/s or N s

  mass is a scalar, so the direction of $\overline{p}$ is nothing but the direction of $\overline{v}$

Example 4.1

Arun has a mass of 60 kg and travels with velocity 1.0 m/s towards Manoj whose mass is 40 kg, and is moving with 1.5 m/s towards Arun. Calculate their momenta. (Plural of momentum)

Solution :

For Arun, momentum = mass × velocity

$$= 60\ kg \times 1.0\ ms^{-1}$$

$$= 60\ kg\,ms^{-1}$$

For Manoj, momentum $= 40\,kg \times \left(-1.5\,ms^{-1}\right)$

$$= -60\,kg\,ms^{-1}$$

–Ve sign, indicates opposite direction, in Physics.

Momenta of Arun & Manoj have the same magnitude (value), but they are in opposite directions.

Example 4.2

A 2 Kg object is allowed to fall freely at t = 0 s. Calculate its momentum at (a) t = 0 (b) t = 1 s and (c) t = 2 s during its free fall.

Solution :

(a) At t = 0, the object is ready to move, but not moving, hence velocity is zero.

Therefore momentum is also zero.

(b) At t = 1s, velocity of the object can be determined by using $v = v_0 + at$

where $v_0 = 0$; $a = g = 9.8\ m/s^2$

$$\therefore \quad v = 0 + 9.8 \times 1 = 9.8\,m/s$$

So, momentum at t = 1s is p = mv

$$p_1 = 2\,kg \times 9.8\,m/s = 19.6\ kg\,ms^{-1}$$

(c) At t = 2s, the velocity of the object will be 19.6 ms^{-1}.

So, the momentum p_2 = (2 kg) (19.6 ms^{-1})

= 39.2 kg ms^{-1}

Thus the momentum of a freely falling object increases, continuously in magnitude and points in the same direction (pointing downwards). Now think what causes the momentum of a freely-falling body to change in magnitude?

Example 4.3

A rubber ball of mass 0.2 kg strikes a rigid wall with a speed of 10 ms^{-1} and rebounds along the original path with the same speed. Calculate the change in the momentum of the ball.

Solution :

Here the momentum of the ball has the same magnitude before and after the impact, but, there is a reversal in its direction. In each case the magnitude of momentum is (0.2 kg) × (10 ms^{-1}) i.e. 2 kg ms^{-1}.

If we choose initial momentum vector to be along +x axis, the final momentum vector will be along –x axis. So p_i = 2 $kgms^{-1}$, p_f = –2 kg ms^{-1}. Therefore the change in momentum of the ball $p_f - p_i$ = (–2 kg ms^{-1}) – (2 kg ms^{-1}) = –4 kg ms^{-1}.

Here negative sign shows that the momentum of the ball changes by 4 kg ms^{-1} in the direction of –x axis. What causes this change in the momentum of the ball?

In actual practice, a rubber ball rebounds from a rigid wall with a speed less than its speed before the impact. In such a case also, the magnitude of the momentum will change.

4.3 SECOND LAW OF MOTION

You now know that a body moving at constant velocity will have constant momentum. Newton's First law of motion suggests that no net external force acts on such a body.

In example 4.2, we have seen that the momentum of a ball falling freely under gravity increases with time. Since such a body falls under the action of gravitational force acting on it, there appears to be a connection between change in momentum of an object, net force acting on it, and the time for which it is acting. Newton's second law of motion gives a quantative relation between these three physical quantities. It states that the rate of change of momentum of a body is directly proportional to the net force acting on the body. Change in momentum of the body takes place in the direction of net external force acting on the body.

This mean that if $\Delta\overline{p}$ is the change in momentum of a body in time Δt due to a net external force $\overline{F}$, we can write

$$\overline{F} \propto \frac{\Delta \overline{p}}{\Delta t} \left(\text{if } \Delta t \to 0, \frac{\Delta p}{\Delta t} = \frac{d\overline{p}}{dt} \right) \tag{4.1}$$

$$\overline{F} = k\frac{\Delta \overline{p}}{\Delta t}$$ where k is a constant of proportionality.

By expressing momentum as a product of mass and velocity, we can rewrite this result as

$$\overline{F} = k\,m\left(\frac{\Delta \overline{v}}{\Delta t}\right)$$

$$\overline{F} = k\,m\,\vec{a}\left(\because \frac{\Delta \overline{v}}{\Delta t} = \overline{a} \right) \tag{4.2}$$

The value of the constant k depends upon the units of m and $\overline{a}$. If these units are chosen such that when the magnitude of m = 1 unit and a = 1 unit the magnitude of $\overline{F}$ is also be 1 unit. Then we can write

$$\overline{F} = m\overline{a}. \tag{4.3}$$

In SI units m = 1 kg, a = 1 m/s^2. Then magnitude of external force

$$F = 1 \text{ kg} \times 1 \text{ ms}^{-2} = 1 \text{ kg ms}^{-2}$$

$$= 1 \text{ unit of force}$$

The unit of force (i.e., 1 kg ms^{-2}) is called newton (N).

Note, the second law of motion gives us a unit for measuring force. The SI unit of force i.e., newton may thus, be defined as the force which will produce an acceleration of 1 ms^{-2} in a mass of 1 kg.

Example 4.4

A ball of mass 0.4 kg starts rolling on the ground at 20 ms^{-1} and comes to a stop after 10 s. Calculate the force which stops the ball, assuming it to be constant in magnitude throughout.

Given m = 0.4 kg, initial velocity u = 20 ms^{-1}, final velocity v = 0 ms^{-1} and t = 10 s. So,

$$\left|\bar{F}\right| = m\left|\bar{a}\right| = \frac{m(\mathrm{v}-u)}{t} = \frac{0.4\,kg\left(-20\,ms^{-1}\right)}{10}$$

$$= -0.8\,kg\,ms^{-2} = -0.8\,N$$

Here negative sign shows that force on the ball is in a direction opposite to that of its motion.

Example 4.5

A constant force of magnitude 50 N is applied to a body of 10 kg moving initially with a speed of 10 m/s. How long will it take the body to stop if the force acts in a direction opposite to its motion.

Solution :

Given m = 10 kg, F = –50 N, u = 10 ms^{-1} and v = 0, we have to calculate t

since $$\mathrm{F = ma = m}\left(\frac{\mathrm{v-u}}{\mathrm{t}}\right)$$

$$-50\,N = 10\,kg\left(\frac{0-10\,ms^{-1}}{t}\right)$$

$$t = \frac{-100\,kg\,ms^{-1}}{-50\,N} = 2s$$

It is important to note here that Newton's second law of motion, as stated here is applicable to bodies having constant mass. Will this law hold for bodies whose mass changes with time, as in the case of a launched rocket?

Intext Questions 4.2

1. Two objects of different masses have the same momentum. Which of them is moving faster?

2. A boy throws up a ball with a velocity V_0. If the ball returns to the thrower with the same velocity, will there be any change in
   (a) momentum of the ball?
   (b) magnitude of the momentum of the ball?
3. When a ball falls from a height, its momentum increases. What causes this increase in momentum?
4. In which case will there be larger change in momentum of the object?
   (a) A 150 N force acts for 0.1 s on 2 kg object initially at rest.
   (b) A 150 N force acts for 0.2 s on a 2 kg object initially at rest.
5. An object is moving at a constant speed in a circular path. Does the object have constant momentum? Give reason for your answer.

4.4 FORCES IN PAIRS

It is the gravitational pull of the earth, which allows an object to accelerate towards the Earth. Does the object also pull the earth? Similarly when we push an Almirah, does the Almirah also push us? If so, why don't we move in the direction of that force. These situations compel us to ask whether a single force such as a push or pull exists? It has been observed that actions of two bodies on each other are always mutual. Here, by action and reaction we mean "forces of interaction". So, whenever two bodies interact, they exert force on each other. One of them is called 'action' and the other is called reaction. Thus, we can say that forces always exist in pairs.

Third Law of Motion

Newton pointed out that, in every case, forces exist because two objects exert a push or pull on each other. A single object can not experience a force by itself. Forces in pairs lead to Newton's 3rd law.

To every action, there is an equal and opposite reaction.

Here by 'action' and 'reaction' we mean force. Thus, when a book placed on a table, exerts some force on the table, the latter, also exerts a force of equal magnitude on the book in the upward direction, as shown in Fig. 4.3. Do the force, $\overline{F}_1$ and $\overline{F}_2$ shown here cancel out? It is important to note that $\overline{F}_1$ and $\overline{F}_2$ are acting on different bodies and therefore, they do not cancel out.

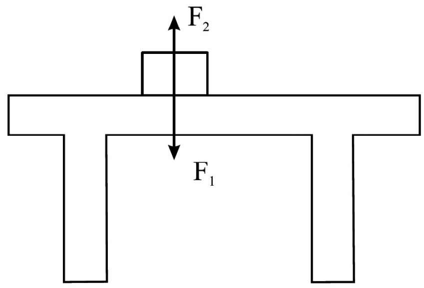


Fig 4.3 : A book placed on a table exerts a force F_1 (equal to its weigh mg) on the table, while the table exerts a force F_2 on the book.

The action and reaction in a given situation appear as a pair of forces. Any one of them can not exist without the other.

If one goes by the literal meaning of words, reaction always follows an action, whereas action and reaction introduced in Newton's 3rd Law exist simultaneously. For this reason it is better to state Newton's third law as ***when two objects interact, the forces exerted by one object on the other is equal in magnitude and opposite in direction to the force exerted by the latter object on the former.***

(or)

If an object 'A' exerts a force (action) on an object B, then object B will exert an equal but opposite force (reaction) on object A.

$$\text{Vectorially } \overline{F}_{AB} = -\overline{F}_{BA} \qquad (4.4)$$

4.4.1 Impulse

From Newton's seond law

$$\overline{F} = m\left(\frac{\overline{v}-\overline{u}}{t}\right) = \frac{m\overline{v}-m\overline{u}}{t}$$

$$\overline{F} = \frac{\overline{p}_f - \overline{p}_i}{\Delta t}$$

$$\overline{F}.\Delta t = \overline{\Delta} p = \text{impulse.} \qquad (4.5)$$

The effect of force applied for a short duration is called impulse. Impulse is defined as the product of force ($\overline{F}$) and the time duration (Δt) for which the force is applied.

Thus, impulse is equal to the change obtained in the linear momentum of the body, due to the application of external force.

Impulse is a vector quantity and its SI unit is kg ms^{-1} or (Ns).

Intext Questions 4.3

1. When a high jumper leaves the ground, where does the force which throws the jumper upwards come from?
2. Identify the action-reaction forces in each of the following situation :

   (a) A man kicks a football (b) Earth pull the Moon (c) A ball hits the wall
3. A person exerts a large force on an Almirah to push it forward, but he is not pushed backward because the Almirah exerts a similar force on him. Is the argument given here correct? Explain.

4.5 CONSERVATION OF MOMENTUM

It has been experimentally shown that if two bodies interact, the vector sum of their momenta remains unchanged, provided the force of mutual interaction is the only force acting on them. The same has been found to be true for more than two bodies interacting with each other. Generally, a number of bodies interacting with each other are said to be forming a system. If the bodies in a system do not interact with bodies outside the system, the system is said to be a closed sytem or an isolated system. "In an isolated system, the vector sum of momenta of bodies remains constant". This is called the law of conservation of momentum.

Here, it follows that the total momentum of the bodies in an isolated system remains unchanged but the momentum of individual bodies may change, in magnitude alone or direction alone or both. You may logically ask : What causes the momentum of individual bodies in an isolated system to change? It is due to mutual interaction and their strengths.

Conservation of linear momentum is applicable in a wide range of phenomena such as collisions, explosions, nuclear reactions, radioactive decay etc.

4.5.1 Conservation of Momentum as a Consequence of Newton's Laws

According to Newton's second law

$$\overline{F}_{ext} = \frac{\Delta \overline{p}}{\Delta t} \tag{4.6}$$

$$\text{if } \overline{F}_{ext} = 0, \text{ then } \frac{\Delta \overline{p}}{\Delta t} = 0 \Rightarrow \Delta \overline{p} = 0 \Rightarrow \overline{p}_{total} = \text{constant} \tag{4.7}$$

This implies that if no external force acts on the body, the change in momentum of the body will be zero. That means, the momentum of the body will remain unchanged. This argument can be extended to a system of bodies as well.

Newton's third law can also be used to arrive at the same result. Consider an isolated system of two bodies A and B which interact with each other for time Δt. If $\overline{F}_{AB}$ and $\overline{F}_{BA}$ are the forces which they exert on each other, then in accordance with Newton's third law

$$\overline{F}_{AB} = -\overline{F}_{BA}$$

$$\frac{\Delta \overline{p}_A}{\Delta t} = -\frac{\Delta \overline{p}_B}{\Delta t}$$

$$\Delta \overline{p}_A + \Delta \overline{p}_B = 0 \qquad \text{(or)}$$

$$\Delta \overline{p}_{total} = 0$$

$$\text{(or)} \qquad \overline{p}_{total} = \text{constant}$$

Momentum is constant, in other words, the momentum of the system is conserved.

4.5.2 A Few Illustrations of Conservation of Momentum

(a) Recoil of a gun : When a bullet is fired from a gun, it moves back, which means gun recoils. The velocity v_2 with which the gun recoils can be determined by using the law of conservation of momentum. Let 'm' be the mass of the bullet being fired from a gun of mass M. If v_1 is the velocity of the bullet, the momentum will said to be conserved if the velocity v_2 of the gun is given by

$$m\overline{v}_1 + M\overline{v}_2 = 0 \qquad (4.8)$$

$$m\overline{v}_1 = -M\overline{v}_2$$

$$\overline{v}_2 = -\frac{m}{M}\overline{v}_1$$

Here, negative sign shows that v_2 is in direction opposite to $\overline{v}_1$. Since m << M, the recoil velocity of the gun will be considerably smaller than the velocity of the bullet.

(b) Collision : In a collision, we may regard the colliding bodies as forming a system. In the abscence of any external force on the colliding bodies, such as the force of friction, the system can be considered to be an isolated system. The forces of interaction between the colliding bodies will not change the total momentum of the colliding bodies.

Collision of the striker with a coin of carrom or collision between the billiard balls may be quite instructive for the sudy of collision between elastic bodies.

Example 4.6

Two trolleys, each of mass m, coupled together are moving with initial velocity 'v'. They collide with three identical stantionary trolleys coupled together and continue moving in the same direction. What will be the velocity of the trolleys after the impact?

Solution :

Let v' be the velocity of the trolleys after the impact.

Momentum before collision = 2 mv

Momentum after collision = $5\ m\,v'$

In accordance with the law of conservation of momentum, we can write

$$2\ mv = 5\ m\,v'$$

$$v' = \frac{2}{5}v$$

(c) Explosion of a bomb : A bomb explodes into fragments with the release of huge energy. Consider a bomb at rest initially which explodes into two fragments A and B. As the momentum of the bomb was zero before the explosion, the total momentum of the two fragments formed will also be zero after the explosion. For this reason, the two fragments will fly off in opposite directions. If the masses of the two fragments are equal, the velocities of the two fragments will also be equal in magnitude.

(d) **Rocket propulsion :** Flight of a rocket is an important practical application of conservation of momentum. A rocket consists of a shell with a fuel tank, which can be considered as one body, the shell is provided with a nozzle through which high pressure gases are made to escape. On firing the rocket, the combution of the fuel produces gases at very high pressure and temperature. Due to their high pressures these gases escape from the nozzle at a high velocity and provide thrust to the rocket to go upward due to the conservation of momentum of the system. If M is the mass of the rocket and *m* is the mass of the gas escaping per second with a velocity v, the change in momentum of the gas in t second = m vt.

If the increase in velocity of the rocket in 't' second is V, the increase in its momentum = MV.

According to the principle of conservation of momentum

$$mvt + MV = 0$$

$$\frac{V}{t} = a = -\frac{mv}{M}$$

i.e., the rocket moves with an acceleration

$$a = -\frac{mv}{M}$$

4.6 FRICTION

You may have noticed that when a batter hits a ball to make it roll along the ground, the ball does not continue to move forever. It comes to rest after travelling some distance. Thus, the momentum of the ball, which was imparted to it during initial push, tends to be zero. We know that some force acting on the ball is responsible for this change in its momentum. Such a force, called the frictional force, exists whenever bodies in contact tend to move with respect to each other. It is the force of friction which has to be overcome when we push or pull a body horizontally along the floor to change its position.

Force of friction is a contact force and always acts along the surfaces in direction opposite to that of the motion of the body. Friction is a force, that opposes the relative motion between the two surfaces which are in contact. It is the property of the surfaces of the object. If the surface is rough, the friction will be more. For this reason deliberate attempts are made to make the surfaces smooth or rough depending upon the requirement.

Friction causes wear and tear and is responsible for loss of mechanical energy in the form of heat, in general. Friction has both advantages as well as disadvantages. Walking, running, driving vehicles and stopping the vehicles by applying breaks are the some of the advantages of the friction. Due to its dual nature in our life, friction is treated as a necessary evil.

There are three types of frictions namely 1. Static friction 2. Kinetic friction and 3. Rolling friction.

4.6.1 Static and Kinetic Friction

We all know that certain minimum force is required to move an object over a surface. To illustrate this point, let us consider a block resting on some horizontal surface, as shown in Fig. 4.4. Let some external force F_{ext} be applied on the block. Initially the block does not move. This is possible only if some other force is acting on the block. The force is called the force of static friction and is represented by symbol t_s. As F_{ext} is increased, f_s also increases and remain equal to F_{ext} in magnitude until it reaches a critical value $f_s^{(max)}$. When F_{ext} is increased further, the block starts to slide and is then subject to kinetic friction. It is common experience that the force needed to set an object in motion is larger than the force needed to keep it moving at constant velocity. For this reason, the maximum value of static friction f_s between a pair of surface in contact will be larger than the force of kinetic friction f_k between them. Fig. 4.5 shows the variation of force & friction with the external force.

For a given pair of surface in contact, you may like to know the factors on which $f_s^{(max)}$ and f_k depend? It is an experimental fact that $f_s^{(max)}$ is directly proportional to the normal force F_N. i.e.,

$$f_s^{(max)} \propto F_N \quad \text{or} \quad f_s^{(max)} = \mu_s F_N \qquad (4.9)$$

where μ_s is called the coefficient of static friction. The normal force F_N of the surface on the block can be found by knowing the force with which the block presses the surface. Refer to Fig 4.4. The normal force F_N on the block will be mg, where m is mass of the block. (Fig. 4.6)

Since $f_s = F_{ext}$ for $f_s \le f_s^{max}$, we can write

$$f_s \le \mu_s F_N . \qquad (4.10)$$

It has also been experimentally found that maximum force of static friction between a pair of surface is independent of the area of contact.

Similarly we can write

$$f_k = \mu_k F_N \qquad (4.11)$$

where μ_k is the coefficient of kinetic friction. In general $\mu_s > \mu_k$. Moreover coefficients μ_s and μ_k are not really constants for any pair of surface such as wood on wood or rubber on concrete, etc. Values of μ_s and μ_k for a given pair of materials depend on the roughness of surfaces, their cleanliness, temperature, humidity etc.

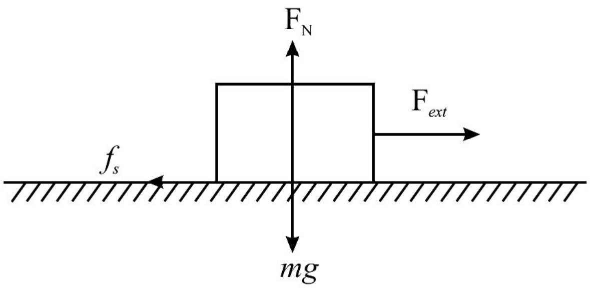


Fig. 4.4 : Forces acting on the block

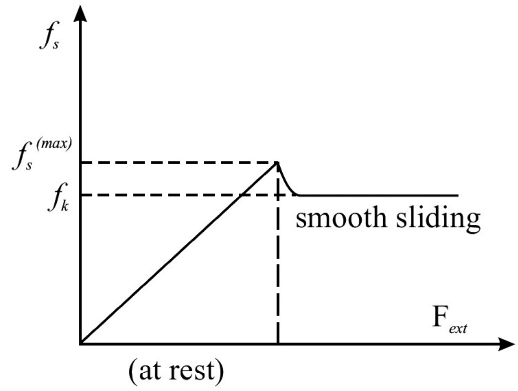


Fig. 4.5 : Variation of force of friction with external force

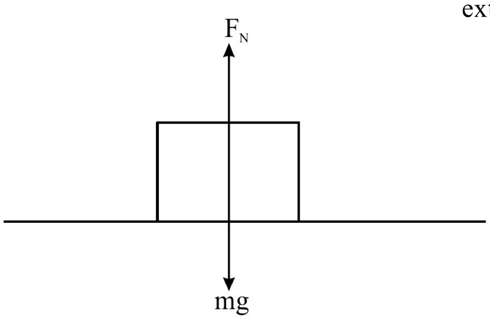


Fig. 4.6 : Normal force on the block

Example 4.7

A 2 kg block is resting on a horizontal surface. The coefficient of static friction between the surface in contact is 0.25. Calculate the maximum magnitude of force of static friction between the surfaces in contact.

Solution :

Here m = 2 kg and $\mu_s = 0.25$

From Eqn. (4.9), we recall that

$$f_s^{(\max)} = \mu_s F_N$$

$$= \mu_s \text{ mg}$$

$$= 0.25 \text{ (2 kg) (9.8 ms}^{-2})$$

$$= 4.9 \text{ N}$$

Example 4.8

A 5 kg block is resting on a horizontal surface for which $\mu_k = 0.1$. What will be the acceleration of the block if it is pulled by a 10 N force acting on it in the horizontal direction?

Solution :

As $f_k = \mu_k F_N$ and $F_N = mg$

$$f_k = \mu_k mg$$

$= (0.1)\ (5\ kg)\ (9.8\ ms^{-2})$

$= 49\ kg\ ms^{-2} = 4.9\ N$

Net force on the block $= F_{ext} - f_k$

$= 10\ N - 4.9\ N$

$= 5.1\ N$

Hence, acceleration $= a = \dfrac{F_{net}}{m} = \dfrac{5.1N}{5\,kg} = 1.02\ ms^{-2}$

4.6.2 Rolling Friction

It is a common experience that pushing or pulling objects such as carts on wheels is much easier. The motion of a wheel is different from sliding motion. It is a rolling motion. The friction in the case of rolling motion is known as rolling friction. For the same normal force, rolling friction is much smaller than sliding friction. For example, when steel wheels roll over steel rails, rolling friction is about $1/100^{th}$ of the sliding friction between steel and steel. Typical values for coefficient of rolling friction μ_r are 0.006 for steel on steel and 0.02 – 0.04 for rubber on concrete.

We would now like you do a simple activity.

Activity 4.1

Place a heavy book or a pile of books on a table and try to push them with your fingers. Next put three or more pencils below the books and now push them again. In which case do you need less force? What do you conclude from your experience?

4.6.3 Methods of Reducing Friction

Wheel is considered to be greatest invention of mankind for the simple reason that rolling is much easier than sliding. Because of this ball bearings are used in machines to reduce friction. In a ball bearing, steel balls are placed between two co-axial cylinders, as shown in Fig. 4.7. Generally one of the two cylinders is allowed to turn with respect to the other. Here the rotation of the balls is almost frictionless. Ball-bearings find applications in almost all types of vehicles and in electric motors such or electric fans etc.

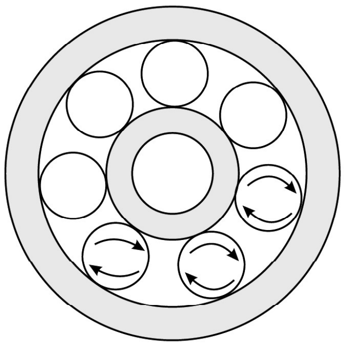


Fig. 4.7 : Balls in the ball-bearing

Use of lubricants such as grease or oil between the surfaces in contact reduce friction considerably. In heavy machines, oil is made to flow over moving parts. It reduces frictional force between moving parts and also prevents them from getting overheated. In fact, the presence of lubricants changes the nature of friction from dry friction to fluid friction, which is considerably smaller than the former.

Flow of compressed and purified air between the surfaces in contact also reduces the friction. It provides a cushioning effect to the moving objects. It also prevents dust and dirt from getting collected on the moving parts.

Fluid Friction : A substance that can flow is called "Fluid". Both liquids and Gases are Fluids. Bodies moving through a fluid also face friction. Shooting stars (meteors) shine because of the heat generated by air friction. Contrary to solid friction, fluid friction depends upon the shape of the bodies. This is why fishes have a special shape and fast moving aeroplanes and vehicles are also given a fish like shape, called stream-line shape. Fluid friction increases rapidly with increase in speed. If a car is to run at a high speed, more fuel will have to be burnt to overcome the increased fluid (air) friction. Car manufactures advise us to drive at a speed of 40 – 45 kmh^{-1} for maximum fuel efficiency.

4.7 THE FREE BODY DIAGRAM TECHNIQUE

Application of Newton's laws to solve problems in mechanics becomes easier by use of the free body diagram technique. A diagram which shows all the forces acting on a body in a given situation is called a free body diagram (FBD). The procedure to draw a free body diagram, is described below:

1. Draw a simple, neat diagram of the system as per the given description.
2. Isolate the object of interest. This object will be called the free body now.
3. Consider all external forces acting on the free body and mark them by arrows touching the free body with their line of action clearly represented.
4. Now apply Netwon's second law $\Sigma F = ma$

   (or $\Sigma F_x = m\ a_x$ and $\Sigma F_y = m\ a_y$)

Remember :

(i) A net force must be acting on the object along the direction of motion.

(ii) For obtaining a complete solution, you must have as many independent equations as the number of unknowns.

Example 4.9

Two blocks of masses m_1 and m_2 are connected by a string and placed on a smooth horizontal surface. The block of mass m_2 is pulled by a force F acting parallel to the horizontal surface. What will be the acceleration of the blocks and the tension in the string connecting the two blocks (assuming it to be horizontal)?

Solution :

Refer to Fig. 4.8. Let 'a' be the acceleration of the blocks in the direction of 'F' and let the tension in the string be T. On applying $\Sigma F = ma$ in the component form to the free body diagram of system of two bodies of masses m_1 and m_2, we get

$$N - (m_1 + m_2)\,g = 0$$

and $$F = (m_1 + m_2)\,a$$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2} \qquad (4.12)$$

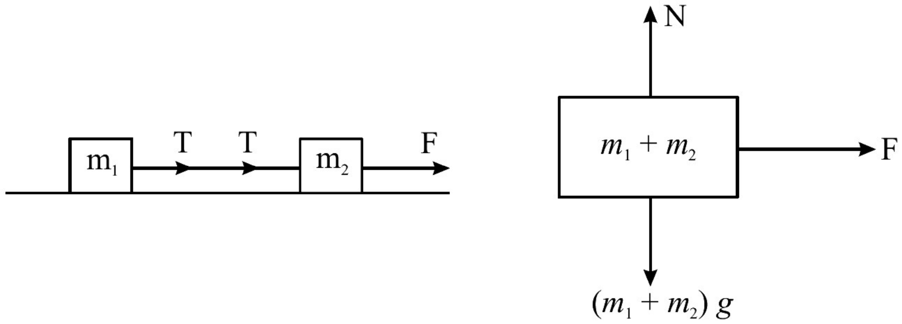


Fig. 4.8 : Free body diagram for two blocks connected by a string

on applying $\Sigma F = ma$ in the component form to the free body diagram of m_1, we get

$$N_1 - m_1 g = 0 \text{ and } T = m_1 a$$

$$T = m_1\left(\frac{F}{m_1 + m_2}\right)$$

$$\Rightarrow \quad T = \left(\frac{m_1}{m_1 + m_2}\right)F \qquad (4.13)$$

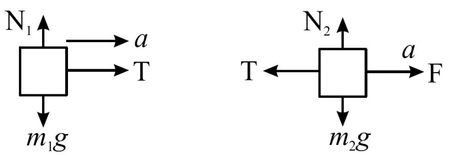


Fig. 4.9

Apply $\Sigma F = ma$ once again to the free body diagram of m_2 and see whether you get the same expressions for a and T.

Example 4.10

Two masses m_1 and m_2 ($m_1 > m_2$) are connected at the two ends of a light inextensible string that passes over a light frictionless fixed pulley. Find the acceleration of the masses and the tension in the string connecting them when the masses are released.

Solution :

Let a be the acceleration of mass m_1 downward. The acceleration of mass m_2 will also be 'a' only but upward (Why?). Let T be the tension in the string connecting the two masses.

On applying $\Sigma F = ma$ to m_1 and m_2 we get

$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$

On solving above two equations for 'a' and T we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \qquad (4.14)$$

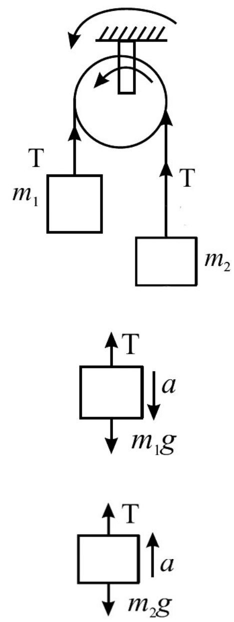


Fig. 4.10

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) a \qquad (4.15)$$

At this stage, you can check the predicted results for the extreme values of the variables (i.e. m_1 and m_2). Either take $m_1 = m_2$ or consider $m_1 >> m_2$, and observe whether the values of 'a' and 'T' match the expected outcomes.

Example 4.11

A trolley of mass M = 10 kg is connected to a block of mass m = 2 kg with the help of massless inextensible string passing over a light frictionless pulley as shown in Fig. 4.11 (a). The co-efficient of kinetic friction between the trolley and the surface (μ_k) = 0.02. Find

(a) acceleration of the trolley and (b) tension in the string

Solution :

Fig. 4.11 (b) and (c) show the free body diagrams of the trolley and the block respectively. Let 'a' be the acceleration of the block and the trolley.

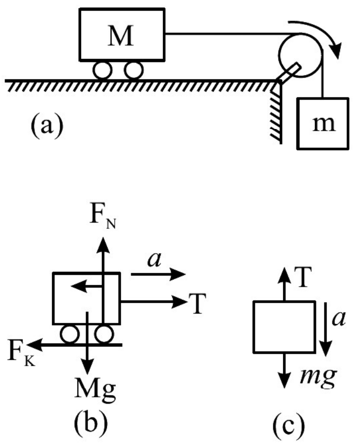


Fig. 4.11

For the trolley F_N = Mg and

$T - f_k = Ma$ where $f_k = \mu_k N = \mu_k Mg$

So $T - \mu_k Mg = Ma$ (1)

For the block $mg - T = ma$ (2)

On adding above two equations we get

$mg - \mu_k Mg = (M + m)\, a$

or

$$a = \frac{mg - \mu_k Mg}{M + m} = \frac{(2kg)(9.8\,ms^{-2}) - (0.02)(10kg)(9.8\,ms^{-2})}{(10+2)kg}$$

$$a = \frac{19.6\,kg\,ms^{-2} - 1.96\,kg\,ms^{-2}}{12\,kg} = 1.47\,ms^{-2}$$

So $a = 1.47\ ms^{-2}$

From equation (2) $T = mg - ma = m\,(g - a)$

$T = 2\ kg\ (9.8\ ms^{-2} - 1.47\ ms^{-2})$

$= 2\ kg\ (8.33\ ms^{-2})$

$T = 16.66\ N$

Intext Questions 4.4

1. A block of mass 'm' is held on a rough inclined surface of inclination θ. Show in a diagram, various forces acting on the block.
2. A force of 100 N acts on two blocks A and B of masses 2 kg and 3 kg respectively placed in contact on a smooth horizontal surfaces as shown in Fig. 4.12. What is the magnitude of force with which block 'A' exerts on B?

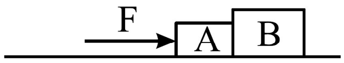


Fig. 4.12

3. What will be the tension in the string when a 5 kg object suspended from it is pulled up with.

   (a) a velocity of 2 ms^{-1}? (b) an acceleration of 2 ms^{-2} ?

4.8 ELEMENTARY IDEAS OF INERTIAL AND NON INERTIAL FRAMES

To study motion in one dimension (ie. in a straight line) a reference point (origin) is enough. But, when it comes to motion in two and three dimensions, we have to use a set of reference lines to specify the position of a point in space. This set of lines is called frame of reference.

Every motion is described by an observer. The description of motion will change with the change in the state of motion of the observer. For example, let us consider a box lying on a railway platform. A person standing on the platform will say that the box is at rest. A person in a train moving with uniform velocity v will say that the box is moving with velocity –v. But, what will be the description of the box by a person in a train having acceleration (a). He/she will find the box is moving with an acceleration (–a). Obviously, the first law of motion is failing for this observer.

Thus a frame of reference is fixed with the observer to describe motion. If the frame is stationary or moving with a constant velocity with respect to the object under study (another frame of reference), then in this frame, law of inertia holds good. Therefore, such frames are called inertial frames. On the other hand, if the observer's frame is accelerating, then we call it non-inertial frame.

For the motion of a body of mass 'm' in a non inertial frame, having acceleration (a), we may apply second law of motion by involving pseudo force. In a rotating body, this force is called centrifugal force.

Intext Questions 4.5

1. A glass half filled with water is kept on a horizontal table in a train. Will the free surface of water remain horizontal as the train starts?
2. When a car is driven too fast around a curve, it skids outwards. How would a passenger sitting inside explain the car's motion? How would an observer standing on a road explain the event?

3. A tiny particle with a mass of 6×10^{-10} kg is in a water suspension which in a centrifuge that is being rotated at an angular speed of $2\pi \times 10^{3}$ rad s^{-1}. The particle is situated at a distance of 4 cm from the axis of rotation. Calculate the net centrifugal force acting on the particle.
4. What must be the angular speed of rotation of earth so that the centrifugal force makes objects fly off from its surface? Take g = 10 ms^{-2}.
5. In the reference frame attached to a freely falling body of mass 2 kg, what is the magnitude and direction of inertial force on the body?

WHAT YOU HAVE LEARNT

- Inertia is the tendency of the body to oppose the change in its state of rest or uniform motion.
- Newton's first law states that a body remains in a state of rest or uniform motion in a straight line as long as net external force acting on it is zero.
- For a single particle of mass m, moving with velocity $\overline{v}$, we define a vector quantity '$\overline{p}$' called the linear momentum $\overline{p} = m\overline{v}$.
- Newton's second law states that the time rate of change of momentum of a body is proportional to net external force acting on the body.
- According to Newton's second law, acceleration produced in a body of constant mass is directly proportional to the net external force acting on the body; F = ma.
- Newton's third law states that if two bodies A and B interact with each other, then the force with which A exerts on body B will equal and opposite to the force which body B exerts on body A.
- According to the law of conservation of momentum, if no external force acts on a system of particles, the total momentum of the system will remain constant regardless of the nature of forces between them.
- Frictional force is the force which acts on the body when it attempts to slide or roll along a surface. The force of friction is always parallel to the surface in contact and opposite to the direction of motion of the object.
- The maximum force of static friction $f_s^{(\max)}$ between a body and a surface is proportional to the normal force F_N acting on the body. This maximum force occurs when the body is on the verge of sliding.
- For a body sliding on some surface, the magnitude of the force of kinetic friction f_k is given by $f_k = \mu_k f_N$. Where μ_k is the coefficient of kinetic friction for the surfaces in contact.
- Use of rollers and ball-bearings reduce friction and associated energy losses considerably as rolling friction is much smaller than kinetic friction. For a given pair of surfaces kinetic friction is further smaller than the maximum static friction between them.

- Newton's laws of motion are applicable only in an inertial frame of reference. For Inertial frame, the coordinate system attached to an isolated object that has zero acceleration.
- For an object to be in static equilibrium, the vector sum of all the forces acting on it must be zero. This is a necessary and sufficient condition for point objects only.

TERMINAL EXERCISE

1. Which of the following will always be in the direction of net external force acting on the body?

   (a) displacement (b) velocity

   (c) acceleration (d) change in momentum

2. When a constant net external force acts on an object which of the following may not change?

   (a) position (b) speed

   (c) velocity (d) acceleration

3. A 0.5 kg ball is dropped from such a height that it takes 4 s to reach the ground. Calculate the change in momentum of the ball.

4. In which case will there be larger change in momentum of a 2 kg object :

   (a) When 10 N force acts on it for 1 s?

   (b) When 10 N force acts on its for 1 m?

   Calculate the change in momentums in each case.

5. A ball of mass 20 kg is lifted with the help of a rope at a constant acceleration 6 ms^{-2}. Calculate air drag on the ball.

6. A load of mass 20 kg is lifted with the help of a rope at a constant acceleration. The load covers a height of 5 m in 2 seconds. Calculate the tension in the rope. ($g = 10\ ms^{-2}$)

7. In a rocket, mass 'm' changes with time. Write down the mathematical form of Newton's law in this case and interpret it physically.

8. A ball of mass 0.1 kg moving at 10 ms^{-1} is deflected by a wall at the same speed in the direction shown in Fig. 4.13. What is the magnitude of the change in momentum of the ball?

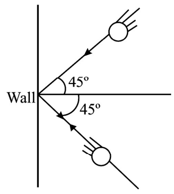


Fig. 4.13

9. Find the average recoil force on a machine gun that is firing 150 bullet per minute, each with a speed of 900 ms^{-1}. Mass of each bullet is 12 g.

10. Explain why, when catching a fast moving ball the hands are drawn back while the ball is being brought to rest.

11. A constant force of magnitude 20 N acts on a body of mass 2 kg, initially at rest, for 2 seconds. What will be the velocity of the body after

(a) 1 second from start? (b) 3 seconds from start?

12. How does a force acting on a block in the direction shown here keep the block from sliding down the vertical wall?

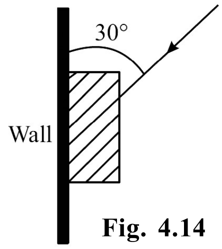


Fig. 4.14

13. A 1.2 kg block is resting on a horizontal surface. The coefficient of static friction between the block and the surface is 0.5. What will be the magnitude and direction of the force of friction on the block when the magnitude of the external force acting on the block in the horizontal direction

(a) 0 N? (b) 4.9 N? (c) 9.8 N?

14. For a block on a surface the maximum force of static friction is 10 N. What will be the force of friction on the block when a 5 N external force is applied to it parallel to the surface on which it is resting?

15. What minimum horizontal force F is required to keep a 5 kg block at rest on an inclined plane of inclination 30°. The coefficient of static friction between the block and the inclined plane is 0.25. ($g = 10\ ms^{-2}$)

16. Two blocks P and Q of masses $m_1 = 2$ kg, $m_2 = 3$ kg respectively are placed in contact with each other on a horizontal frictionless surface. Some external force F = 10 N is applied to the block 'P' in the direction parallel to the surface. Find the following.

(a) acceleration of the blocks

(b) force with which the block P exerts on block Q.

17. Two blocks P and Q of masses $m_1 = 2$kg and $m_2 = 4$ kg are connected to a third block R of mass M as shown in Fig. 4.15. For what maximum value of M will the system be in equilibrium? The frictional force acting on each block is half the force of normal reaction on it.

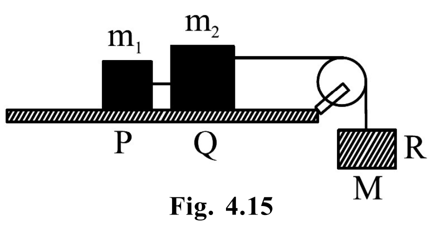


Fig. 4.15

18. A 2 kg block is pushed up on an inclined plane of inclination θ = 37° imparting it a speed of 20 ms^{-1}. How much distance will the block travel before coming to rest? The coefficient of kinetic friction between the block and the inclined plane is $\mu_k = 0.5$.

Take $g = 10\ ms^{-2}$, sin 37° = 0.6, cos 37° = 0.8.

19. The linear momentum of a particle as a function of time 't' is given by p = at + b, where a and b are positive constants. What is the force acting on the particle.

20. On a smooth horizontal surface a block A of mass 10 kg is kept. On this block a second block B of mass 5 kg kept. The coefficient friction between the two blocks is 0.2. A horizontal force of 20 N is applied on the lower block as shown in figure. What is the force of friction between the blocks? ($g = 10\ ms^{-2}$). [$F = (m_1 + m_2)\ a$]

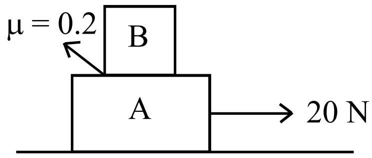


Fig. 4.16

21. A person of mass 60 kg is in a lift, standing on a weighing maching. What are the readings of the weighing machine in the following situations.

   (a) Lift moving upwards with an acceleration of $1.2\ ms^{-2}$.

   (b) Lift moving downwards with an acceleration of $1.2\ ms^{-2}$.

   (c) Lift moving upwards with a uniform velocity of $5\ ms^{-1}$.

   (d) Lift falling freely.

22. Two blocks of mass 3 kg and 4 kg are suspended at the ends of massless string passing over a frictionless pulley. Find the acceleration of the system and the tension in the string? ($g = 10\ ms^{-2}$)

23. A fixed pulley with a smooth grove has a light string passing over it with a 5 kg attached on one side and a 4 kg on the other side. Another 4 kg is hung from the other 4 kg as shown with another light string. If the system is released from rest, find the common acceleration? ($g = 10\ ms^{-2}$)

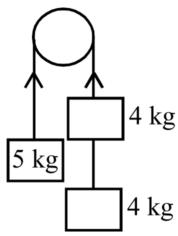


Fig. 4.17

24. Two people are pulling a rope in opposite direction each with a force F. Find the tension in the rope?

25. Explain the role of friction in the case of bicycle brakes. What will happen if few drops of oil are put on the rim?

ANSWERS TO INTEXT QUESTIONS

4.1

1. No. not in all situations. During the circular motion force acts perpendicular to the direction of motion (velocity)

2. Inertial mass

3. Yes, in uniform circular motion

4. A force can change the state of the object. It can also deform the objects.

5. Both are relative, which means that they can be defined with respect to surroundings of some other objects.

6. Imaginary co-ordinate system attached to an object.

4.2

1. P = mv = constant

   mv = constant

   $$v \propto \frac{1}{m}$$

   object of smaller mass moves faster.

2. (a) yes (b) No

3. p = mv : m remains same but v increases due to gravitational attraction, as the object falls freely.

4. $$F = \frac{\Delta p}{\Delta t} \quad \Rightarrow \Delta p = f \times \Delta t$$

   In case (b) the product of F and Δt is larger.

5. No, speed is constant but the velocity of the object changes due to change in direction. Hence momentum will not be a constant. Direction of velocity is in the direction of tangent drawn to the circle at a given point, at a given juncture.

4.3

1. The jumper is thrown upwards by the force which the ground exerts on the jumper, according to Newton's third law of motion i.e., action = – reaction. The force exert by the jumper on the ground is the action.

2. (a) Kicking the football is the action and the force which the football exerts on the man will be the reaction.

   (b) The force with which earth pulls on the moon is action and the force with which the moon exerts on the earth will be its reaction.

   (c) The force with which the ball exerts on the wall is action and the force with thich wall exerts on the ball is its reaction.

3. No, The argument is not correct. The almirah moves when the push by the person exceeds the frictional force between the almirah and the floor. He doesn't get pushed backward due to a large force of frcition that he experiences due to the floor. On a shippery surface, he will not be able to push the almirah forward.

4.4

1.

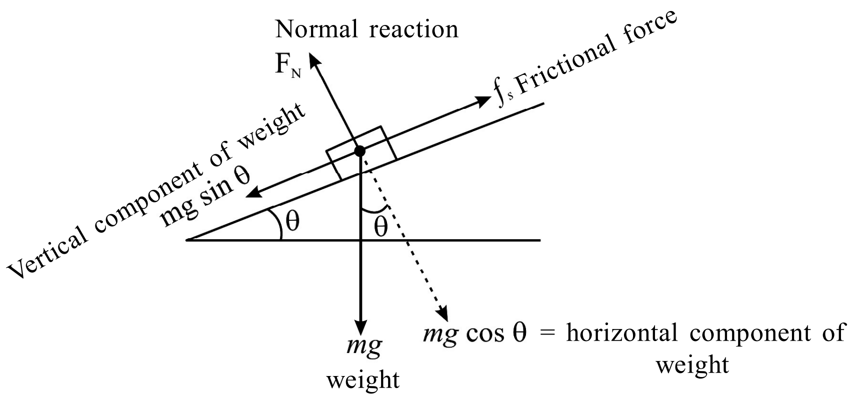
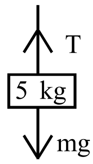


Fig. 4.18

2. $$a = \frac{F}{m_1 + m_2} = \frac{100N}{(2+3)kg} = 20 ms^{-2}$$

$F = m_1 a = 2 \times 20 = 40$ N

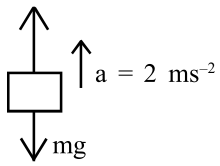
3. (a)



T = mg = (5 × 9.8) N

T = 49.0 N

(b)



T – mg = ma

T = m (g + a)

= 5 kg (9.8 + 2)

= 5 (11.8) N

T = 59 N

4.5

1. No, when train accelerates with acceleration 'a', the total force acting on water in the frame of reference attached to the train is

F = mg – ma

The surface of the water takes up position along the resultant force F shown in the diagram.

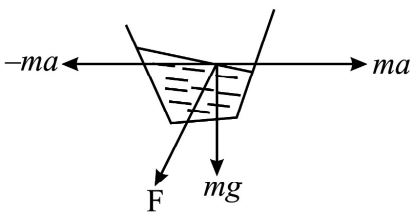


Fig. 4.19

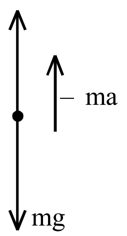
2. To the passenger sitting inside, a centrifugal force $\left(\frac{-mv^2}{r}\right)$ acts on the car. To an observer standing on the road, the car experiences a centripetal force given by $\frac{mv^2}{r}$.
   In both the cases v is larger for lesser 'r'.

3. $$F = m\omega^2 r = \left(6 \times 10^{-12}\, kg\right)\left(2\pi \times 10^3\, rad\, s^{-1}\right)^2 \times \left(0.04\, m\right)$$
   $$= 9.6 \times 10^{-4}\ \text{N}.$$

4. Centrifugal force = weight of the object
   $$\frac{mv^2}{r} = mg \quad \Rightarrow \quad v = \sqrt{rg}$$
   but $\quad v = r\omega \quad \Rightarrow \omega = v/r$
   $$\omega = \frac{\sqrt{rg}}{r} = \sqrt{\frac{g}{r}}$$

5. Zero (as it a case of free fall of a body)

ANSWERS TO TERMINAL EXERCISE

1. (d)
2. (a) If internal forces developed within the material counter back the external force. It happens in case of force applied on a wall.
   (b) If force is applied at right angles to the direction of motion of the body, the force changes the direction of motion of body keeping the speed constant.
3. v = u + at
   v = 0 + (9.8) × 4 $\Rightarrow$ v = 40 ms^{-1}
   $$\Delta P = m\left(v - u\right) = 0.5\left(4.0 - v\right) = 20\, kg\, ms^{-1}$$
4. ΔP = FΔt. When 10 N acts for 1s.
5. 

   $\therefore$ force of air drag = mg – ma
   = m (g – a)
   = 0.2 kg (9.8 – 6) ms^{-2}
   = 0.2 × 3.8 kg ms^{-2}
   = 0.76 N

6. T = m (g + a)

from s = ut + 1/2 at^2

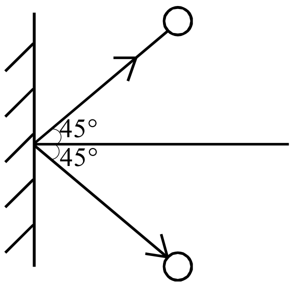
s = 0 + 1/2 × a × 2^2 $\Rightarrow$ 5 = 1/2 × a × 4

$$a = \frac{5}{2} = 2.5\, ms^{-2}$$

$$\therefore \quad T = 20\,kg\,(10+2.5)\,ms^{-2} = 20\,kg \times 12.5\,ms^{-2}$$

T = 250 kg ms^{-2}

7. $F = \frac{d}{dt}(mv)$. As rocket is a variable mass system, we can write. $F = m\frac{dv}{dt} + v\frac{dm}{dt}$.

8. 

We consider the horizontal components of velocities as both u, v are same, change in velocity (Δv) = 2u cosθ.

ΔP = m Δ v

= 0.1 × 2 × 10 × cos 45°

$$\Delta P = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}\, kg\, ms^{-1}$$

9. $$F = \left(\frac{mv}{t}\right) \times n = \frac{12 \times 10^{-3}\, kg \times 900\, ms^{-1} \times 150}{60\,s}$$

$$= \frac{\cancel{12}^{3} \times 9 \times \cancel{15} \times 10^{-3} \times 10^{3}}{\cancel{60}_{4}} = 27N$$

10. Impulse = Force × time

by increasing the time, the force acting on the hands can be decreased.

11. F = 20 N, m = 2 kg $a = F/m = \frac{20}{2} = 10\, ms^{-2}$

(a) $v = u + at \Rightarrow v = 0 + 10 \times 1 \Rightarrow v = 10\, m/s$

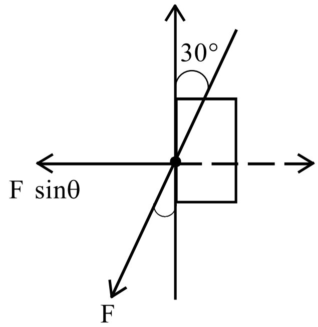
(b) Force acts for t = 2s only, after 2 seconds it continues with same velocity (if friction is zero)

$\therefore$ v after t = 2 s is

v = 0 + 10 × 2 = 20

$\therefore$ v = 20 m/s.

12.



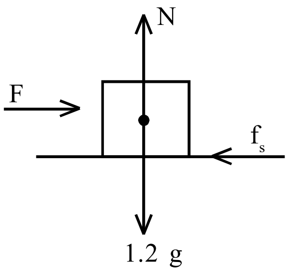
Normal reaction, N = F sin θ

Frictional force F = μ_s N

F = μ_s F sin θ

if the frictional force F is greater than the weight of the object, then the block doesn't slide.

13.



N = mg = 1.2 kg; g = 10 ms^{-2}

∴ N = 12 N ; μ_s = 0.5

$f_s = \mu_s N$

$f_s = 0.5 \times 12 = 6$ N

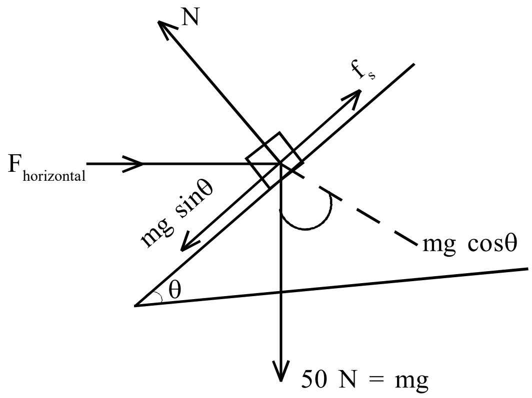
(a) = 0 N

(b) $4.9N < 6N \quad \Rightarrow \quad friction = 4.9\,N$

(c) 9.8 N is greater than 6 N, so it will have kinetic friction $f_k = \mu_k$ N, which will be less than $f_s = \mu_s$ N.

14. f_s = 10 N, applied force is 5 N, which is less than the maximum frictional force. Therefore friction, in this case will be 5 N only.

15.



If F is the external force the Fcosθ is the external force parallel to the incline plane. F sinθ will add upto to the normal reaction.

∴ N = mg cosθ + F sinθ

frictional force $f_s = \mu_s N = \mu_s$(mg cosθ + F sinθ)

when F cosθ = μ_s(mg cosθ + F sinθ), the object will be in equilibrium

$$F \times \frac{\sqrt{3}}{2} = 0.25\left(50 \times \frac{\sqrt{3}}{2} + F \times \frac{1}{\sqrt{2}}\right)$$

$$\frac{\sqrt{3}F}{2} - \frac{F}{4\sqrt{2}} = 50 \times 0.25\frac{\sqrt{3}}{2}$$

$$F\left[\frac{\sqrt{3}}{2}-\frac{1}{4\sqrt{2}}\right]=25\times 0.25\times\sqrt{3}$$

$$F\left[\frac{1.732}{2}-\frac{0.25}{1.414}\right]=625\times 1.732\times 10^{-2}$$

F [0.866 – 0.176] = 10.825

F (0.69) = 10.825

$$F=\frac{10.825}{0.69}=15.6\,N$$

16. $a=\dfrac{F}{m_1+m_2}=\dfrac{10}{5}=2\,ms^{-2}$

F = ma = 3 × 2 = 6 N

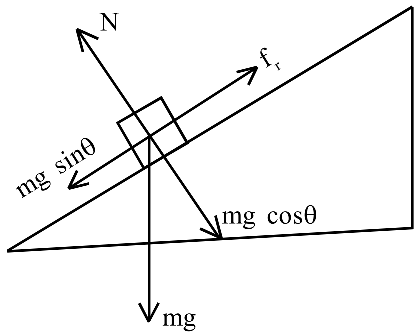
17. 6a = M ____ (1)

$$f_{r_1}=\frac{m_1 g}{2}=\frac{20}{2}=10\,N$$

$$f_{r_2}=\frac{m_2 g}{2}=\frac{40}{2}=20\,N$$

$$f_{r_1}+f_{r_2}=Mg \quad\Rightarrow\quad 30\,N=M\times 10 \quad\Rightarrow M=3\,kg.$$

18. $a=m(\sin\theta+\mu\cos\theta)$



$$a=10(\sin 37^\circ+0.5\cos 37^\circ)$$

$$=10(0.6+0.5\times 0.8)$$

a = 10 ms^{-2}

$v^2 - u^2 = 2as$

$0-(20)^2 = 2 \times 10 \times s$

$$s=\frac{400}{200}=20\,m$$

19. $p = at+b$; $F=\dfrac{dp}{dt}=a$

20. $f = (m_1 + m_2)a \quad \Rightarrow \quad 20 = 15a$

$$a = \frac{20}{15} = \frac{4}{3} ms^{-2}$$

$$f_r = m_2 a = 5 \times \frac{4}{3} = 6.6 N$$

21. (a) $W = m(g+a) = 60\,(9.8+1.2) = 60 \times 11 = 660 N$

(b) $W = m(g-a) = 60(9.8-1.2) = 60(8.6) = 516 N$

(c) $a = 0 \quad W = mg = 60 \times 9.8 = 588 N$

(d) W = m (g – a) = m (g – g) = 0.

22. $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = \left(\frac{4-3}{7}\right) \times 10 = 1.4 ms^{-2}$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g = \left(\frac{2 \times 3 \times 4}{7}\right) \times 10 = 34.2 N$$

23. For 5 kg block T – 50 = 5a

For other two blocks 80 – T = 8a

on solving $a = \frac{30}{13} = 2.4 ms^{-2}$

24. T = F.

25. Due to the friction between the blades of brakes of the cycle and the rim, the cycle stops, as soon as the brakes are applied. When oil is placed on the rim, its surface becomes smooth, due to which friction becomes less.

5

WORK ENERGY AND POWER

INTRODUCTION

Newton's Laws of motion move around a term "Force". Force leads to motion, brings change in the size and shape of the object. In this chapter you are going to learn the concepts of Work and Energy. Though we use the word "work" in many daily situations like office work, typing and domestic works like cooking, it is quite different with respect to physics. You also come to a conclusion that energy is necessary for doing work. The term work is related to the Force, displacement and the angle between these two vector quantities. We also know that there are many types of energy in the nature such as solar energy, wind energy, geothermal energy etc. Both energy and work are interchangeable and at some point of time, they appear to be equivalent. The amount of energy spent or the amount of work done in a given interval of time is known as **'power'.**

OBJECTIVES

After studying this lesson, you should be able to

- *define work done by a force and give unit of work;*
- *calculate the work done by an applied force;*
- *state work-energy theorem;*
- *define power of a system;*
- *calculate the work done by gravity when a mass moves from one point to another;*
- *explain the meaning of energy;*
- *obtain expressions for a gravitational potential energy and elastic potential energy;*
- *apply the principle of conservation of energy for physical systems;*
- *apply the laws of conservation of momentum and energy in elastic collisions.*

5.1 WORK

The word work has different meaning for different people. A labour claims that he did a lot of work in carrying a bag on his head for a long distance. Similarly your father may claim that he had to do lot of work in clearing the office files. The technical meaning of work is

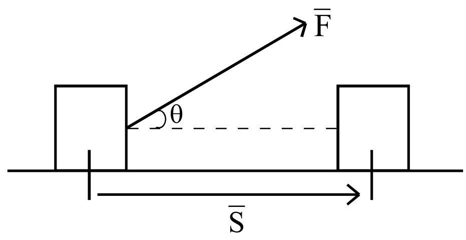


Fig. 5.1 : Work done in moving a block by applying a Force $\overline{F}$ by a displacement $\overline{S}$

not always the same as the common meaning. The work is said to be done when a constant force **F** acting on the body displaces it by a displacement **S,** as shown in Fig. 5.1.

Work is defined as the dot product of $\bar{\mathbf{F}}$ and $\bar{\mathbf{S}}$

$$W = \bar{\mathbf{F}}.\bar{\mathbf{S}} = FS\cos\theta = (F\cos\theta)S \qquad (5.1)$$

Where θ is the angle between $\bar{\mathbf{F}}$ and $\bar{\mathbf{S}}$.

It can also be defined as the product of displacement and the component of force acting in the direction of displacement.

If S = 0 then W = 0. That is no work is done by a force, whatever its magnitude, if there is no displacement of the object.

Sclar product (dot product) of two vectors is a scalar. Hence, work is a scalar.

Activity 5.1

You and your friends may try to push the wall of a room. Irrespective of the applied force, the wall will not move. Thus we say that no work is done.

The unit of work is defined using Eqn. (5.1). If the applied force is in newton and displacement is in metre, then the unit of work is joule.

(Unit of Force) × (Unit of displacement) = newton . metre = Nm

This unit is given a special name, **joule,** and is denoted by J.

One joule is defined, as the work done by a force of one newton when it produces a displacement of one metre. Joule is the SI unit of work.

Example 5.1

Find the dimensional formula of work.

Solution :

$$W = \text{Force} \times \text{displacement}$$
$$= \text{Mass} \times \text{Acceleration} \times \text{displacement}$$
$$\text{Dimension of work} = [M] \times [LT^{-2}] \times [L]$$
$$= [ML^2T^{-2}]$$

In electrical measurements, kilowatt-hour (kW h) is used as unit of work. It is related to joule as

$$1 \text{ kW h} = 3.6 \times 10^6 \text{ J}$$

You will study the details of this unit later in this lesson.

Example 5.2

A force of 6 N is applied on an object at an angle of 60° with the horizontal. Calculate the work done in moving the object by 2 m in the horizontal direction.

Solution :

From Eqn. (5.1) we know that

$$\begin{aligned} W &= FS \cos\theta \\ &= 6 \times 2 \times \cos 60^\circ \\ &= 6 \times 2 \times (½) \\ &= 6\ J \end{aligned}$$

Example 5.3

A person lifts 5 kg potatoes from the ground floor to a height of 4 m to bring it to first floor. Calculate the work done.

Solution :

Since the potatoes are lifted, work is being done against gravity. Therefore, we can write

$$\begin{aligned} \text{Force} &= mg \\ &= 5\ kg \times 9.8\ ms^{-2} \\ &= 49\ N \\ \text{Work done} &= 49 \times 4\ (N\ m) \\ &= 196\ J \end{aligned}$$

5.1.1 Different Cases of Work done by a Force

As you have seen work is defined as $W = FS \cos\theta$, where θ is the angle between force and displacement. So, in addition to force and displacement, the angle between them is also important in determing the nature of work.

$$W = FS \cos\theta$$

1. If $\theta = 0^\circ$ then $\cos\theta^\circ = 1$

$\therefore\ W = FS$

In this case work is positive. Ex: work done by the engine of a car during it speeding up. In this case both force and displacement are in the same direction.

2. If $\theta = 180^\circ$, then $\cos 180^\circ = -1$

$\therefore\ W = -\ FS$

Here the work done by the force is negative. Eg: work done by the engine of a car while brakes are applied. In this case force and displacement are opposite to each other.

Not only for $\theta = 180^\circ$, work is negative for all the θ values lying between 180º and 270°.

3. If $\theta = 90^\circ$, then $\cos 90^\circ = 0$

$W = 0$

Eg : A porter, carrying a suitcase on his head, walk along a railway platform for a long distance, said to be done no work at all. Though his muscular energy is doing work.

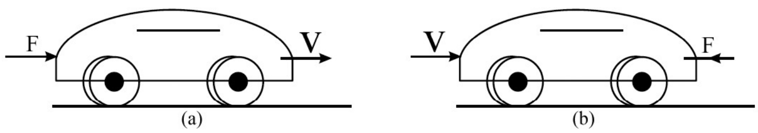


Fig. 5.2 : A car is moving on a horizontal road. (a) A force F is applied in the direction of the moving car. It gets acceleratged. (b) A force F is applied in opposite direction so that the car comes to rest after some distance

5.1.2 Work done by the force of Gravity

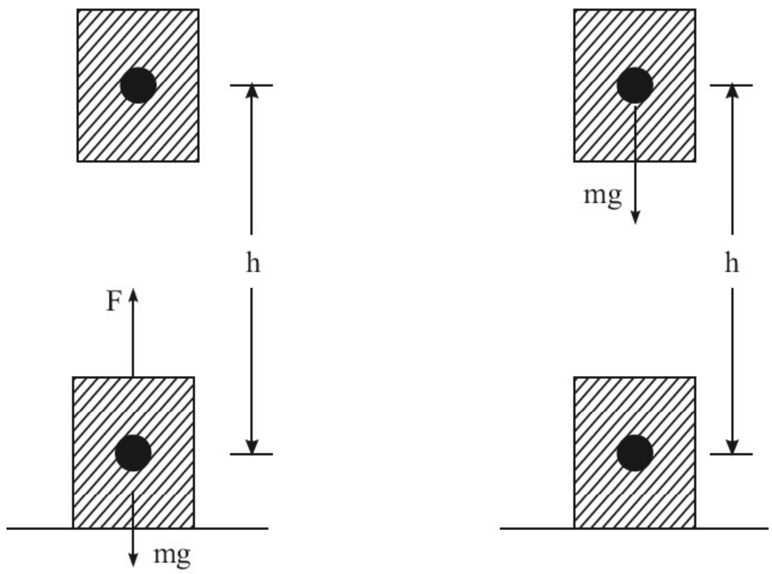
Fig. 5.3 (a) shows a mass 'm' being lifted to a height 'h' and Fig 5.3 (b) shows the same mass being lowered by a distance h. The weight of the object is mg. You may recall from the previous lesson that weight is a force.

In Fig. 5.3 (a), the work is done against the downward force mg and the displacement is upward ($\theta = 180°$).

Therefore $\qquad W = FS \cos 180°$

$$= mgh\,(-1)$$

$$\therefore\ W = -\,mgh \qquad (5.2)$$



(a) The object is lifted up against the force of gravity

(b) The object is lowered towards the earth

Fig 5.3

In the Fig. 5.3 (b), the mass is being lowered. The force mg and the displacement are in the same direction ($\theta = 0$)

$$\therefore\ W = FS \cos 0°$$

$$W = +\,mgh \qquad (5.3)$$

You must be very careful in interpreting the results obtained above. When the object is lifted up, the work done by the gravitational force is negative but the work done by the person in lifting the object is positive. When the object is being lowered, the work done by the gravitational force is positive but the work done by the person in lowering the object is negative. In both of these cases, it is assumed that the object is being moved without acceleration i.e., with a constant velocity.

Intext Questions 5.1

1. When a particle rotates in a circle, centripetal force acts on it. Calculate the work done by this force on the particle.
2. Give one example of each of the following. Work done by a force is
   (a) zero (b) negative (c) positive
3. A bag of grains of mass 2 kg, is lifted through a height of 5m.
   (a) How much work is done by the lift force?
   (b) How much work is done by the force of gravity?
4. A force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N produces a displacements $\overline{\mathbf{S}} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\,\mathrm{m}$. Calculate the work done.
5. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N acts on a particle to give a displacement $\overline{\mathbf{S}} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\,\mathrm{m}$.
   (a) Calculate the magnitude of displacement
   (b) Calculate the magnitude of force.
   (c) How much work is done by the force?

5.2 WORK DONE BY A VARIABLE FORCE

You have so far studied the cases where the force acting on the object is constant. This may not always be true. In some cases, the force responsible for doing work may keep varying with time. Let us now consider a case in which the magnitude of force F(x) changes with the position x of the object. Let us now calculate the work done by a variable force. Let us assume that the displacement is from x_i to x_f, where x_i and x_f are the initial and final positions. In such a situation, work is calculated over a large number of small intervals of displacements Δx. In fact, Δx is taken so small that the force F(x) can be assumed to be constant over each such interval. The work done during a small displacements Δx is given by

$$\Delta W = F(x)\,\Delta x \qquad (5.4)$$

$F(x)\,\Delta x$ is numerically equal to the small area shown shaded in the Fig. 5.4 (a). The total work done by the force between x_i and x_f is the sum of all such areas (area of all strips added together):

$$W = \Sigma \Delta W$$

$$= \Sigma F\,(x)\,\Delta x \qquad (5.5)$$

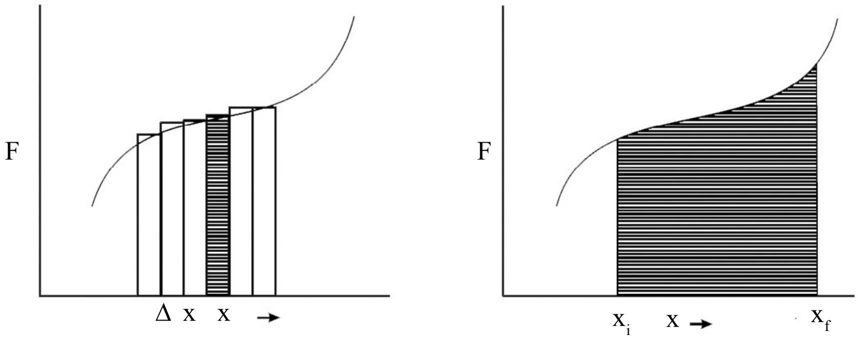


Fig. 5.4 : A varying force F moves the object from the initial position x_i to final position x_f. The variationof force with distance is shown by the solid curve (arbitrary) and work done is numerically equal to the shaded area.

The width of the strips can be made as small as possible so that the areas of all strips added together are equal to the total area enclosed between x_i and x_f. It will give the total work done by the force between x_i and x_f :

$$W = \sum_{\lim \Delta x \to 0} F(x)\Delta x \qquad (5.6)$$

When Δx value tends zero, the work done by a varying force can be expressed as a difinite integral of force over a displacement from $x = x_i$ to $x = x_f$

$$\therefore \qquad W = \int_{x_1}^{x_2} F(x)\,dx\,. \qquad (5.7)$$

Example 5.4

A force F = –kx acts on a particle along the x-axis. Find the work done by the force in displacing the particle from x = a to x = 2a. Here k is a positive constant

Solution :

$$W = \int_{x=a}^{x=2a} F(x)\,dx$$

$$= \int_{x=a}^{2a} (-kx)\,dx = -k\int_{x=a}^{x=2a} x\,dx$$

$$= -k\left[\frac{x^{1+1}}{1+1}\right]_a^{2a} = -k\left[\frac{x^2}{2}\right]_a^{2a} \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{-k}{2}\left[(2a)^2 - a^2\right] = \frac{k}{2}\left[4a^2 - a^2\right] = \frac{-3a^2}{2}$$

5.2.1 Work done by a Spring

A very simple example of a variable force is the force exerted by a spring. Let us derive the expression for work done in this case.

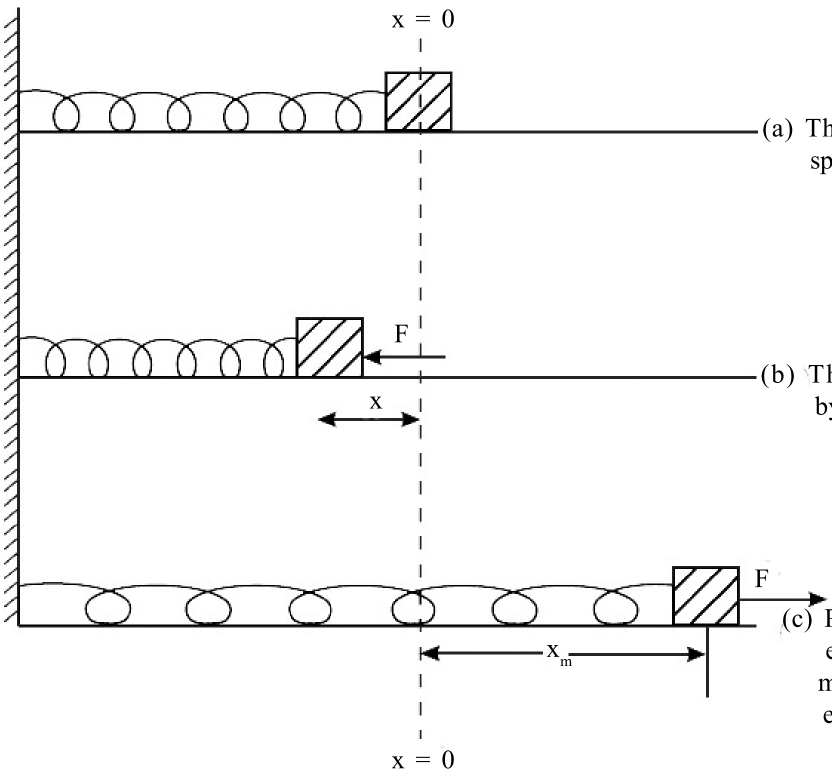


Fig. 5.5 : A spring-mass system whose one end is rigidly fixed and mass m, rests on a smooth horizontal surface.

Fig. 5.5 (a) shows the equilibrium position of a light spring whose one end is attached to a rigid wall and the other end is attached to a block of mass m. The system is placed on a smooth horizontal table. We take x-axis along the horizontal direction. Let mass m be at position x = 0. The spring is now compressed (or elongated) by an external force **F.** An internal force $\mathbf{F}_s$ is called into play in the spring due to its elastic property. This force $\mathbf{F}_s$ keeps increasing with increasing x and becomes equal to **F** when the compression (or elongation) is maximum at $x = x_m$.

According to Hooke's law (true for small x only), $|\mathbf{F}_s| = kx$, where k is known as spring constant. Since the direction of $\mathbf{F}_s$ is always opposite to compression (or extension), it is written as :

$$\mathbf{F} = \mathbf{F}_s = -k\mathbf{x} \qquad (5.8)$$

Let us now calculate the work done and also examine, if it is positive or negative. In the event of compression of the spring, the external force **F** is directed towards left and the displacement **x** is also towards left. Hence, the work done by the **external force** is positive. However, for the same direction of displacement, the restoring force generated in the spring is towards right, i.e. **F** and **x** are oppositely directed. The work done by the **spring force** is negative. You can yourself examine the case of extension of the spring and arrive at the same result: ***"the work done by the external force is positive but the work done by the spring force is negative and its magnitude is*** **(½)** $\mathbf{kx_m^2}$ ***".***

A simple calculation can be done to derive an expression for the work done. At x = 0, the force **F**s = 0. As x increases, the force **F**s increases and becomes equal to **F** when x = xm. Since the variation of the force is linear with displacement, the average force during compression (or extension) can be approximated to $\left(\frac{0 + F_s}{2}\right) = \frac{F_s}{2}$. The work done by the force is given by

$$W = \text{force} \times \text{displacement}$$

$$= \frac{F_s}{2}.x,$$

But | Fs | = k | xm |. Hence

$$W = \frac{1}{2}kx_m \times x_m$$

$$= \frac{1}{2}kx_m^2 \qquad (5.9)$$

The work done can also be obtained graphically. It is shown in Fig. 5.6.

The area of the shaded triangle is:

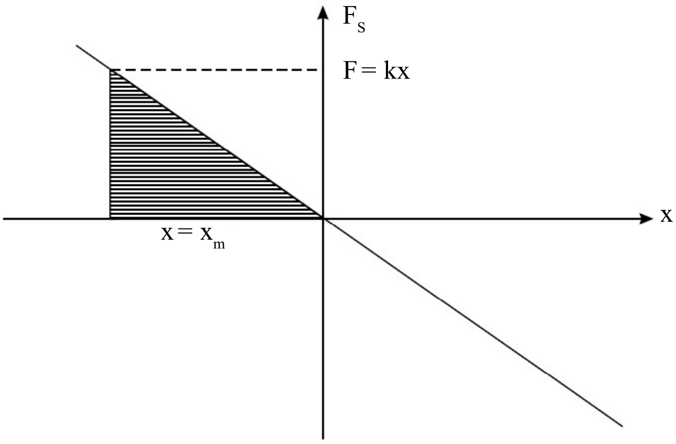


Fig. 5.6 : The work done is numerically equal to the area of the shaded triangle.

$$\frac{1}{2}\text{ base } \times \text{ height}$$

$$W = \frac{1}{2}x_m \times kx_m$$

$$= \frac{1}{2}kx_m^2 \qquad (5.10)$$

This is the same as that obtained analytically in equation (5.9)

The same can be obtained by using the formula $W = \int_{x=0}^{x=x_m} F(x)\,dx.$

here dx is the small elongation in the spring

$$W = \int_{x=0}^{x=x_m} kx\,dx = k\int_{x=0}^{x=x_m} x\,dx = k\left[\frac{x_m^2}{2}\right]_0^x$$

$$\therefore \quad W = \frac{1}{2}kx_m^2$$

Activity 5.2

Measuring spring constant

Suspend the spring vertically, as shown in Fig. 5.7 (a). Now attach a block of mass m to the lower end of the spring. On doing so, the spring extends by some distance. Measure the extension. Suppose it is s, as shown in Fig 5.7 (b). Now think why does the spring not extend further. This is because the spring force (restoring force) acting upwards balances the weight mg of the block in equilibrium state. You can calculate the spring constant by putting the values in

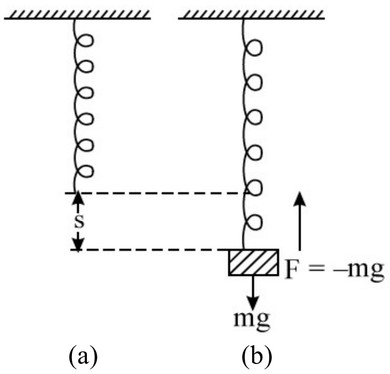


Fig. 5.7 : Extension in a spring under a load

$$F_s = k.s$$

or $$mg = k.s.$$

Thus, $$k = \frac{mg}{s} \tag{5.11}$$

Intext Questions 5.2

1. Define spring constant. Give its SI unit.
2. A force of 10 N extends a spring by 1cm. How much force is needed to extend this spring by 5 cm? How much work will be done by this force?
3. A graph is plotted between a variable force (on y-axis) acting on an object and the displacement caused along x-axis. The area under this curve indicates...

5.3 WORK AND KINETIC ENERGY

As you know, the capacity to do work is called energy. If a system has energy, it has ability to work. An automobile like car utilizes the chemical energy of its fuel to run the car. The energy possessed by a moving object is called Kinetic Energy.

If an object of mass 'm' has a velocity $\vec{v}$, then its kinetic energy K is

$$K = \frac{1}{2}m(\vec{v}.\vec{v}) = \frac{1}{2}mv^2 \tag{5.12}$$

Kinetic Energy is a scalar quantity.

If velocity is zero, then there is no Kinetic Energy.

The above equation can be obtained as follows.

Consider a force F acting on a object of mass m, increased the velocity from u to v in a time interval 't'. During this period, the object covers a distance 's'

by using $$v^2 - u^2 = 2as \tag{5.13}$$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting in F = ma, we get

$$F = m\left(\frac{v^2 - u^2}{2s}\right) \tag{5.14}$$

Work done by the force is

$$W = F.s. = m\left(\frac{v^2 - u^2}{2s}\right) \times s$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \tag{5.15}$$

$$W = KE_{final} - KE_{initial}$$

$$W = \Delta KE \tag{5.16}$$

Work done by the fore is equal to the change in Kinetic Energy. This is also known as **'Work - Energy theorem'.**

If the initial velocity 'u' is zero, then the work done by the force $W = \frac{1}{2}mv^2$, which is nothing but the Kinetic Energy of the object.

$$\therefore \quad KE = \frac{1}{2}mv^2$$

Work - Energy Theorem

The work - energy theorem states that the work done by the resultant of all forces acting on a body is equal to the change in its kinetic energy of the body.

Example 5.5

A body of mass 10 kg is initially moving with a speed of 4.0 ms^{-1}. A force of 30 N is now applied on the body for 2 seconds.

(i) What is the final speed of the body after 2 seconds?

(ii) How much work has been done during this period?

(iii) What is the initial kinetic energy?

(iv) What is the final kinetic energy?

(v) What is the distance covered during this period?

(vi) Show that the work done is equal to the change in kinetic energy?

Solution :

(i) Force (F) = ma

or $a = F/m$

$= 30/10$

$= 3 \text{ m s}^{-2}$

The final speed $v_2 = v_1 + at$

$= 4 + (3 \times 2) = 10 \text{ m s}^{-1}$

(ii) The distance covered in 2 seconds:

$$s = ut + \frac{1}{2}at^2$$

$$= (4 \times 2) + \frac{1}{2}(3 \times 4)$$

$$= 8 + 6 = 14 \text{ m}$$

Work done $W = F \times S$

$= 30 \times 14 = 420 \text{ J}$

(iii) The initial Kinetic Energy

$$K_1 = \frac{1}{2}mv_1^2$$

$$= \frac{1}{2}(10 \times 16) = 80 \text{ J}$$

(iv) The final kinetic energy

$$K_2 = \frac{1}{2} m v_2^2$$

$$= \frac{1}{2}\left(10 \times 100\right) = 500 \text{ J}$$

(v) The distance covered as calculated above = 14m

(vi) The change in kinetic energy is:

$$K_2 - K_1 = (500 - 80) = 420 \text{ J}$$

As may be seen, this is same as wok done.

Intext Questions 5.3

1. Is it possible for a particle to have a negative value of kinetic energy? Why?
2. What happens to the kinetic energy of a particle if.
   a) The speed v of the particle is made 2v.
   b) The mass 'm' of the particle is made $\frac{m}{2}$
3. A particle moving with a kinetic energy of 3.6J collides with a spring of force constant 180 Nm^{-1}. Calculate the maximum compression of the spring.
4. If an external force does 375J of work in compressing a spring, how much work is done by the spring itself?

5.4 THE CONCEPT OF POTENTIAL ENERGY

In the previous section we have discussed that a moving object has kinetic energy associated with it. The term potential energy indicates a stored energy which inturn is the outcome of the work done against the wish of the object. A compressed or stretched spring, water stored in a overhead tank or by a dam, a flying object and an arrow ready to be released by a bow have potential energies. We also come across potential energy in electrostatics.

Potential energy is the stored energy by virtue of the position or configuration of a body. Familiar example is the gravitational potential energy possessed by a body in gravitational field. Let us understand it now.

5.4.1 Potential energy in Gravitational Field

Suppose that a person lifts a mass m from a given height h_1 to a height h_2 above the earth's surface. Let us also assume that the value of acceleration due to gravity remains constant. The mass has been displaced by a distance $h = (h_2 - h_1)$ against the force of gravity. The magnitude of this force is mg and it acts downwards. Therefore, the work done by the person is

$$W = \text{force} \times \text{distance}$$

$$= mgh \qquad (5.17)$$

The work is positive and is stored in mass m as energy. This energy by virtue of the position in space is called **gravitational potential energy**. It has capacity to do work. If this mass is left free, it will fall down and during the fall it can be made to do work. For example, it can lift another mass if properly connected by a string, which is passing over a pulley.

The selection of the initial height h_1 is arbitrary. The important concept is the change in height, i.e. $(h_2 - h_1)$. We, therefore, say that the point of **zero potential energy** is arbitrary. Any point in space can be chosen as a point of zero potential energy. Normally, a point on the surface of the earth is assumed to be the reference point with zero potential energy.

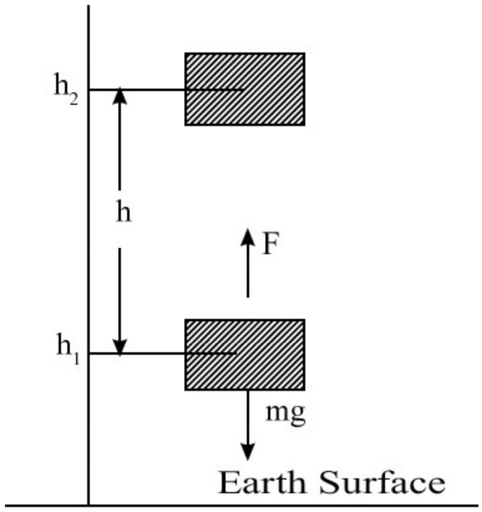


Fig. 5.8 : Object of mass m originally at height h_1 above the earth's surface is moved to a height h_2.

5.4.2 Potential energy of springs

The work done in elongating a spring, against its wish is stored as the potential energy of that spring.

$$F = -kx \qquad (5.18)$$

$$W = \int_{x=0}^{x=x} F.dx \qquad (5.19)$$

$$W = \int -kx.dx = \frac{1}{2}kx^2$$

This work stored as elastic potential energy.

$$U = \frac{1}{2}kx^2 \qquad (5.20)$$

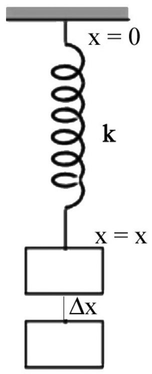


Fig. 5.9 : Elastic potential Energy in the case of a spring - mass system

When the spring is released, it bounces back and the elastic potential energy of the spring is converted into the kinetic energy. The same principle is followed in a loaded gun and a stretched bow to release an arrow.

5.5 CONSERVATION OF ENERGY

We see various forms of energy, in our daily life. Examples are Electrical Energy, Thermal Energy, Gravitational Energy, Chemical Energy and Nuclear Energy etc. These forms of energy are very closely related in the sense that one can be changed to another. There is a very fundamental law about energy. It is known as **Law of Conservation of Energy**. It states, "**The total energy of an isolated system always remains constant.**" The energy may change its form. It can be converted from one form to other. But the total energy of the system remains unchanged. In an isolated system, if there is any loss of energy of one form, there is a gain of an equal amount of another form of energy. Thus energy is neither created nor destroyed.

The universe is also an isolated system as there is nothing beyond this. It is therefore said that the total energy of the universe always remains constant in spite of the fact that variety of changes are taking place in the universe every moment. It is a law of great importance. It has led to many new discoveries in science and it has not been found to fail.

In a Thermal Power Station, the chemical energy of coal is changed into electrical energy. The electrical energy runs machines. In these machines, the electrical energy changes into mechanical energy, light energy or thermal energy.

The law of conservation of energy is more general than we can think of it. It applies to systems ranging from big planets and stars to the smallest nuclear particles.

(a) Conservation of mechanical energy during the free fall of a body

We now wish to test the validity of the law of conservation of energy in case of mechanical energy, which is of immediate interest.

Let us suppose that an object of mass m lying on the ground is lifted to a height h. The work done is mgh, which is stored in the object as potential energy. This object is now allowed to fall freely. Let us calculate the energy of this object when it has fallen through a distance h_1. The height of the object now above the earth surface is $h_2 = h - h_1$ (Fig 5.10). At this point P, the potential energy = mgh_2.

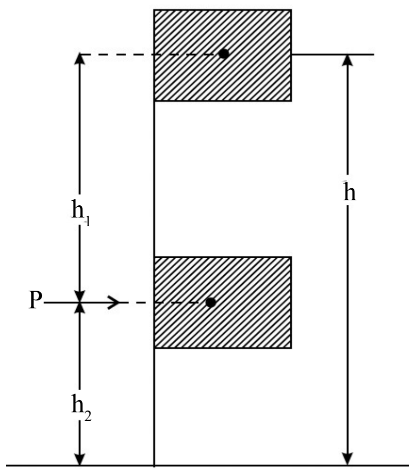


Fig. 5.10 : Mass m is lifted to a height h from earth's surface. It is then lowered to a height h_2 at point P. The total energy at P is same as that at the highest point.

When the object falls freely, it gets accelerated and gains in speed. We can calculate the speed of the object when it has fallen through a height h_1 from the top positions using the equation

$$v^2 = u^2 + 2gs \qquad (5.21)$$

where u is the initial speed at the height h_1, i.e. u = 0 and s = h_1. Then, we have

$$v^2 = 2gh_1$$

The kinetic energy at point P is given by

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{m}{2} \times 2gh_1$$

$$= mgh_1 \quad (5.22)$$

The total energy at the point P is

Kinetic Energy + Potential Energy = $mgh_1 + mgh_2$

$$= mgh \quad (5.23)$$

This is same as the potential energy at the highest point. **Thus, the total Energy is conserved.**

(b) Conservation of Mechanical Energy for a Mass Oscillating on a Spring

Fig. 5.11 shows a spring whose one end is fixed to a rigid wall and the other end is connected to a wooden block lying on a smooth horizontal table. This free end is at x_0 in the relaxed position of the spring. A block of mass m moving with speed v along the line of the spring collides with the spring at the free end, and compresses it by xm. This is the maximum compression. At x_0, the total energy of the spring mass system is $\frac{1}{2}mv^2$. It is the kinetic energy of the mass. The potential energy of the spring is zero. At the point of extreme compression, the potential energy of the spring is $\frac{1}{2}kx_m^2$ and the kinetic energy of the mass is zero. The total energy now is $\frac{1}{2}kx_m^2$. Obviously, this means that

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 \quad (5.24)$$

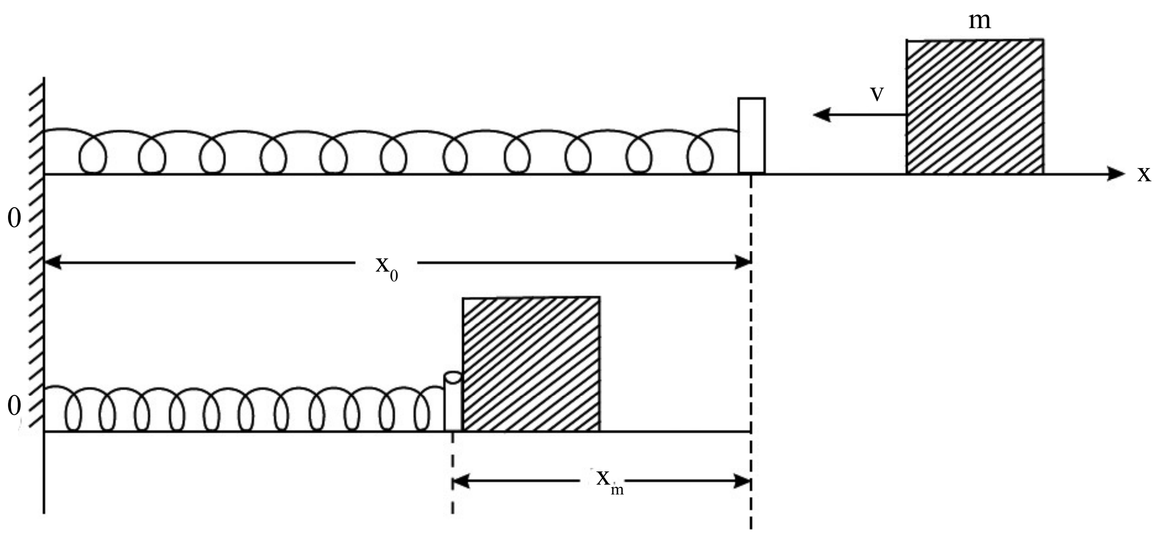


Fig. 5.11 : A block of mass m moving with velocity v on a horizontal surface collides with the spring. The maximum compression is x_m.

K.E + P.E (Before collision) = K.E. + P.E. (After collision)

$$\frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx_m^2 \qquad (5.25)$$

i.e., the total energy is conserved.

Conservation of mass-energy in nuclear reactions

Nuclear energy is different from other forms of energy in the sense that it is not obtained by the transformation of some other form of energy. On the contrary, it is obtained by transformation of mass into energy.

Hence, in nuclear reactions, the law of conservation of mass and the law of conservation of energy merge into a single law of conservation of mass-energy.

Example 5.6

A block of mass 0.5 kg slides down a smooth curved surface and falls through a vertical height of 2.5m to reach a horizontal surface at B Fig 5.12. On the basis of energy conservation, calculate, (i) the energy of the block at point A, and (ii) the speed of the block at point B.

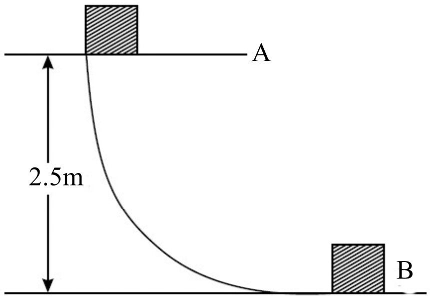


Fig. 5.12 : A block slides on a curved surface. The total energy at A (potential only) gets converted into total energy at B (kinetic only)

Solution :

(i) Potential energy at A

$= mgh = (0.5) \times (9.8) \times 2.5$ J

$= 4.9 \times 2.5$ J $= 12.25$ J

The kinetic energy at A = 0 and

Total Energy = 12.25 J

(ii) The total energy of the block at A must be the same as the total energy at B.

The total energy (P.E. + K.E.) at A = 12.25 J

The total energy (P.E. + K.E.) at B = $\frac{1}{2}mv^2$

Since P.E. at B is zero, the total energy is only K.E.

$$\therefore \quad \frac{1}{2}mv^2 = 12.25$$

$$v^2 = \frac{12.25 \times 2}{0.5} = 12.25 \times 4$$

$$v^2 = 49.00$$

Hence $\quad v = 7.0 \text{ ms}^{-1}$

Note: This can also be obtained from the equations of motion :

$$v^2 = v_0^2 + 2gx$$

$$= 0 + 2 \times 9.8 \times 2.5$$

$$v^2 = 49$$

$$v = 7 \text{ ms}^{-1}$$

5.6 POWER

You have already learnt to calculate the work done by a force. In all such situations, we did not consider whether the work is done in one second or in one hour. In our daily life, however, the time taken to perform a particular work is important. For example, a man may take several hours to load to truck with cement bags, whereas a machine or a crane may do this work in much less time. Therefore, it is important to know the rate at which work is done or the rate at which energy is delivered to do a particular work.

"The rate of doing work or the rate of utilizing the energy is called power."

$$\text{Power} = \frac{\text{Work done}}{\text{time taken}} = \frac{\text{Energy delivered}}{\text{time taken}}$$

Mathematically, we can write

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t} \qquad (5.26)$$

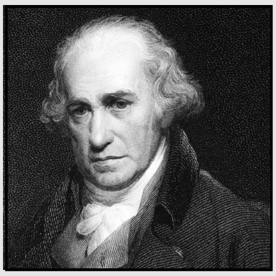
If the rate of doing work is not constant, this rate may vary. In such cases, we may define instantaneous power P.

$$P = \lim_{\Delta t \to 0}\left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt} \qquad (5.27)$$

- Power can also be written as the dot product of force and velocity $P = \overline{\mathbf{F}} \cdot \overline{\mathbf{V}}$
- SI unit of power is joule / second. It is also known as watt (W), which is in the rememberance of James Watt, who invented a Steam Engine.

James Watt (1736–1819)

Scottish inventor and mechanical engineer, James Watt is renowned for improving the efficiency of a steam engine. This paved the way for industrial revolution. He, introduced horse power as the unit of power. SI unit of power watt is named in his honour. Some of the important inventions by James Watt are : a steam locomotive and an attachment that adapted telescope to measure distances.



The larger unit of power is horsepower (hp)

$$1 \text{ hp} = 746 \text{ W}$$

The unit of power (watt) is used to define a new unit of work (energy). One such unit of work is kilowatt-hour. This unit is commonly used in the unit of electrical consumption.

$$1 \text{ kwh} = 1 \times 10^3 \times \text{ J/s} \times 60 \times 60 \text{s}$$

$$1 \text{kwh} = 36 \times 10^5 \text{J} = 3.6 \text{ MJ (mega joules)}$$

- Power can also be defined as the following, in various situation.

$$P = \frac{KE}{\text{time}} = \frac{1/2\, mv^2}{t} \qquad (5.28)$$

$$P = \frac{\text{Potential energy}}{\text{time}} = \frac{mgh}{t} \qquad (5.29)$$

Example 5.7

A truck is loaded with sugar bags. The total mass of the load and the truck together is 100,000 kg. The truck moves on a winding path up a mountain to a height of 700 m in 1 hour. What average power must the engine produce to lift the material?

Solution :

$$W = mgh$$
$$= (100{,}000 \text{ kg}) \times (9.8 \text{ m s}^{-2} \times 700 \text{ m})$$
$$= 9.8 \times 7 \times 10^7 \text{ J}$$
$$= 68.6 \times 10^7 \text{J}$$

Time taken = 1 hour = 60 × 60 s

= 3600 s

Average Power, P = W / t

$$= \frac{68.6 \times 10^7 \text{ J}}{3600 \text{ s}}$$

$$= 1.91 \times 10^5 \text{ watt}$$

We know that 746 W = 1 hp

$$P = \frac{1.91 \times 10^5}{746} = 2.56 \times 10^2 = 256 \text{hp}$$

Example 5.8

Hydroelectric power generation uses falling water as a source of energy to turn turbine blades and generate electric power. In a power station 1000×10^3 kg water falls through a height of 51 m in one second.

(i) Calculate the work done by the falling water?

(ii) How much power can be generated under ideal conditions?

Solution :

(i) The potential energy of the water at the top is converted into work in moving the balance of the turbine.

$$PE = Work = mgh$$

$$= (1000 \times 10^3 \text{ kg}) \times (9.8\text{ms}^{-2}) \times (51 \text{ m})$$

$$PE = W = 500 \times 10^6 \text{ J}$$

(ii) The work done per second is given by

$$P = \frac{W}{t} = \frac{500 \times 10^6 \text{ J}}{1\text{s}}$$

$$= 500 \times 10^6 \text{ watt}$$

$$PE = 500 \text{ MW}$$

Intext Questions 5.4

1. A body of mass 100 kg is lifted through a distance of 8 m in 10 s. Calculate the power of the lifter.
2. Convert 10 horse power into kilo watt.
3. A car of mass 1000 kg is moving at a speed of 90 kmh^{-1}. Brakes are applied and the car stops at a distance of 15 m from the breaking point. What is the average force applied by brakes? If the car stops in 25 s after breakings, calculate the average power of the brakes.
4. An engine of car applies a force of 4000 N to make it to move at a speed of 72 kmph. What is the power of the engine.

5.7 CONSERVATIVE AND NON-CONSERVATIVE (DISSIPATIVE) FORCES

(a) Conservative forces : We have seen that the work done by the gravitational force acting on an object depends on the product of the weight of the object and its vertical displacement. If an object is moved from a point A to a point B under gravity, Fig. 5.13, the work done by gravity depends on the vertical separation between the two points. It does not depend on the path followed to reach B starting from A. When a force obeys this rule, it is called **a conservative force**. Some of the examples of conservative forces are gravitational force, elastic force and electrostatic force.

A conservative force has a property that the work done by a conservative force is independent of path. In Fig 5.13 (a)

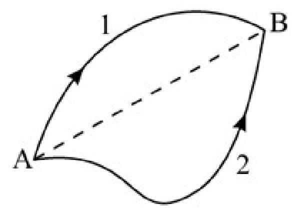
W_{AB} (along 1) = W_{AB} (along 2)

Fig. 5.13 (b) shows the same two positions of the object. The object moves from A to B along the path 1 and returns back to A along the path 2. By definition, the work done by a conservative force along path 1 is equal and opposite to the work done along the path 2.

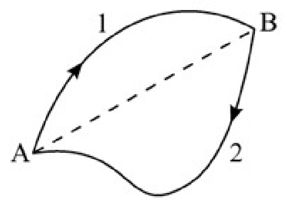
W_{AB} (along 1) = $-W_{BA}$ (along 2)

or $W_{AB} + W_{BA} = 0$

This result brings out an important property of the conservative force in ***that the work done by a conservative force on an object is zero when the object moves around a closed path and returns back to its starting point.***



(a) The object is moved from A to B along two different paths



(b) It is taken from A to B along path 1 and brought back to A along path 2

Fig. 5.13

(b) Non-conservative Forces : The force of friction is a good example of a non-conservative force. Fig. 5.14 shows a rough horizontal surface. A block of mass m is moving on this surface with a speed v at the point A.

After moving a certain distance along a straight line, the block stops at the point B. The block had a kinetic energy $E = \frac{1}{2}mv^2$ at the point A. It has neither kinetic energy nor potential energy at the point B. It has lost all its energy. Do you know where did the energy go? It has changed its form. Work has been done against the frictional force or we can say that force of friction has done negative work on the block. The kinetic energy has changed to thermal energy of the system. The block with the same kinetic energy E is now taken from A to B through a longer path 2. It may not even reach the point B. It may stop much before reaching B. This obviously means that more work has to be done along this path. Thus, it can be said that the work done depends on the path.

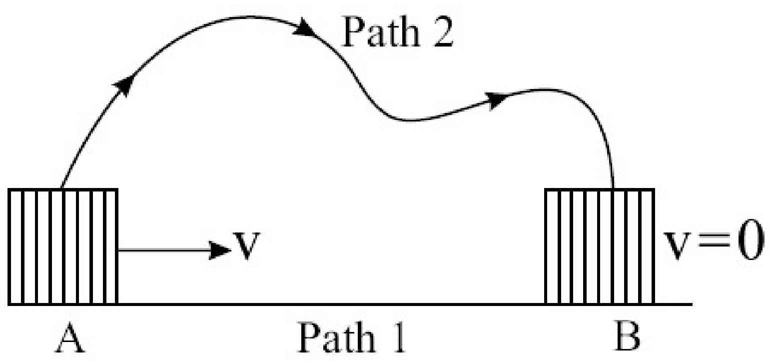


Fig. 5.14 : A block which is given an initial speed v on a rough horizontal surface, moves along a straight line path 1 and comes to rest at B. It starts with the same speed at A but now moves along a different path 2.

Intext Questions 5.5

1. ABC is a triangle where AB is horizontal and BC is vertical. The length of the sides AB = 3 m, BC = 4 m and AC = 5 m (see Fig. 5.15). A block of mass 2 kg is at A. What is the change in potential energy of the block when

   (a) it is taken from A to B

   (b) from B to C

   (c) from C to A

   (d) How much work is done by gravitational force in moving the mass form B to C (positive or Negative work)?

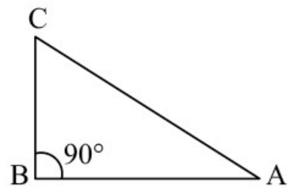


Fig. 5.15

2. A ball of mass 0.5 kg is at A at a height of 10m above the ground (see Fig. 5.16). Solve the following questions by applying work-energy principle. In free fall

   (a) What is the speed of the ball at B?

   (b) What is the speed of the ball at the point C?

   (c) How much work is done by gravitational force in bringing the ball from A to C (give proper sign)?

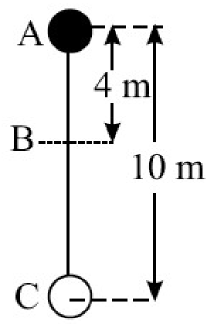


Fig. 5.16

3. A block at the top of an inclined plane slides down. The length of the plane BC = 2 m and it makes an angle of 30° with horizontal (see Fig. 5.17). The mass of the block is 2 kg. The kinetic energy of the block at the point B is 15.6 J. How much of the potential energy is lost due to non-conservative forces (friction). How much is the magnitude of the frictional force?

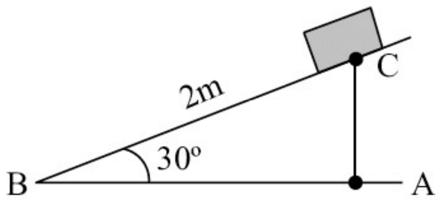


Fig. 5.17

4. The Figure 5.18 shows two curves A and B between energy E and displacement x of the bob of a simple pendulum. Which one represents the P.E. of the bob and why?

5. When non-conservative forces work on a system, does the total mechanical energy remain constant?

6. When a conservative force does positive work on a body, what happens to the potential energy of the body.

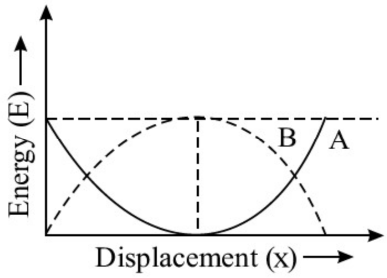


Fig. 5.18

5.8 ELASTIC AND INELASTIC COLLISIONS

In daily life we come across many collisions between vehicles, a hammer and a nail, between a ball and a bat etc. Not only in visible objects, collisions are common among invisible atoms, particles, nuclei.

In the absence of external forces, when two bodies interact for a small interval of time, it is termed as 'collision' between those two bodies.

Let us start with a collision of two balls and to make the analysis simpler, let there be a **"head - on"** or **"central collision"**. In such collisions, colliding bodies move along the line joining their centres. The collisions are of two types.

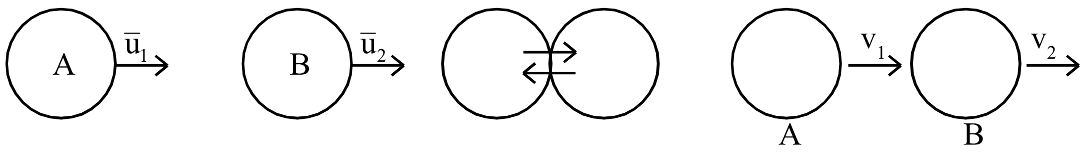
(i) Perfectly Elastic Collision : A collision in which both the law of conservation of linear momentum and the law of conservation of kinetic energy are followed i.e., the total momentum and total kinetic energy are same before and after the collision.

(ii) Perfectly inelastic Collision : A collision in which Law of Conservation of linear momentum is followed but not the law of conservation of kinetic energy. In these type of collisions colliding objects stick together, after the collision and move with a common velocity.

In both the types of collisions law of conservation of energy holds good.

5.8.1 Elastic Collision

Let two balls A and B having masses m_1, and m_2 respectively collide "head-on" as shown in Fig. 5.19. Let $\bar{u}_1$, $\bar{u}_2$ be the velocities of the two balls before collision and $\bar{v}_1$, $\bar{v}_2$ be their final velocities after the collision. Assume that $u_1 > u_2$.



Before the Collision After the Collision

Fig. 5.19 : Schematic representation of an Elastic Head-on Collision.

Now applying law of conservation of linear momentum. total momentum before collision = total momentum after the collision.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 (u_1 - v_1) = m_2 (u_2 - v_2) \qquad (5.30)$$

From the law of conservation of kinetic energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 \left(u_1^2 - v_1^2\right) = m_2 \left(u_2^2 - v_2^2\right) \qquad (5.31)$$

Divide equation 5.29 with equation 5.28.

We get $\quad u_1 + v_1 = u_2 + v_2$

or $\quad u_1 - u_2 = v_2 - v_1 \qquad (5.32)$

It indicates that the relative velocity of approach is equal to the relative velocity of seperation from 5.30, we can write.

$$v_2 = u_1 - u_2 + v_2 \qquad (5.33)$$

Substituting equation 5.31 in equation 5.28, we get

$$m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2\ m_2u_2$$

$$v_1(m_1 + m_2) = u_2(m_1 - m_2) + 2\ m_2u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad (5.34)$$

Similarly by substituting $v_1 = v_2 + u_2 - u_1$ from equation 5.30 in equation 5.28, we get

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_2 \qquad (5.35)$$

Now, we discuss few special cases.

Case - I

Suppose that the two balls have same mass i.e., $m_1 = m_2 = m$ in equations 5.32 and 5.33

We get

$$v_1 = u_2$$

$$v_2 = u_1$$

That is, if two identical balls collide "head-on", their velocities, after the collision, get interchanged.

Now think what would happen if one of the ball is at rest before the collision.

Let B be at rest so that $u_2 = 0$. Then

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

If $m_1 = m_2$ then $v_1 = 0$ and $v_2 = u_1$

After the collision, A comes to rest and B moves with the velocity of A before collision.

Similar conclusion can be drawn about the kinetic energy of the balls after collision. Complete loss of kinetic energy or partial loss of kinetic energy $(m_1 + m_2)$ by A is same as the gain in the kinetic energy of B. These facts have very important applications in nuclear reactors in solwing down neutrons.

Case - II

The second interesting case is that of collision of two particles of unequal masses.

(i) Let us assume that m_2 is very large compared to m_1 and particle B is initially at rest.

$$m_2 >> m_1 \qquad u_2 = 0$$

Then, the mass m_1 can be neglected in comparison to m_2.

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = -u_1 \;(\because m_1 \text{ can be neglected in comparison to } m_2 \,;\, m_1 + m_2 = m_2)$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$$v_2 = 0 \;\left(\because \;\; m_1 << m_2 \rightarrow \frac{m_1}{m_2} \rightarrow 0\right)$$

After the collision, the heavy particle continues to be at rest. The light particle returns in its path velocity equal to its initial velocity.

This is what happens when a child hits a wall with a ball.

These results find applications in physics of atoms, as for example in the case where an α - particle hits a heavy nucleus such as uranium.

5.8.2 Coefficient of Restitution (e)

The Coefficient of restitution is defined as the ratio of relative velocity of seperation to the relative velocity of approach.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \qquad (5.36)$$

Where u_1, u_2 are the velocities of two objects before the collision and v_1, v_2 are the velocities after the collision.

e is a unitless and dimensionless quantity.

'e' depends on the nature of the colliding objects. For a perfectly elastic collision, e = 1, and e = 0 for a perfectly inelastic collision. But, the practical value of e lies between 0 and 1.

5.8.3 Determination of Coefficient of restitution

To determine the coefficient of restitution between any two materials, one is taken as a ball and other is taken as a plate. Ball is allowed to fall on the plate from a height h_1. Let the ball rebounds to a height h_2 after the collision.

Then the coefficient of restitution is given by, $e = \sqrt{\frac{h_2}{h_1}}$ (5.37)

Intext Questions 5.6

1. Two hard balls collide when one of them is at rest.

   (a) Is it possible that both of them remain at rest after collision?

   (b) Is it possible that one of them remains at rest after collision?

2. There is a system of three identical balls A, B and C on a straight line as shown here. B and C are in contact and at rest. A moving with a velocity v collides "head-on" with B. After collision, what will be the velocities of A, B and C separately? Explain.

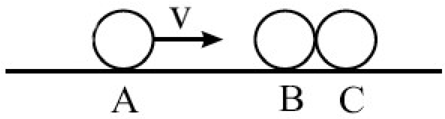


Fig. 5.20

3. Ball A of mass 2 kg collides head-on with ball B of mass 4 kg. A is moving in + x direction with speed 50 ms^{-1} and B is moving in –x direction with speed 40 ms^{-1}. What are the velocities of A and B after collision? The collision is elastic.

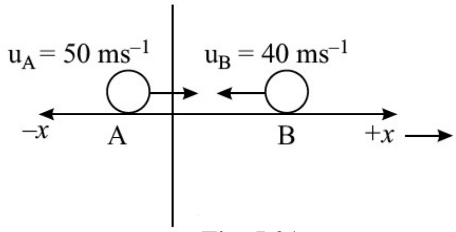


Fig. 5.21

4. A bullet of mass 1 kg is fired and gets embedded into a block of mass 1 kg initially at rest. The velocity of the bullet before collision is 90 m/s.

   (a) What is the velocity of the system after collision?

   (b) Calculate the kinetic energies before and after the collision?

   (c) Is it an elastic collision or inelastic collision?

   (d) How much energy is lost in collision?

5. In an elastic collision between two balls, does the kinetic energy of each ball change after collision?

6. When two balls undergo a perfect inelastic collision, what is their relative velocity after the collision?

7. What is the nature of forces involved in an elastic collision?

8. In which type of collision, the mechanical energy is converted into other form of energy?

9. A ball falling from a height on to the ground, rebounds after the collision. Which scenarios described above demonstrates conservation of linear momentum?

WHAT YOU HAVE LEARNT

- Work done by a constant force F is

$$W = \overline{\mathbf{F}}.\overline{\mathbf{S}} = FS\cos\theta$$

Where θ is the angle between $\overline{\mathbf{F}}$ and $\overline{\mathbf{S}}$

- Work is a scalar quantity. Its S.I. unit is Joule (J).
- Work is numerically equal to the area under the force, displacement graph.
- Work done by elastic force obeying Hooke's law is $W = \frac{1}{2}kx^2$

  Where k is a force (spring) constant.
- The sign of W is positive for the external force acting on the spring and negative for the restoring force offered by the spring. x is the compression or elongation of the spring.
- The unit of 'k' is newton per meter (Nm^{-1})
- Mechanical energy of a system exists in two forms (i) kinetic energy and (ii) potential energy.
- Kinetic energy is the energy possessed by a moving object. It is given by

$$KE = \frac{1}{2}mv^2$$

- Potential energy is a stored energy possessed by an object by virtue of its position and configuration
- Gravitational potential energy PE = mgh.
- Energy is a scalar quantity. Its units are same as that of work.
- The Work - Energy Theorem states that work done by a resultant force is equal to the change in the kinetic energy of the object.

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K_f - K_i = \Delta K$$

- Power is the time rate of doing work (or) the time rate of utilizing energy.

$$P = \frac{W}{t} = \frac{E}{t}\text{; units J/s or watt (W)}$$

- Work done by a conservative force on a particle is equal to the change in mechanical energy of the particle, that is change in kinetic energy + the change in potential energy. In other words the mechanical energy is conserved under conservative forces.

$$\Delta E = (\Delta E)_{KE} + (\Delta E)_{PE}$$

- Work done by a conservative force on an object is zero for a round trip of the object i.e., work done by a conservative force is independent of path. It depends only on its initial and final positions.

- Energy stored in a compressed or a stretched spring is known as elastic potential energy, and it is given by $PE_{elastic} = \frac{1}{2}kx^2$.
- Law of conservation of Energy: Energy neither be created nor destroyed, but it can be transformed from one type to another type. The total energy of an isolated system, always remains constant.
- Law of conservation of linear momentum always holds good in any type of collision. But law of conservation of kinetic energy hold good in elastic collisions only.
- Law of conservation of energy holds good for all types of collisions.
- The coefficient of restitution is defined as the ratio of relative velocity of seperation to relative velocity of approach.

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

- e = 1 for perfectly elastic collision.

  e = 0 for perfectly inelastic collision.
- The practical values of e lies between '0' and '1'

TERMINAL EXERCISE

1. If two particles have the same kinetic energy, are their momenta also same? Explain.
2. A particle in motion collides with another one at rest. Is it possible that both of them are at rest after collision?
3. Does the total mechanical energy of a system remain constant when dissipative forces work on the system?
4. A child throws a ball vertically upwards with a velocity 20 ms^{-1}.

   (a) At what point is the kinetic energy maximum?

   (b) At what point is the potential energy maximum?
5. A block of mass 3 kg moving with a velocity 20m s^{-1} collides with a spring of force constant 1200 N m^{-1}. Calculate the maximum compression of the spring.
6. What will be the compression of the spring in question 5 at the moment when kinetic energy of the block is equal to twice the elastic potential energy of the spring?
7. The power of an electric bulb is 60 W. Calculate the electrical energy consumed in 30 days if the bulb is lighted for 12 hours per day.
8. 1000 kg of water falls every second from a height of 120 m. The energy of this falling water is used to generate electricity. Calculate the power of the generator assuming no losses.

9. The speed of a 1200 kg car is 90 km h^{-1} on a highway. The driver applies brakes to stop the car. The car comes to rest in 3 seconds. Calculate the average power of the brakes.

10. A 400 g ball moving with speed 5 ms^{-1} has elastic head-on collision with another ball of mass 600 g initially at rest. Calculate the speed of the balls after collision.

11. A bullet of mass 10 g is fired with an initial velocity 500 ms^{-1}. It hits a 20 kg wooden block at rest and gets embedded into the block.

    (a) Calculate the velocity of the block after the impact

    (b) How much energy is lost in the collision?

12. An object of mass 6 kg is resting on a horizontal surface. A horizontal force of 15 N is constantly applied on the object. The object moves a distance of 100 m in 10 seconds.

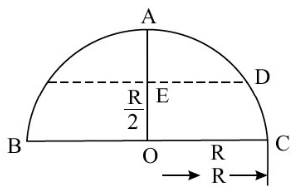
    (a) How much work does the applied force do?

    (b) What is the kinetic energy of the object after 10 seconds?

    (c) What is the magnitude and direction of the frictional force (if there is any)?

    (d) How much energy is lost during motion?

13. A, B, C and D are four point on a hemispherical cup placed inverted on the ground. Diameter BC = 50 cm. A 250 g particle at rest at A, slide down along the smooth surface of the cup. Calculate it's



    (a) Potential energy at A relative to B.

    (b) Speed at the point B (Lowest point).

    (c) Kinetic and potential energy at D.

    Do you find that the mechanical energy of the block is conserved? Why?

14. The force constant of a spring is 400 N/m. How much work must be done on the spring to stretch it (a) by 6.0 cm (b) from x = 4.0 cm to x = 6.0 cm, where x = 0 is the relaxed position of the spring.

15. The mass of a car is 1000 kg. It starts from rest and attains a speed of 15 ms^{-1} in 3.0 seconds. Calculate

    (a) The average power of the engine.

    (b) The work done on the car by the engine.

16. A ball of mass 0.6 kg moving with a speed of 2 m/s collides with a ball of mass 0.8 kg. If the collision is an elastic collision, find the velocities of the two balls after the collision?

17. A ball of mass 0.2 kg is allowed to fall on the plane surface from a height of 1 m. After the collision, ball rebounds to a height of 0.64 m. What is the value of coefficient of restitution between the ball and surface.

18. A bomb at rests explodes into two fragments of masses 1 kg, 2 kg respectively. If the speed of the small fragment is 200 ms^{-1}, then what is the speed of the other fragment?

19. A machine gun can fire 240 bullets per minute. The speed of the each bullet is 500 m/s power of the gun is 2.5 kW. Find the mass of the each bullet.

20. A passenger of mass 70 kg with a bag of mass 30 kg on his head climbing a staircase in a Railway Station to reach his platform. The height of each step is 30 cm and the number of steps in the staircase are 30. Calculate the work done by the passenger against the gravitational attraction. ($g = 10ms^{-2}$)

ANSWERS TO INTEXT QUESTIONS

5.1

1. $W = FS \cos \theta$

   $W = FS \cos 90° = 0$

   Hence, No work is done by the force.

2. (a) When there is no displacement even after the application of Force.

   Ex: A boy trying to push a big wall.

   (b) When the angle between force and displacement is in between 180° to 270°.

   Ex: Work done by the frictional force.

   (c) When the angle between the force and displacement is less than 90°.

   Ex: Player kicking the foot ball towards the goal post.

3. (a) $W = mgh = +\ 98J$

   (b) $W = mgh = -\ 98J$

4. $\mathbf{F} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)N,\ S = \left(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right)m$

$$W = F.S = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right).\left(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right)$$

$$= 2 \times -1\left(\hat{\mathbf{i}}.\hat{\mathbf{i}}\right) + 3 \times 2\left(\hat{\mathbf{j}}.\hat{\mathbf{j}}\right)$$

$$= \left(2 \times -1\right) + \left(3 \times 2\right)$$

$$W = -2 + 6 = 4$$

$$W = 4J$$

5. $F = \left(5\hat{i} + 3\hat{j}\right)N; \bar{\mathbf{S}} = \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)m$

(a) $|S| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\,m$

(b) $|F| = \sqrt{5^2 + 3^2} = \sqrt{34} = 5.83\,N$

(c) $W = \bar{\mathbf{F}}.\bar{\mathbf{S}} = \left(5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right).\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)$

$= 15 + 12 = 27$

$\therefore \quad W = 27\,J$

5.2

1. Spring constant (k) is defined as the restoring force per unit elongation. Its unit is Nm^{-1}.

2. $k = \dfrac{10\,N}{10^{-2}\,m} = 1000\,Nm^{-1}$

As $F = kx$ for $x = 5$ cm $= 5 \times 10^{-2}$ m

$F = 1000 \times 5 \times 10^{-2}$

$F = 50$ N

3. Work done by the force.

5.3

1. $K = \dfrac{1}{2}mv^2$. No, it can't have negative value at all, because neither m nor v^2 can be negative.

2. (a) KE becomes 4 times

(b) KE becomes half

3. $PE = \dfrac{1}{2}4x^2 = 3.6J$

$$x^2 = \frac{2 \times 3.6}{k} = \frac{2 \times 3.6}{180} = 0.04\,m$$

$x = 0.2$ m $= 20$ cm.

4. – 375J

5.4

1. PE = mgh = 100 × 9.8 × 8

   t = 10 s

$$\text{Power} = \frac{mgh}{t} = \frac{100 \times 9.8 \times 8}{10}\,\text{W} = 784\,\text{W}$$

2. $$10\,\text{hp} = 10 \times 746\,\text{watts} = \frac{10 \times 746}{1000}\,\text{Kilo watts}$$

$$= 7.46 \text{ KW}$$

3. $v^2 - u^2 = 2$ as $u = 0$; $u = 90\,\text{km/h} = 25\,\text{ms}^{-1}$

$$a = \frac{v^2}{2s} = \frac{25 \times 25}{2 \times 15} = 20.83\,\text{ms}^{-2}$$

F = ma = 1000 × 20.83 = 20830 N.

$$P = \frac{W}{t} = \frac{20830}{25} = 12498\,\text{W}$$

4. $$P = F.V = 4000 \times \left(\frac{\cancel{72}^{4} \times 5}{\cancel{18}}\,\text{m/s} \right)$$

$$= 4000 \times 20 = 80000 = 80 \text{ KW}$$

5.5

1. (a) No change

   (b) Change in PE = mgh = 2 × 9.8 × 4 = 78.4 J

   (c) $\Delta PE = 78.45$

   (d) –78.4 J

2. (a) $\Delta PE = mgh = 0.5 \times 9.8 \times 4 = 19.6\,\text{J}$

$$\text{KE at B} = \frac{1}{2} mv^2 = 19.6\,\text{J}$$

$$v^2 = \frac{19.6 \times 2}{0.5} = 78.4$$

$$v = \sqrt{78.4} = 8.85\,\text{ms}^{-1}$$

(b) $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14ms^{-1}$

(c) W = mgh = + 49 J.

3. $BC = 2m \; ; \; \sin 30° = \dfrac{AC}{BC} = \dfrac{AC}{2}$

$$AC = 2\sin 30° = 2 \times \frac{1}{2} = 1m$$

Δ PE from C to B = mgh = 2 × 9.8 × 1 = 19.6 J

KE at B = 15.6 J

Energy lost = (19.6 – 15.6) J = 4 J.

4 J = F × S = F × 2

Frictional force F = 2N.

4. When the bob of simple pendulum oscillates its KE is max at x = 0 and minimum at $x = x_m$.

The PE is minimum at x = 0 and maximum at $x = x_m$.

Hence A represents the PE curve.

5. No

6. PE decreases du = – fdx

$U_f - U_i = - dW$

5.6

1. (a) No, it violates law of conservation of linear momentum.

(b) Yes.

2. $v_A = 0 \; ; \; v_B = 0 \; ; \; v_c = v$

In order to obey laws of conservation of (1) linear momentum and (2) total kinetic energy.

3. By using equations 5.34 & 5.35

$$v_A = -35ms^{-1}$$

$$v_B = 20\,m/s$$

4. (a) 1.76 ms^{-1} (b) 81 J and 1.58 J

(c) Inelastic collision (d) 79.42 J

5. Yes

6. Zero
7. conservative forces
8. Inelastic collision
9. Ball falling from a height and then bouncing back after hitting the ground.

ANSWERS TO TERMINAL EXERCISE

1. No, $KE = \frac{p^2}{2m}$; $KE_1 = KE_2$

$$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2} \Rightarrow p_1 \neq p_2 \text{ as } m_1 \neq m_2$$

2. No
3. No
4. (a) at initial point
   (b) at maximum height
5. $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$3 \times (20)^2 = 1200 \times x^2$$

$$3 \times 400 = 1200x^2 \Rightarrow x = 1m$$

6. 0.707 m
7. $P = \frac{E}{t} \Rightarrow E = Pt$

$$= 60 \times 30 \times 12 \times 60 \times 60 \text{ joules}$$

$$E = \frac{60 \times 30 \times 12 \times 60 \times 60}{36 \times 10^5} \text{KWh}$$

$$= 21.6 \text{ KWh} = 21.6 \text{ units}$$

8. $P = \frac{mgh}{t} = \frac{1000 \times 9.8 \times 120}{1} = 1176000 \text{ J/s}$

$$\simeq 1.2 \text{ mega Watt}$$

9. $W = \frac{1}{2}mv^2 = \frac{1}{2} \times 1200 \times \left(\frac{90 \times 5}{18}\right)^2$

$$= \frac{1}{2} \times 1200 \times (25)^2$$

$$= 600 \times 625 = 375000 \text{ J}$$

$$P = \frac{W}{t} = \frac{375000}{3} = 125000\,W$$

$$= 125 \text{ KW.}$$

10. $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 = \frac{(-200)}{1000} \times 5 = -1 \text{ m/s}$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1 = \frac{(2 \times 400)}{1000} \times 5 \text{ ms}^{-1} = 4\,\text{ms}^{-1}$$

11. (a) $m_1 u_1 = (m_1 + m_2) v$

$$v = \frac{m_1 u_1}{m_1 + m_2} = \frac{10^{-2} \times 500}{10^{-2} + 20} = \frac{5}{20 + 0.01}\ ;\ 0.25\,\text{ms}^{-1}$$

(b) $\Delta KE = \frac{1}{2}\left[m_1 u_1^2 - (m_1 + m_2) v^2\right] = 1249.4\text{J}$

12. (a) 500 J

(b) 1200 J

(c) 3 N opposite to the direction of motion

(d) 300 J

13. (a) mgh = 0.625 J

(b) $PE = \frac{1}{2}mv^2$

(c) 0.313 J

14. (a) $k = 400$ n/m $\quad W = \frac{1}{2}kx^2 = \frac{1}{2} \times 400 \times (0.06)^2$

(b) $W = \frac{1}{2}\left[x_2^2 - x_1^2\right] = 0.4 \text{ J}$

15. (a) $W = \dfrac{\frac{1}{2}mv^2}{t} = \dfrac{1}{2} \times \dfrac{1000 \times 15 \times 15}{3} = 37.5\,kW$

(b) $W = p \times t = 1.125 \times 10^5\ J$

16. $v_1 = 0.28\,m/s \quad v_2 = 1.72\,m/s$

(Hint $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$; $u_1 - u_2 = v_1 - v_2$)

17. $e = 0.8 \left(\because \quad e = \sqrt{\dfrac{h_2}{h_1}} \right)$

18. $-100\,m/s\ \left(0 = m_1v_1 + m_2v_2\right)$

19. $m = 5\,g \left(\because \quad P = \dfrac{\frac{1}{2}\ mnv^2}{t} \right)$

20. $W = (m_1 + m_2)\ gh$

$= 100 \times 10 \times \left(0.3\,m\right) \times 30$

$= 1000 \times 9$

$= 9\ KJ$

6

SYSTEM OF PARTICLES & ROTATORY MOTION

INTRODUCTION

So far you have learnt about the motion of a single object, usually taken as a point mass. This simplification is quite useful for learning the laws of mechanics. But in real life, objects consist of very large number of particles. A tiny pebble contains millions of particles. Do we then write millions of equations, one for each particle? Or is there a simpler way? While discovering answer to this question you will learn about centre of mass and moment of inertia, which plays the same role in rotational motion as does mass in translational motion.

You will also study an important concept of physics, the angular momentum. If no external force acts on a rotating system, its angular momentum in conserved. This has very important implications in physics. It enables us to understand how a swimmer is able to somersault while diving from a diving board into the water below.

OBJECTIVES

After studying this lesson, you should be able to

- *differentiate Rigid Body from other objects;*
- *define centre of mass and centre of gravity of a rigid body;*
- *explain the types of motions that a rigid body can have;*
- *differentiate translational motion from Rotational motion;*
- *define moment of inertia and state theorems of parallel and perpendicular axes;*
- *define torque and find the direction of rotation produced by it;*
- *write the equations of motion of a rigid body;*
- *state the principle of conservation of angular momentum;*
- *calculate the velocity acquired by a rigid body at the end of its motion on an inclined plane.*

6.1 RIGID BODY

As mentioned earlier, point masses are ideal constructs, brought in for simplicity in discussion. But in reality, all objects have finite size and these are called extended bodies. When extended bodies interact with each other and the distances between them are very large compared to their sizes, their enormous sizes can be ignored and they may be treated as point masses. Though the sizes of planets, stars and other celestial objects are very large, we consider them as point objects due to their distances from others. While dealing universal law of gravitation, all the objects including celestial objects are treated as point masses. But when we have to consider the rotation of a body about an axis, the size of the body becomes important. When we represent a bigger object like a football, we consider the centre of mass as the representative of all the mass particles of the football.

When we consider the rotatory motion of an object, we treat the object as a system of particles.

An extended body can be considered as a Rigid body, is an ideal concept. As such, there are no rigid bodies in our vicinity. A body in which the relative distance between any two points of it doesn't change, even after the application of huge force is called as a **"Rigid Body"** (or) A body which does not lose its shape or size even after the application of a deforming force. For practical purposes we consider many objects as rigid objects by neglecting the deformation found in them. E.g.: A cricket ball, steel wheel, earth and the moon.

Can water in a bucket be considered a rigid body? Obviously, water in a bucket cannot be a rigid body because it change shape as bucket is moved around.

You may now like to check what you have understood about a rigid body.

Intext Questions 6.1

1. A frame is made of six wooden rods. The rods are firmly attached to each other. Can this system be considered as a rigid body?
2. Can a heap of sand be considered as a rigid body? Explain your answer.

6.2 CENTRE OF MASS (CM) OF A RIGID BODY

Suppose, you want to lift a meter scale, horizontally with the help of a strong thread. Where do you tie the meter scale? If some mass is tied at one of the ends of the meter scale, then you try to lift the scale. Can you lift it horizontally with the thread tied at the previous position?

Answer to both the questions are as follows.

1. If you want to lift the meter scale of uniform thickness, then you are, obviously, going to tie the thread at the midpoint i.e., at 50 cm division of the scale, to lift it horizontally.
2. When some mass is attached to the same scale, you can't lift it horizontally, without changing the position of the knot of the thread.

In the first situation, though you tie at a single point, you are able to lift the whole scale and you feel the total mass of the scale. If we cut a very small part of the scale at 50^{th} division, then you won't feel the same mass as that of the scale.

So, **"Centre of mass (CM)** is the point of a rigid body, where the whole mass of it appears to be concentrated".

Suppose, equal masses are attached, at both the ends of the scale, then the position of the CM will remain at the midpoint of the scale.

Centre of mass simplifies the motion of the object, though the motions of individuals mass points are very complex. Let's start with a two particle system. Let two infinitesimally small particles of masses m_1 and m_2 are placed at the position x_1 and x_2 respectively from the origin, as shown in the diagram.

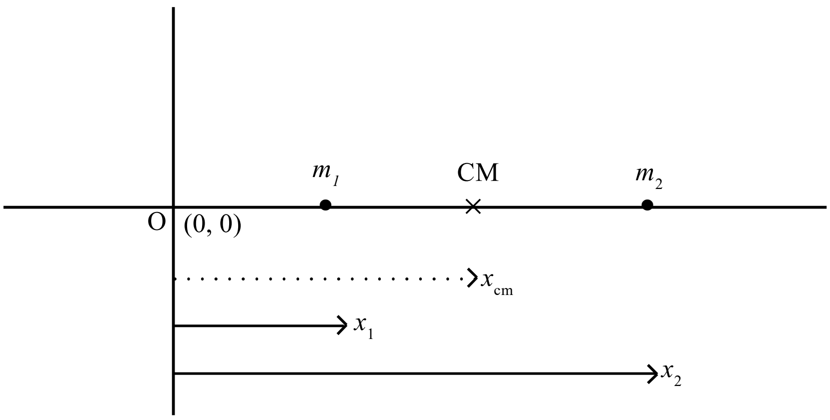


Fig. 6.1 : Centre of mass of two particle system.

Let the position of the CM of this system be X_{cm} from the origin 'O'.

The position of the CM is given by

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \tag{6.1}$$

If we have n number of mass particles with masses m_1, m_2, m_3 ...... m_n respectively, placed along the X - axis, at a distances of x_1, x_2, x_3, ... x_n respectively from the origin, the position of the CM is given by

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n} \tag{6.2}$$

$$X_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M} \sum m_i x_i$$

Where $M = \sum m$ = total mass of the system

Suppose that we have a 2 dimensional system of particles, lying in a plane XY. Positions of mass particles m_1, m_2, m_3 .... m_n are (x_1, y_1), (x_2, y_2) .... (x_n, y_n) respectively.

The position of CM of this 2D object (system) is given by

$$\left(X_{cm}, Y_{cm}\right) = \left(\frac{\sum m_i x_i}{\sum m_i}, \frac{\sum m_i y_i}{\sum m_i}\right) \tag{6.3}$$

Similarly, for 3D object, having the particles of masses m_1, m_2, m_3 .... m_n at the coordinates (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n), the position of the CM is given by

$$\left(X_{cm}, Y_{cm}, Z_{cm}\right) = \left(\frac{\sum m_i x_i}{\sum m_i}, \frac{\sum m_i y_i}{\sum m_i}, \frac{\sum m_i z_i}{\sum m_i}\right) \tag{6.4}$$

Interms of position vectors of the individual mass particles, the position of the CM is also expressed in the form of a position vector. Let $\overline{r}_1$, $\overline{r}_2$, $\overline{r}_3$........$\overline{r}_n$ be the position vectors of masses m_1, m_2, m_3 ...... m_n respectively. The position vector of the CM is given by

$$\overline{r}_{cm} = \frac{m_1\overline{r}_1 + m_2\overline{r}_2 + ...m_n\overline{r}_n}{m_1 + m_2 + m_3 + ...+ m_n} \tag{6.5}$$

For a continuous mass distribution such as a metal sphere, it is impossible to identify the individual particles and their individual positions with respect to the origin. In such cases, the summation is replaced by the Mathematical integration and the position of the CM is given by

$$\left(X_{cm}, Y_{cm}, Z_{cm}\right) = \left(\frac{1}{M}\int xdm, \frac{1}{M}\int ydm, \frac{1}{M}\int zdm\right) \tag{6.6}$$

Example 6.1

Mass of the earth is 81 times that of the Moon. The distance between the Earth and the Moon is 3.84×10^5 km. Find the position of the centre of mass of the Earth-Moon system, from the centre of the Earth.

Solution :

Let the mass of the moon be 'm'

Mass of the Earth = 81 m

Let us imagine that the Earth is positioned at the origin.

$x_1 = 0$

$x_2 = 3.84 \times 10^5$ km

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$X_{cm} = \frac{81m\times0 + m\times3.84\times10^5}{81m + m}$$

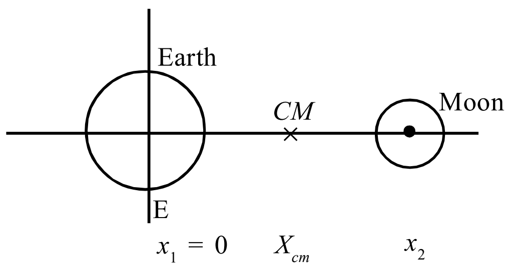


Fig. 6.2

$$X_{cm} = \frac{m\times3.84\times10^5}{82m} = \frac{3.84\times10^5}{82} km \simeq 4700\,km$$

Example 6.2

Suppose four masses, 1.0 kg, 2.0 kg, 3.0 kg and 4.0 kg are located at the corners of a square of side 1.0 m. Locate its centre of mass?

Solution :

We can always make the square lie in a plane. Let this plane be the (*x, y*) plane. Further, let us assume that one of the corners coincides with the origin of the coordinate system and the sides are along the *x* and *y* axes. The coordinates of the four masses are : m_1 (0, 0), m_2 (1.0, 0), m_3 (1.0, 1.0) and m_4 (0, 1.0), where all distances are expressed in metres as shown in Fig. 6.3.

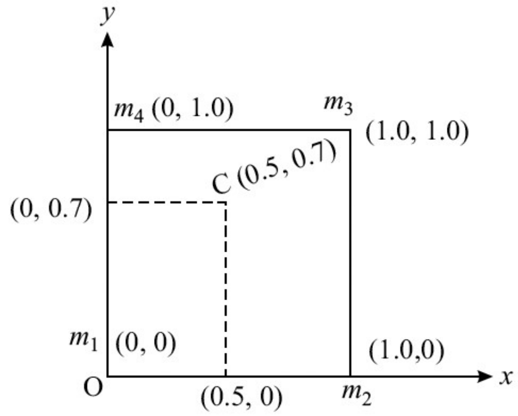


Fig. 6.3 : Locating CM of four masses placed at the corners of a square

From Eqn. (6.3), we get

$$x = \frac{1.0\times0+2.0\times1.0+3.0\times1.0+4.0\times0}{1.0+2.0+3.0+4.0} m = 0.5\ m$$

and $$y = \frac{1.0\times0+2.0\times0+3.0\times1.0+4.0\times1.0}{1.0+2.0+3.0+4.0} m = 0.7m$$

The CM has coordinates (0.5 m, 0.7 m) and is marked C in Fig. 6.3. Note that the CM is not at the centre of the square although the square is a symmetrical figure.

What could be the reason for the CM not being at the centre? To discover answer to this question, calculate the coordinates of CM if all masses are equal.

Example 6.3

Position vectors of two objects of mass 1 kg and 3 kg are $\left(2\bar{\mathbf{i}}+5\bar{\mathbf{j}}+13\bar{\mathbf{k}}\right)m$ and $\left(-6\bar{\mathbf{i}}+4\bar{\mathbf{j}}-2\bar{\mathbf{k}}\right)m$. Find the position of vector of the CM of this system

$$\bar{r}_{cm} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1 + m_2}$$

$$= \frac{1\left(2\bar{\mathbf{i}}+5\bar{\mathbf{j}}+13\bar{\mathbf{k}}\right)+3\left(-6\bar{\mathbf{i}}+4\bar{\mathbf{j}}-2\bar{\mathbf{k}}\right)}{1+3}$$

$$= \frac{1}{4}\left(2\bar{\mathbf{i}}+5\bar{\mathbf{j}}+13\bar{\mathbf{k}}-18\bar{\mathbf{i}}+12\bar{\mathbf{j}}-6\bar{\mathbf{k}}\right)m$$

$$\bar{r}_{cm} = \frac{1}{4}\left(-16\bar{\mathbf{i}}+17\bar{\mathbf{j}}+7\bar{\mathbf{k}}\right)m$$

Why should we define CM so precisely?

Recall that the rate of change of displacement is velocity, and the rate of change of velocity is acceleration.

From the defination of CM of a system of particles of masses m_1, m_2 .... m_n placed at positions x_1, x_2 .... x_n respectively along the X-axis from the origin.

$$X_{cm} = \frac{m_1x_1 + m_2x_2 .....+ m_nx_n}{M}$$

$$MX_{cm} = m_1x_1 + m_2x_2 + ...+ m_nx_n$$

on differention with respect to time, we get

$$M\frac{dx_m}{dt} = m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt} + ...+ m_n\frac{dx_n}{dt} \qquad (6.7)$$

$$\text{but } \frac{dx}{dt} = \text{velocity}$$

$$MV_{cm} = m_1v_1 + m_2v_2 + ...+ m_nv_n \qquad (6.8)$$

The rate of change of velocity is acceleration

$$M\frac{dv_{cm}}{dt} = m_1\frac{dv_1}{dt} + m_2\frac{dv_2}{dt} + ...+ m_n\frac{dv_n}{dt}$$

$$Ma_{cm} = m_1a_1 + m_2a_2 + ...+ m_na_n \qquad (6.9)$$

but ma = force.

$$\therefore\ F_{cm} = f_1 + f_2 + ...+ f_n \qquad (6.10)$$

It is equivalent to apply an external force on the CM of instead of applying individual forces on all the mass particles.

Here $F_{cm} = F_{external} = Ma_{cm}$

In the case of a rigid body, there is no effect of internal forces on the motion of the CM as they cancel each other in pairs.

When a bomb is thrown, it follows a parabolic path, as shown in Fig. 6.4. In the middle of its path, bomb explodes due to the internal forces. Fragments of the exploded bomb may travel in different directions. But, the remnant center of mass continues as a projectile and follows parabolic path, till it falls on the ground.

This is the proof for the motion of CM under the influence of external force only. Here, gravity is the external force acting on the bomb.

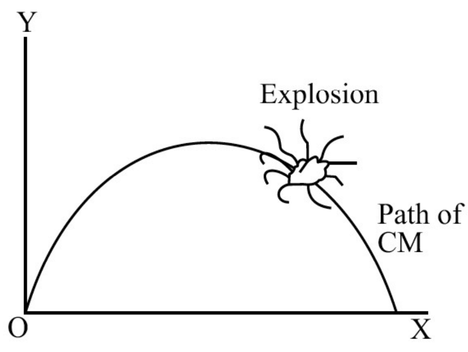


Fig. 6.4 : Centre of mass of a projectile

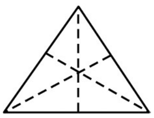
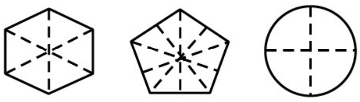
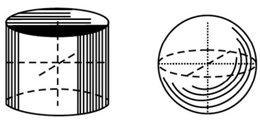
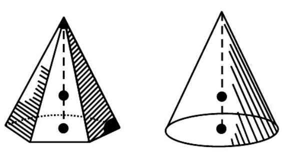
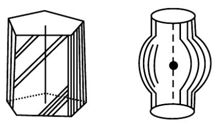
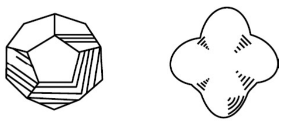
6.2.1 Characteristics of CM

1. The position of the CM only depends on the mass of the particles and their relative positions. That is to say that it depends only on the mass distribution.
2. The algebraic sum of moments of mass (mass × relative distance) above the CM is zero. $\Sigma m_i x_i = 0$.
3. It is not necessary to have any mass (matter) at the centre of mass. The centre of mass of a metal ring is at its geometrical centre, where there is no material (mass).
4. The position of the CM is independent of the chosen frame of reference.

6.2.2 CM of some bodies

The position of centre of mass of extended bodies cannot be easily calculated because of very large number of particles constituting the body have to be considered. The fact that all the particles of a rigid body have same mass and are uniformly distributed makes things somewhat simpler. If the body is regular in shape and possesses some symmetry, say it is cylindrical or spherical, the calculation is a little bit simplified. But even such calculations are beyond the scope of this course. However, keeping in mind the importance of CM, we give in Table 6.1 the position of CM of some regular, symmetrical bodies.

Table - 6.1 : Centres of Mass of some regular symmetrical bodies

Figure	Position of Centre of Mass
	*Triangular Plate* Point of intersection of the three medians.
	*Regular polygon and circular plate* At the geometrical centre of the figure.
	*Cylinder and sphere* At the geometrical centre of the figure.
	*Pyramid and cone* On line joining vertex with centre of base and at h/4 of the height measured from the base.
	*Figure with axial symmetry* Some point on the axis of symmetry.
	*Figure with centre of symmetry* At the centre of symmetry.

Two things must be remembered about the centre of mass : (i) It may be outside the body as in case of a ring. (ii) When two bodies revolve around each other, they actually revolve around their common centre of mass. For example, stars in a binary system revolve around their common centre of mass. The Earth-Sun system also revolves around its common centre of mass. But since mass of the Sun is very large as compared to the mass of earth, the centre of mass of the system is very close to the centre of the Sun.

Now it is time to check your progress.

Intext Questions 6.2

1. The grid shown here has particles A, B, C, D and E respectively have masses 1.0 kg, 2.0 kg, 3.0 kg, 4.0 kg and 5.0 kg. Calculate the coordinates of the position of the centre of mass of the system Fig. 6.5.

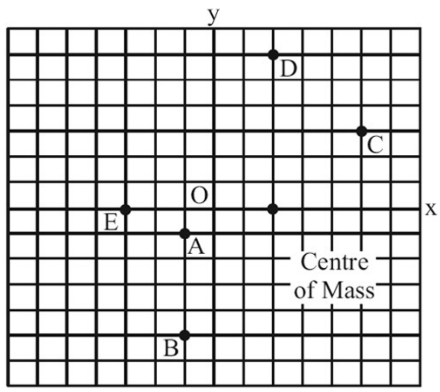


Fig. 6.5

2. If three particles of masses m_1 =1 kg, m_2 = 2 kg, and m_3 = 3 kg are situated at the corners of an equilateral triangle of side 1.0 m, obtain the position coordinates of the centre of mass of the system.
3. Show that the ratio of the distances of the two particles from their common centre of mass is inversely proportional to the ratio of their masses.
4. There is no external force acting on the CM of a rigid body. Will there be any change in the total momentum of all the particles of the system.

6.3 CENTRE OF GRAVITY

In a system of particles, all the particles are having some finite mass, hence, they are attracted by the Earth due to gravity. The Force with which a mass is attracted by the Earth is its weight.

$$F = ma$$

$$\text{When } a = g \quad F = W$$

$$\therefore \quad W = mg \qquad (6.11)$$

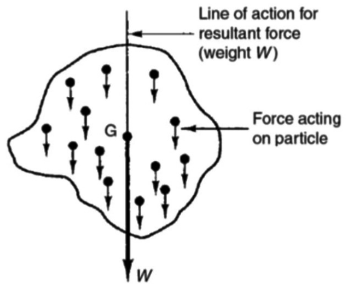


Fig. 6.6 : Resultant weight acting along the centre of gravity

All the weights are forces acting on the particles are parallel and pointed towards the Earth. Since weight is a vector, resultant of all the vectors acts through a point. This point, where the total weight of the system appears to be concentrated is called as "Centre of Gravity".

For a reasonably small objects of uniform thickness, placed in the uniform gravitational field, the centre of gravity coincides with its centre of mass. CM describes the motion of the system in a simple manner. Whereas, the centre of gravity, describes the stability of the object.

When the resultant weight or the force vector passes through the base of the object, the object is more stable. That is the reason why vehicles with broad base and less height are more stable.

- A girl trying to walk on a rope, tied between the two poles uses, a long stick in her hands to balance, so that the total weight (line of action) passes through the thin rope.
- Racing cars are small in height and broader in width.
- A mountain climber bends forward while climbing the mountain so that his weight vector lies in between his legs.

6.4 TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY : A COMPARISION

When a rigid body moves in such a way that all its particles move along parallel paths (Fig. 6.7), its motion is called **translational** motion. Since all the particles execute identical motion, its centre of mass must also be tracing out an identical path. Therefore, the translational motion of a body may be characterised by the motion of its centre of mass. We have seen that this motion is given by

$$M\,\mathbf{a} = \mathbf{F}_{ext}$$

Do you now see the advantage of defining the centre of mass of a body? With its help, the translational motion of body can be described by an equation for a single particle having mass equal to the mass of the whole body. It is located at the centre of mass and is acted upon by the sum of all the external forces which are acting on the rigid body. To understand the concept clearly, perform the following activities.

Activity 6.1

Take a wooden block. Make two or three marks on any of its surfaces. Now keep the marked surface in front of you and push the block along a horizontal floor. Note the paths traced by the marks. All these marks have paths parallel to the floor and, therefore, parallel to one another Fig. 6.7. You can easily see that the lengths of the paths are also equal.

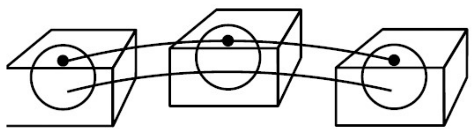


Fig. 6.7 : A wooden block moving along the floor performs translational motion.

Activity 6.2

Let us now perform another simple experiment. Take a cylindrical piece of wood. On its plane face make a mark or two. Now roll the cylinder slowly on the floor, keeping the plane face towards you. You would notice that the mark such as A in Fig. 6.8, has not only moved parallel to the floor,

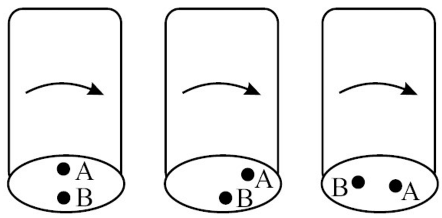


Fig. 6.8 : Rolling motion of a cylinder: The point A has not only moved parallel to the floor but also performed circular motion

but has also performed circular motion. So, the body has performed both translational and rotational motion.

While the general motion of a rigid body consists of both translation and rotation, it cannot have translational motion if one point in the body is fixed; it can then only rotate. The most convenient point to fix for this purpose is the CM of the body.

You might have seen a grinding stone (the chakki). The handle of the stone moves in a circular path. All the points on the stone also move in circular paths around an axis passing through the centre of the stone Fig. 6.9.

The motion of a rigid body in which all its constituent particles describe concentric circular paths is known as rotational motion.

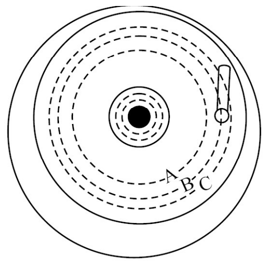


Fig. 6.9 : Pure rotation of a grinding stone

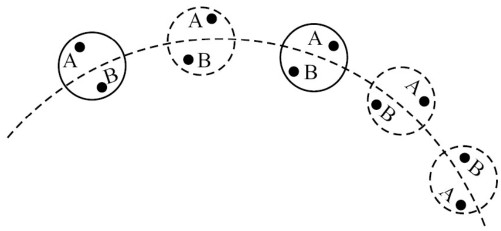


Fig. 6.10 : Rotation of the earth

We have noted above that translational motion of a rigid body can be described by an equation similar to that of a single particle. You are familiar with such equations. Therefore, in this lesson we concentrate only on the rotational motion of a rigid body. The rotational motion can be obtained by keeping a point of the body fixed so that it cannot have any translational motion. For the sake of mathematical convenience, this point is taken to be the CM. The rotation is then about an axis passing through the CM. A good example of rotational motion is the earth's rotation about its own axis Fig. 6.10. You have studied in earlier lessons that the mass of the body plays a very important role. It determines the acceleration acquired by the body for a given force. Can we define a similar quantity for rotational motion also? Let us find out.

6.4.1 Angular displacement (θ)

In a rotatory motion, all the particles of the object undergo circular motion around the axis of rotation. The distance between the particle and the axis of rotation (o) is the radius vector. The angle displaced by the radius vector during the period '*t*' is called angular displacement.

From Fig. 6.11 $\angle AOB = \theta$

Its unit is radian. For a complete rotation $\theta = 2\pi$ radian.

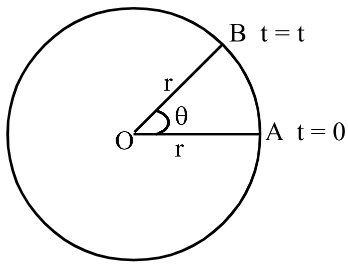


Fig. 6.11

6.4.2 Angular Velocity & Angular Acceleration

The rate of angular displacement of the particle is Angular Velocity (ω)

$$\omega = \frac{angle}{time} = \frac{\theta}{t} \qquad (6.12)$$

unit of ω is radian / second

ω is a vector quantity. Its direction can be determined by using right hand thumb rule.

If the particle completes 'n' revolutions in 't' seconds then $\omega = \frac{2\pi n}{t}$

The rate of change of angular velocity is called Angular acceleration (α)

$$\alpha = \frac{\Delta\omega}{t} \tag{6.13}$$

or $$\alpha = \frac{\omega - \omega_0}{t}$$

SI unit of α is radian / (second)^2. Angular acceleration is a vector quantity.

The relationship between Linear Velocity (v) and angular velocity (ω) is

$$v = r\omega \tag{6.14}$$

The relationship between linear acceleration and angular acceleration (α) is

$$a = r\alpha \tag{6.15}$$

6.5 MOMENT OF INERTIA

Let C be the centre of mass of a rigid body. Suppose it rotates about an axis through this point Fig. 6.12.

Suppose particles of masses m_1, m_2, m_3...are located at distances r_1, r_2, r_3...from the axis of rotation and are moving with speeds v_1, v_2, v_3 respectively. Then particle 1 has kinetic energy (½) $m_1v_1^2$. Similarly, the kinetic energy of particle of mass m_2 is (½) $m_2v_2^2$. By adding the kinetic energies of all the particles, we get the total energy of the body. If T denotes the total kinetic energy of the body, we can write

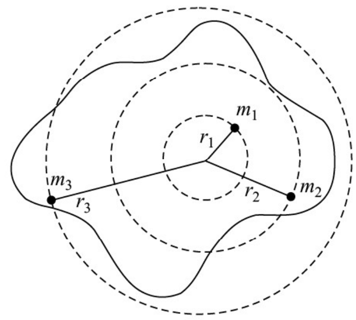


Fig. 6.12 : Rotation of a plane lamina about an axis passing through its centre of mass

$$T = (1/2)m_1v_1^2 + (1/2)m_2v_2^2 + ...$$

$$= \sum_{i=1}^{i=n}\left(\frac{1}{2}\right)m_iv_i^2 \tag{6.16}$$

where $\sum_{i=1}^{i=n}$ indicates the sum over all the particles of the body.

You have studied that angular speed (ω) is related to linear speed (v) through the equation $v = r\omega$. Using this result in Eqn. (6.16), we get

$$T = \sum_{i=1}^{i=n} \left(\frac{1}{2}\right) m_i \left(r_i \omega\right)^2 \tag{6.17}$$

Note that we have not put the subscript i with ω because all the particles of a rigid body have the same angular speed. Eqn. (6.17) can now be rewritten as

$$T = \frac{1}{2}\left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega^2$$

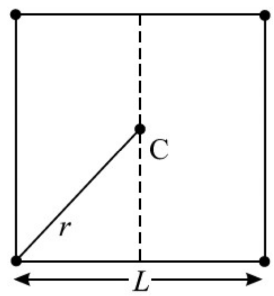
$$= \frac{1}{2} \mathrm{I}\omega^2 \tag{6.18}$$

The quantity $$I = \sum_i m_i r_i^2 \tag{6.19}$$

is called the ***moment of inertia*** of the body.

Example 6.4

Four particles of mass m each are located at the corners of a square of side L. Calculate their moment of inertia about an-axis passing through the centre of the square and perpendicular to its plane.



Solution :

Simple geometry tells us that the distance of each particle from the axis of rotation is. Therefore, we can write

$$I = mr^2 + mr^2 + mr^2 + mr^2$$

$$I = 4mr^2$$

$$I = 4m\left(\frac{L}{\sqrt{2}}\right)^2$$

$$= 2mL^2$$

It is important to remember that ***moment of inertia*** is defined with reference to an axis of rotation. Therefore, whenever you mention moment of inertia, the axis of rotation must also be specified. In the present case, I is the moment of inertia about an axis passing through the centre of the square and normal to the plane containing four equal masses as shown in figure. The moment of inertia is expressed in kg m^2.

The moment of inertia of a rigid body is often written as

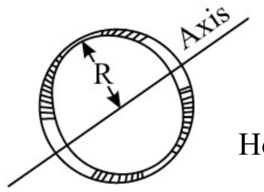
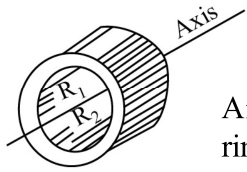
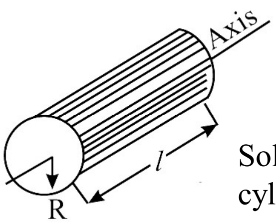
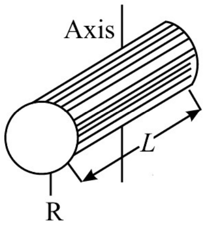
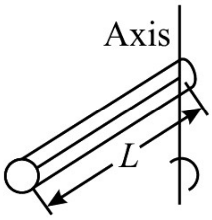
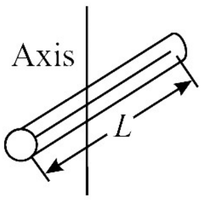
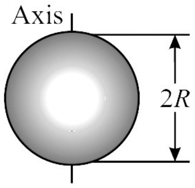
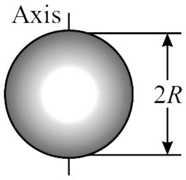
$$I = M\,K^2 \tag{6.20}$$

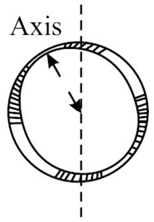
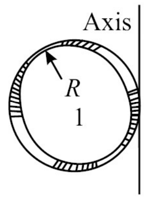
where M is the total mass of the body and K is called the ***radius of gyration*** of the body. ***The radius of gyration is that distance from the axis of rotation where the whole mass of the body can be assumed to be placed to get the same moment of inertia which the body actually has.*** It is important to remember that the moment of inertia of a body about an axis depends on the distribution of mass around that axis. If the distribution of mass changes, the moment of inertia will also change. This can be easily seen from Example 6.4. Suppose we place additional masses at one pair of opposite corners of amount m each. Then the moment of inertia of the system about the axis through C and perpendicular to the plane of square would be

$$\begin{aligned} I &= mr^2 + 2mr^2 + mr^2 + 2mr^2 \\ &= 6mr^2 \end{aligned}$$

Note that moment of inertia has changed from $2mL^2$ to 3 mL^2.

Table - 6.2 : Moments of inertia of a few regular and uniform bodies.

 Hoop about central axis $I = MR^2$	 Angular cylinder (or ring) about cylinder axis $I = \frac{M}{2}\left(R_1^2 + R_2^2\right)$
 Solid cylinder about cylinderical axis $I = \frac{MR^2}{2}$	 Solid cylinder (or disk about a central diameter $I = \frac{MR^2}{4} + \frac{M^2l^2}{12}$
 Thin rod about an axis passing through its centre and normal to its length $I = \frac{ML^2}{12}$	 Thin rod about an axis passing through one end and perpendicular to length $I = \frac{MR^2}{3}$
 Solid shpere about any diameter $I = \frac{2MR^2}{5}$	 Thin spherical shell about any diameter $I = \frac{2MR^2}{3}$

 Hoop about any diameter $I = \frac{MR^2}{2}$	 Hoop about any tangent line $I = \frac{3MR^2}{2}$

Refer to Eqn.(6.18) again and compare it with the equation for kinetic energy of a body in linear motion. Can you draw any analogy? You will note that in rotational motion, the role of mass has been taken over by the moment of inertia and the angular speed has replaced the linear speed.

A. Physical significance of moment of inertia : *The physical significance of moment of inertia is that it performs the same role in rotational motion that the mass does in linear motion.*

Just as the mass of a body resists change in its state of linear motion, the moment of inertia resists a change in its rotational motion. This property of the moment of inertia has been put to a great practical use. Most machines, which produce rotational motion have as one of their components a disc which has a very large moment of inertia. Examples of such machines are the steam engine and the automobile engine. The disc with a large moment of inertia is called a ***flywheel***. To understand how a flywheel works, imagine that the driver of the engine wants to suddenly increase the speed. Because of its large moment of inertia, the flywheel resists this attempt. It allows only a gradual increase in speed. Similarly, it works against the attempts to suddenly reduce the speed, and allows only a gradual decrease in the speed. Thus , the flywheel, with its large moment of inertia, prevents jerky motion and ensures a smooth ride for the passengers.

We have seen that in rotational motion, angular velocity is analogous to linear velocity in linear motion. Since angular acceleration (denoted usually by α) is the rate of change of angular velocity, it must correspond to acceleration in linear motion.

B. Equations of motion for a uniformly rotating rigid body : Consider a lamina rotating about an axis passing through O and normal to its plane. If it is rotating with a constant angular velocity ω, as shown, then it will turn through an angle θ, in t seconds such that

$$\theta = \omega t \qquad (6.21\ [a])$$

However, if the lamina is subjected to constant torque (which is the turning effect of force), it will undergo a constant angular acceleration. The following equations describe its rotational motion:

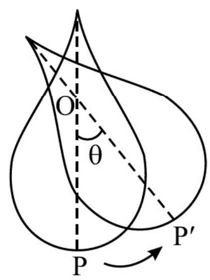


Fig. 6.13 : Rotation of a lamina about a fixed nail

$$\omega_f = \omega_i + \alpha t \qquad (6.21\ [b])$$

Where ω_i is initial angular velocity and ω_f is final angular velocity.

Similarly, we can write

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (6.21\ [c])$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta \qquad (6.21\ [d])$$

where θ is angular displacement in t seconds.

On a little careful scrutiny, you can recognise the similarity of these equations with the corresponding equations of kinematics for translatory motion.

Example 6.5

A wheel of a bicycle is free to rotate about a horizontal axis Fig. 6.14. It is initially at rest. Imagine a line OP drawn on it. By what angle would the line OP move in 2 s if it had a uniform angular acceleration of 2.5 rads^{-2}.

Solution :

Angular displacement of line OP is given by

$$\theta = \omega_0 t + (1/2)\alpha t^2$$

$$= 0 + (1/2) \times (2.5\,\text{rad}\,\text{s}^{-2}) \times 4\text{s}^2$$

$$\theta = 5 \text{ rad}$$

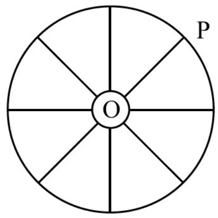


Fig. 6.14 : Rotation of bicycle wheel

We have mentioned above that for rotational motion of a rigid body, its CM is kept fixed. However, it is just a matter of convenience that we keep CM fixed. But many a time, we consider points other than the CM. That is, a point in the body which can also be kept fixed and the body rotated about it. But then the axis of rotation will pass through this fixed point. The moment of inertia about this axis would be different from the moment of inertia about an axis passing through the CM. The relation between the two moments of inertia can be obtained using the theorems of moment of inertia.

6.5.1 Theorems of moment of inertia

There are two theorems which connect moments of inertia about two axes; one of which is passing through the CM of the body. These are :

(i) the theorem of parallel axes, and

(ii) the theorem of perpendicular axes.

Let us now learn about these theorems and their applications.

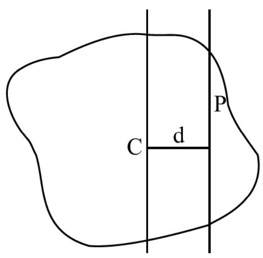


Fig. 6.15 : Parallel axes through CM and another point P

(i) Theorem of parallel axes : Suppose the given rigid body rotates about an axis passing through any point P other than the centre of mass. The moment of inertia about this axis can be found from a knowledge of the moment of inertia about a parallel axis through the centre of mass. Theorem of parallel axis states that *the moment of inertia about an axis parallel to the axis passing through its centre of mass is equal to the moment of inertia about its centre of mass plus the product of mass and square of the perpendicular distance between the parallel axes*. If I denotes the required moment of inertia and I_C denotes the moment of inertia about a parallel axis passing through the CM, then

$$I = I_C + M\,d^2 \qquad (6.22)$$

where M is the mass of the body and d is the distance between the two axes (Fig. 6.15). This is known as the **theorem of parallel axes.**

(ii) Theorem of perpendicular axes : Let us choose three mutually perpendicular axes, two of which, say x and y are in the plane of the body, and the third, the z–axis, is perpendicular to the plane Fig. 6.16. The perpendicular axes theorem states that ***the sum of the moments of inertia about x and y axes is equal to the moment of inertia about the z–axis.***

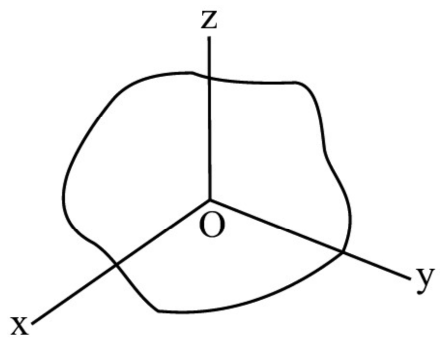


Fig. 6.16 (a) : Theorem of perpendicular axes

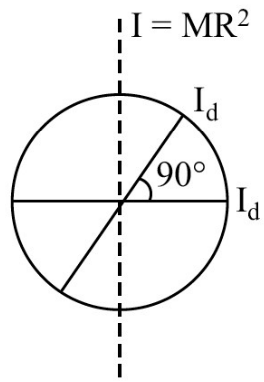


Fig. 6.16 (b) : Moment of inertia of a hoop

That is,

$$I_z = I_x + I_y \qquad (6.23)$$

We now illustrate the use of these theorems by the following example. Let us take a hoop shown in Fig. 6.16. From Table 6.2 you would recall that moment of inertia of a hoop about an axis passing through its centre and perpendicular to the base is $M\,R^2$, where M is its mass and R is its radius. The theorem of perpendicular axes tells us that this must be equal to the sum of the moments of inertia about two diameters which are perpendicular to each other as well as to the central axis. The symmetry of the hoop tells us that the moment of inertia about any diameter is the same as about any other diameter. This means that all the diameters are **equivalent** and any two perpendicular diameters may be chosen. Since the moment of inertia about each is the same, say I_d, Eqn.(6.23) gives

$$M\,R^2 = 2\,I_d$$

and therefore

$$I_d = (½)\,MR^2$$

So, the moment of inertia of a hoop about any of its diameter is (½) $M R^2$.

Let us now take a point P on the rim. Consider a tangent to the hoop at this point which is parallel to the axis of the hoop. The distance between the two axes is obviously equal to R. The moment of inertia about the tangent can be calculated using the theorem of parallel axes. It is given by

$$I_{\tan} = M R^2 + M R^2 = 2 M R^2.$$

It must be mentioned that many of the entries in Table 6.2 have been computed using the theorems of parallel and perpendicular axes.

6.6 TORQUE AND COUPLE

Activity 6.3

Have you ever noticed that it is easy to open the door by applying force at a point far away from the hinges? What happens if you try to open a door by applying force near the hinges? Carry out this activity a few times. You would realise that much more effort is needed to open the door if you apply force near the hinges than at a point away from the hinges. Why is it so? Similarly, for turning a screw we use a spanner with a long handle. What is the purpose of keeping a long handle? Let us seek answers to these questions now.

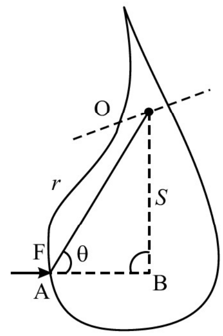


Fig. 6.17 : Rotation of a body

Suppose O is a fixed point in the body and it can rotate about an axis passing through this point as shown in Fig. 6.17.

Let a force of magnitude F be applied at the point A along the line AB. If AB passes through the point O, the force F will not be able to rotate the body. The farther is the line AB from O, the greater is the ability of the force to turn the body about the axis through O. The ***turning effect of a force is called torque.*** Its magnitude is given by

$$\tau = Fs = Fr\sin\theta \qquad (6.24\ [a])$$

where s is the distance between the axis of rotation and the line along which the force is applied.

The units of torque are newton-metre, or Nm. The torque is actually a vector quantity. The vector from of Eqn. (6.24 [a]) is

$$\tau = \mathrm{r}\times\mathrm{F} \qquad (6.24\ [b])$$

which gives both magnitude and direction of the torque. What is the direction in which the body would turn? To discover this, we recall the rules of vector product (refer to lesson 3) : τ is perpendicular to the plane containing vectors **r** and **F**, which is the plane of paper here (Fig. 6.18). If we extend the thumb of the right hand at right angles to the fingers and curl the fingers so as to point from **r** to **F** through the smaller angle, the direction in which thumb points is the direction of τ.

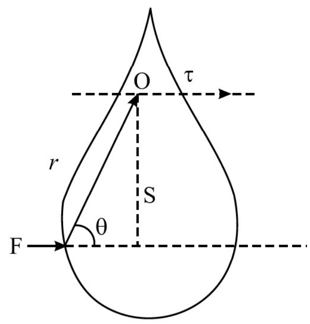


Fig. 6.18 : Right hand thumb rule

Apply the above rule and show that the turning effect of the force in Fig. 6.18 *is normal to the plane of paper downwards. This corresponds to clockwise rotation of the body.*

Example 6.6

Fig. 6.19 shows a bicycle pedal. Suppose your foot is at the top and you are pressing the pedal downwards. (i) What torque do you produce? (ii) Where should your foot be for generating maximum torque?

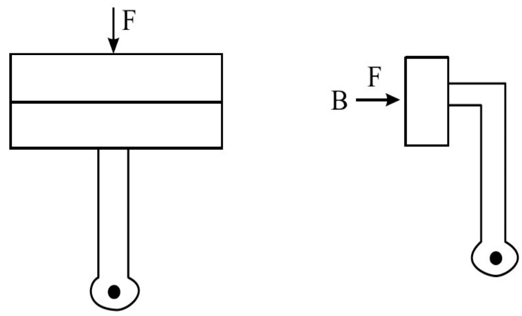


Fig. 6.19 : A bicycle pedal (a) at the top when $\tau = 0$; (b) when τ is maximum

Solution :

(i) When your foot is at the top, the line of action of the force passes through the centre of the pedal. So, $\theta = 0$, and $\tau = Fr\sin\theta = 0$.

(ii) To get maximum torque, $\sin\theta$, must have its maximum value, that is θ must be 90º. This happens when your foot is at position B and you are pressing the pedal downwards.

If there are several torques acting on a body, the net torque is the vector sum of all the torques. Do you see any correspondence between the role of torque in the rotational motion and the role of force in the linear motion? Consider two forces of equal magnitude acting on a body in opposite directions (Fig. 6.20). Assume that the body is free to rotate about an axis passing through O. The two torques on the body have magnitudes

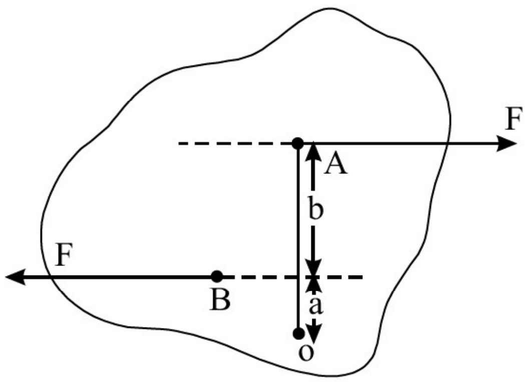


Fig. 6.20 : Two opposite forces acting on body

$$\tau_1 = (a+b)F$$

and $$\tau_2 = aF\ .$$

You can verify that the turning effect of these torques are in the opposite directions. Therefore, the magnitude of the net turning effect on the body is in the direction of the larger torque, which in this case is τ_1:

$$\tau = \tau_1 - \tau_2 = \mathrm{b}\mathrm{F} \qquad (6.25)$$

We may therefore conclude that ***two equal and opposite forces having different lines of action are said to form a couple whose torque is equal to the product of one of the forces and the perpendicular distance between them.***

There is another useful expression for torque which clarifies its correspondence with force in linear motion. Consider a rigid body rotating about an axis passing through a point

O Fig. 6.21. Obviously, a particle like P is rotating about the axis in a circle of radius r. If the circular motion is non–uniform, the particle experiences forces in the radial direction as well as in the tangential direction. The radial force is the centripetal force $m\omega^2 r$, which keeps the particle in the circular path. The tangential force is required to change the magnitude of v, which at every instant is along the tangent to the circular path. Its magnitude is ma, where a is the tangential acceleration. ***The radial force does not produce any torque.*** *Do you know why? The tangential force produces a torque of magnitude* $ma\ r$. Since $a = r\alpha$, where α is the angular acceleration, the magnitude of the torque is $mr^2\alpha$. If we consider all the particles of the body, we can write

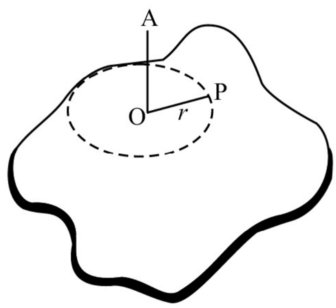


Fig. 6.21 : A rigid body rotating about on axis

$$\tau = \sum_{i=0}^{i=n} m_i r_i^2 \alpha = \left(\sum_i m_i r_i^2\right)\alpha \tag{6.26}$$

$$= \mathrm{I}\alpha$$

because α is same for all the particles at a given instant.

The similarity between this equation and **F** = $m\ a$ shows that τ performs the same role in rotational motion as **F** does in linear motion. A list of corresponding quantities in rotational motion and linear motion is given in Table 6.3. With the help of this table, you can write any equation for rotational motion if you know its corresponding equation in linear motion.

Table - 6.3 : Corresponding quantities in rotational and translational motions

Translational Motion		**Rotation about a Fixed Axis**	
Displacement	x	Angular displacement	θ
Velocity	$v = \dfrac{dx}{dt}$	Angular velocity	$\omega = \dfrac{d\theta}{dt}$
Acceleration	$a = \dfrac{dv}{dt}$	Angular acceleration	$\alpha = \dfrac{d\omega}{dt}$
Mass	M	Moment of inertia	I
Force	$F = m\ a$	Torque	$\tau = I\alpha$
Work	$w = \int F dx$	Work	$W = \int \tau d\theta$
Kinetic energy	$\frac{1}{2} Mv^2$	Kinetic energy	$(\frac{1}{2}) I\omega^2$
Power	P = Fv	Power	$P = \tau\omega$
Linear momentum	Mv	Angular momentum	$I\omega$

With the help of Eqn.(6.26) we can calculate the angular acceleration produced in a body by a given torque.

Example 6.7

A uniform disc of mass 1.0 kg and radius 0.1m can rotate about an axis passing through its centre and normal to its plane without friction. A mass less string goes round the rim of the disc and a mass of 0.1 kg hangs at its end (Fig. 6.22). Calculate (i) the angular acceleration of the disc, (ii) the angle through which the disc rotates in one second, and (iii) the angular velocity of the disc after one second. Take g = 10 ms$^{-2.}$

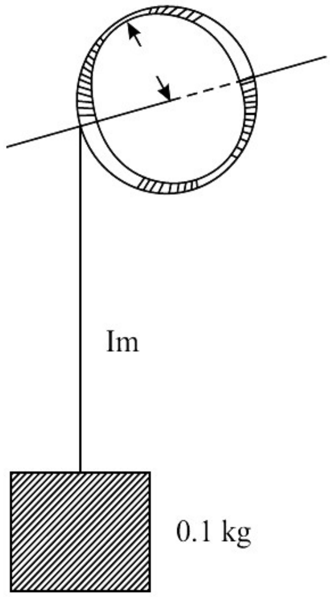


Fig. 6.22

Solution :

(i) If R and M denote the radius and mass of the disc, from Table 6.2, we recall that its moment of inertia is given by $I = (½)\ M R^2$. If F denotes the magnitude of force (= $m\ g$) due to the mass at the end of the string then $\tau = FR$. Eqn. (6.26) now gives

$$\alpha = \tau / I = FR / I = 2F / MR$$

$$= \frac{2\times(0.1\text{kg})\times(10\text{ms}^{-2})}{(1.0\text{kg})\times(0.1\text{m})} = 20\,\text{rad}\,\text{s}^{-2}$$

(ii) For angle ¸ through which the disc rotates, we use Eqn. (6.21 [c]). Since the initial angular velocity is zero, we have

$$\theta = (1/2)\times 20\times 1.0 = 10\,\text{rad}$$

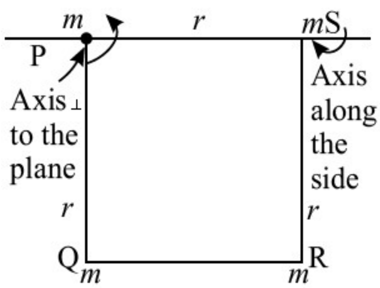
(iii) For the velocity after one second, we have

$$\omega = \alpha t = 20\times 1.0 = 20\,\text{rad s}^{-1}$$

Now, you may like to check your progress. Try the following questions.

Intext Questions 6.3

1. Four particles, each of mass **m**, are fixed at the corners of a square whose each side is of length r. Calculate the moment of inertia about an axis passing through one of the corners and perpendicular to the plane of the square. Calculate also the moment of inertia about an axis which is along one of the sides. Verify your result by using the theorem of perpendicular axes.



2. Calculate the radius of gyration of a solid sphere if the axis is a tangent to the sphere. (You may use Table 6.2).
3. What is the angular velocity of the minutes hand of a clock?
4. A bus driver has to replace a punctured tyre, for which he is attaching a long pipe to the spanner and applying the force at its end. Explain for using the long pipe.
5. Give few examples for couple of forces that we apply in our daily life.
6. Tossing of a coin is the example for the application of Torque or moment of a couple explain.
7. What are the units of moment of Inertia?

6.7 ANGULAR MOMENTUM

From Table 6.3 you may recall that rotational analogue of linear momentum is angular momentum. To understand its physical significance, we would like you to do an activity.

Activity 6.4

If you can get hold of a stool which can rotate without much friction, you can perform an interesting experiment. Ask a friend to sit on the stool with her arms folded. Make the stool rotate fast. Measure the speed of rotation. Ask your friend to stretch her arms and measure the speed again. Do you note any change in the speed of rotation of the stool? Ask her to fold her arms once again and observe the change in the speed of the stool.

Let us try to understand why we expect a change in the speed of rotation of the stool in two cases: sitting with folded and stretched hands. For this, let us again consider a rigid body rotating about an axis, say z – axis through a fixed point O in the body. All the points of the body describe circular paths about the axis of rotation with the centres of the paths on the axis and have angular velocity ω. Consider a particle like P at distance r_i from the axis Fig. 6.23. Its linear velocity is $v_i = r_i\omega$ and its momentum is therefore $v_i = r_i\omega$. ***The product of linear momentum and the distance from the axis is called angular momentum, denoted by L.*** If we sum this product for all the particles of the body, we get

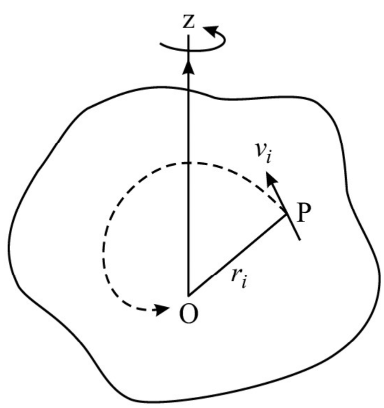


Fig. 6.23 : A rigid body rotating about an axis through 'O'

$$L = \sum_i m_i \omega r_i \; r_i = \left(\sum_i m_i r_i^2\right)\omega$$

$$= I\omega \qquad (6.27)$$

Remember that the angular velocity is the same for all the particles and the term within brackets is the moment of inertia. Like the linear momentum, the angular momentum is also a vector quantity. Eqn. (6.23) gives only the component of the vector **L** along the axis of rotation. It is important to remember that *I* must refer to the same axis. The unit of angular momentum is kg m^2 s^{-1}

Recall now that the rate of change of ω is α and I $\alpha = \tau$. Therefore, ***the rate of change of angular momentum is equal to torque.*** In vector notation, we write the equation of motion of a rotating body as

$$\frac{dL}{dt} = \tau = I\frac{d\omega}{dt} = I\alpha \tag{6.28}$$

6.7.1 Conservation of angular momentum

Eqn. (6.28) shows that ***if there is no net torque acting on the body,*** $\frac{dL}{dt} = 0$ ***This means that there is no change in angular momentum, i.e. the angular momentum is constant. This is the principle of conservation of angular momentum.*** Along with the conservation of energy and linear momentum, this is one of the most important principles of physics.

The principle of conservation of angular momentum allows us to answer questions such as : How the direction of toy umbrella floating in air remains fixed? The trick is to make it rotate and thereby impart it some angular momentum. Once it goes in air, there is no torque acting on it. Its angular momentum is then constant. Since angular momentum is a vector quantity, its constancy implies fixed direction and magnitude. Thus, the direction of the toy umbrella remains fixed while it is in air.

In the case of your friend on the rotating stool; when no net torque acts on the stool, the angular momentum of the stool and the person on it must be conserved. When the arms are stretched, she causes the moment of inertia of the system to increase. Eqn. (6.19) then implies that the angular velocity must decrease. Similarly, when she folds her arms, the moment of inertia of the system decreases. This causes the angular velocity to increase. Note that the change is basically caused by the change in the moment of inertia due to change in distance of particles from the axis of rotation.

Let us look at a few more examples of conservation of angular momentum. Suppose we have a spherical ball of mass *M* and radius *R*. The ball is set rotating by applying a torque on it. The torque is then removed. When there is no external torque, whatever angular momentum the ball has acquired must be conserved. Since moment of inertia of the ball is (2/5) $M R^2$ (Table 6.2), its angular momentum is given by

$$L = \frac{2}{5}MR^2\omega \tag{6.29}$$

where ω is its angular velocity. Imagine now that the radius of the ball somehow decreases. To conserve its angular momentum, ω must increase and the ball must rotate faster. This is what really happens to some stars, such as those which become pulsars.

What would happen if the radius of the ball were to increase suddenly?

You can again use Eqn. (6.19) to show that if R increases, ω must decrease to conserve angular momentum. If instead of radius, the moment of inertia of the system changes somehow, ω will change again. For an interesting effect of this kind see Box below.

The length of the day is not constant

Scientists have observed very small and irregular variations in the period of rotation of the earth about its axis, i.e. the length of the day. One of the causes that they have identified is weather. Due to changes in weather, large scale movement in the air of the earth's atmosphere takes place. This causes a change in the mass distribution around the axis of the earth, resulting in a change in the moment of inertia of the earth. Since the angular momentum of the earth $L = I\ \omega$ must be conserved, a change in I means a change in rotational speed of the earth, or in the length of the day.

Acrobats, skaters, divers and other sports persons make excellent use of the principle of conservation of angular momentum to show off their feats. You must have seen divers jumping off the diving boards during swimming events in national or international events such as Asian Games, Olympics or National meets. At the time of jumping, the diver gives herself a slight rotation, by which she acquires some angular momentum. When she is in air, there is no torque acting on her and therefore her angular momentum must be conserved. If she folds her body to decrease her moment of inertia Fig. 6.24 her rotation must become faster. If she unfolds her body, her moment of inertia increases and she must rotate slowly. In this way, by controlling the shape of her body, the diver is able to demonstrate her feet before entering into pool of water.

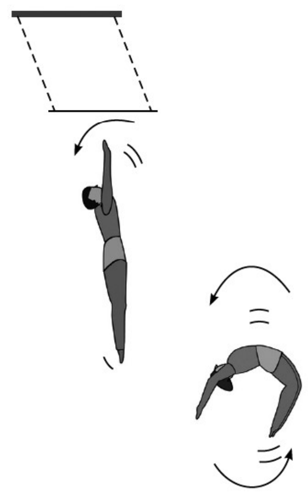


Fig. 6.24 : Diver, Sommer saulting after jumping off the diving boards.

Example 6.8

Shiela stands at the centre of a rotating platform that has frictionless bearings. She holds a 2.0 kg object in each hand at 1.0 m from the axis of rotation of the system. The system is initially rotating at 10 rotations per minute. Calculate (a) the initial angular velocity in rad s^{-1} (b) the angular velocity after the objects are brought to a distance of 0.2 m from the axis of rotation, and (c) change in the kinetic energy of the system. (d) If the kinetic energy has increased, what is the cause of this increase? (Assume that the moment of inertia of Shiela and platform I_{SP} stays constant at 1.0 kg m^2.)

Solution :

(a) 1 rotation = 2π radian

$\therefore$ initial angular velocity $\omega = \dfrac{10 \times 2\pi \text{ radian}}{60\text{s}} = 1.05 \text{ rad s}^{-1}$

(b) The key idea here is to use the law of conservation of angular momentum. The initial moment of inertia $I_i = I_{SP} + mr_i^2 + mr_i^2$

$$= 1.0 kg\, m^2 + (2.0 kg) \times (1m^2) + (2.0 kg) \times (1m^2)$$

= 5.0 kg m^2

After the objects are brought to a distance of 0.2 m, final moment of inertia.

$$I_t = I_{SP} + mr_f^2 + mr_f^2$$

= 1.0 kg m^2 + 2.0 kg × $(0.2)^2$ m^2 + 2.0 kg × $(0.2)^2$

= 1.16 kg m^2.

Conservation of angular momentum requires that

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i\omega_i}{I_f}$$

$$= \frac{(5.0\,\text{kgm}^2)1.05\,\text{rad s}^{-1}}{1.16\,\text{kgm}^2}$$

= 4.5 rad s^{-1}

Suppose the change in kinetic energy of rotation is ΔE. Then

$$\Delta E = \frac{1}{2} I_f\, \omega_f^2 - \frac{1}{2} I_i\, \omega_i^2$$

$$= \frac{1}{2} \times 1.16\text{kgm}^2 \times (4.5)^2 \left(\text{rad s}^{-1}\right)^2 - \frac{1}{2} \times 5.0\text{kgm}^2 \times (10.5)^2 \left(\text{rad s}^{-1}\right)^2$$

= 9.05 J

Since final kinetic energy is higher than the initial kinetic energy, there is an increase in the kinetic energy of the system.

(d) When Shiela pulls the objects towards the axis, she does work on the system. This work goes into the system and increases its kinetic energy.

Intext Questions 6.4

1. A hydrogen molecule consists of two identical atoms, each of mass *m* and separated by a fixed distance *d*. The molecule rotates about an axis which is halfway between the two atoms, with angular speed ω. Calculate the angular momentum of the molecule.

2. A uniform circular disc of mass 2.0 kg and radius 20 cm is rotated about one of its diameters at an angular speed of 10 rad s^{-1}. Calculate its angular momentum about the axis of rotation.

3. A wheel is rotating at an angular speed ω about its axis which is kept vertical. Another wheel of the same radius but half the mass, initially at rest, is slipped on the same axle gently. These two wheels then rotate with a common speed. Calculate the common angular speed.

4. It is said that the earth was formed from a contracting gas cloud. Suppose some time in the past, the radius of the earth was 25 times its present radius. What was then its period of rotation on its own axis?

6.8 SIMULTANEOUS ROTATIONAL AND TRANSLATIONAL MOTIONS

We have already noted that if a point in a rigid body is not fixed, it can possess rotational motion as well as translational motion. The general motion of a rigid body consists of both these motions. Imagine the motion of an automobile wheel on a plane horizontal surface. Suppose you are observing the circular face Fig. 6.25. Fix your attention at a point P and at the centre C of the circular face. Remember that the centre of mass of the wheel lies at the centre of its axis and C is the end point of the axis. As it rolls, you would notice that point P rotates round the point C. The point C itself gets translated in the direction of motion. So the wheel has both the rotational and translational motions. If point C or the centre of mass gets translated with velocity v_{cm}, the kinetic energy of translation is

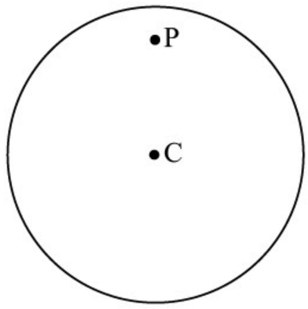


Fig. 6.25

$$(KE)_{tr} = \frac{1}{2} M v_{cm}^2 \qquad (6.30)$$

where M is the mass. And if ω is the angular speed of rotation, the kinetic energy of rotation is

$$(KE)_{rot} = \frac{1}{2} I \omega^2 \qquad (6.31)$$

where I is the moment of inertia. The total energy of the body due to translation and rotation is the sum of these two kinetic energies. An interesting case, where both translational and rotational motion are involved, is the motion of a body on an inclined plane.

Example 6.9

Suppose a rigid body has mass M, radius R and moment of inertia I. It is rolling down an inclined plane of height h Fig. 6.26. At the end of its journey, it has acquired a linear speed v and an angular speed ω. Assume that the loss of energy due to friction is small and can be neglected. Obtain the value of v in terms of h.

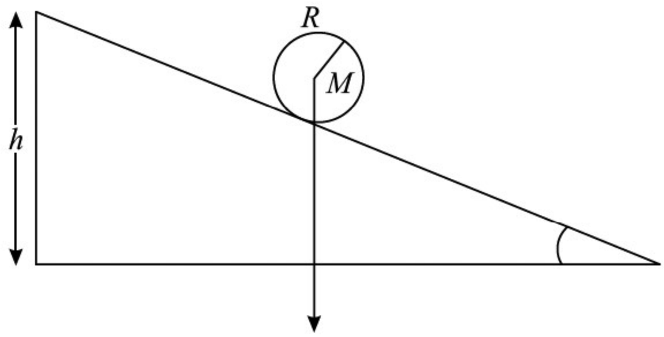


Fig. 6.26 : Motion of a rigid body on an inclined plane

Solution :

The principle of conservation of energy implies that the sum of the kinetic energies due to translation and rotation at the foot of the inclined plane must be equal to the potential energy that the body had at the top of the inclined plane. Therefore,

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh \qquad (6.32)$$

If the motion is pure rolling and there is no slipping, we can write $v = R\omega$. Inserting this expression is Eqn. (6.32), we get

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = Mgh \qquad (6.33)$$

To take a simple example, let the body be a hoop. Table 6.2 shows that its moment of inertia about its own axis is MR^2. Eqn.6.33 then gives

$$\frac{1}{2}Mv^2 + \frac{1}{2}\frac{MR^2v^2}{R^2} = Mgh \qquad (6.34)$$

or

$$v = \sqrt{gh} \qquad (6.35)$$

Do you notice anything interesting in this equation? ***The linear velocity is independent of mass and radius of the hoop. Its means that a hoop of any material and any radius rolls down with the same speed on the inclined plane.***

Intext Questions 6.5

1. A solid sphere rolls down a slope without slipping. What will be its velocity in terms of the height of the slope?
2. A solid cylinder rolls down an inclined plane without slipping. What fraction of its kinetic energy is translational? What is the magnitude of its velocity after falling through a height h?

3. A uniform sphere of mass 2 kg and radius 10cm is released from rest on an inclined plane which makes an angle of 30^0 with the horizontal. Deduce its (a) angular acceleration, (b) linear acceleration along the plane, and (c) kinetic energy as it travels 2m along the plane.

Secret of Pulsars

An interesting example of the conservation of angular momentum is provided by pulsating stars. These are called pulsars. These stars send pulses of radiation of great intensity towards us. The pulses are periodic and the periodicity is extremely precise. The time periods range between a few milliseconds to a few seconds. Such short time periods show that the stars are rotating very fast. Most of the matter of these stars is in the form of neutrons. (The neutrons and protons are the building blocks of the atomic nuclei.) These stars are also called neutron stars. These stars represent the last stage in their life. The secret of their fast rotation is their tiny size. The radius of a typical neutron star is only 10 km. Compare this with the radius of the Sun, which is about 7×10^5 km. The Sun rotates on its axis with a period of about 25 days. Imagine that the Sun suddenly shrinks to the size of a neutron star without any change in its mass. In order to conserve its angular momentum, the Sun will have to rotate with a period as short as the fraction of a millisecond.

WHAT YOU HAVE LEARNT

- A rigid body can have rotational and translational motions separately as well as combinely.
- The equations of translational motion for a rigid body may be written in the same form as for a single particle interms of motion of its centre of mass.
- In a rigid body, the relative distance between its particles does not change even after the application of external force.
- In reality, there is no rigid body in the universe. But, we consider many things like train rails, wheels, metal spheres as rigid objects.
- The point of the object where the total mass appears to be concentrated is called "Centre of Mass".
- Centre of mass simplifies the study of motion of the rigid objective or a system of particles.
- It is not necessary to have mass at the centre of mass.
- The point of the object, where the total weight of it appears to be concentrated is called centre of gravity.
- Centre of gravity deals with the stability of the object.

- In a pure rotatory motion, all the particles of a rigid object go around the axis of rotation (fulcrum) in concentric circular paths. All particles have same angular velocity, but with different linear velocities.
- If a point in the rigid body is fixed, then it possess only rotational motion.
- The moment of inertia about an axis of rotation is defined as $I = \sum_{i=1}^{n} m_i r_i^2 = MK^2$.

  Where M = total mass; K = radius of gyration.
- The moment of Inertia plays the same role in rotational motion as does the mass in linear motion.
- Moment of a force (Torque) is given by

$$\tau = \overline{\mathbf{r}} \times \overline{\mathbf{F}}$$

  It is also called as the "turning effect of a force".
- Two equal, opposite and parallel forces constitute a couple. The magnitude of turning effect, when the couple acts, is defined as the product of one of the forces and the perpendicular distance between the two parallel forces.
- The application of an external torque changes the angular momentum of the body.
- Angular momentum L = I ω.
- When no net torque acts on a body, the angular momentum of the body remains constant

  if $\tau_{ext} = 0$ then L = I ω = Constant

  or $I_1\omega_1 = I_2\omega_2$

* Law of conservation of angular momentum is applied in spring board diving, scating and somersault.
* When a cylindrical or a spherical body rolls down an inclined plane without slipping, its speed is independent of its mass and radius.

TERMINAL EXERCISE

1. The weight *Mg* of a body is shown, generally, as acting at the centre of mass of the body. Does this mean that the earth does not attract other particles?
2. Is it possible for the centre of mass of a body to lie outside the body? Give two examples to justify your answer?
3. In a molecule of carbon monoxide (CO), the nuclei of the two atoms are 1.13×10^{-10}m apart. Locate of the centre of mass of the molecule.
4. A grinding wheel of mass 5.0 kg and diameter 0.20 m is rotating with an angular speed of 100 rad s^{-1}. Calculate its kinetic energy. Through what distance would it have to be dropped in free fall to acquire this kinetic energy? (Take g =10.0 ms^{-2}).

5. A wheel of diameter 1.0 m is rotating about a fixed axis with an initial angular speed of 2 rev s^{-1}. The angular acceleration is 3 rev s^{-2}.
   (a) Compute the angular velocity after 2 seconds.
   (b) Through what angle would the wheel turned during this time?
   (c) What is the tangential velocity of a point on the rim of the wheel at $t = 2$ s?
   (d) What is the centripetal acceleration of a point on the rim of the wheel at $t = 2$ s?
6. A wheel rotating at an angular speed of 20 rads^{-1} is brought to rest by a constant torque in 4.0 seconds. If the moment of inertia of the wheel about the axis of rotation is 0.20 kg m^2, calculate the work done by the torque in the first two seconds.
7. Two wheels are mounted on the same axle. The moment of inertia of wheel A is 5×10^{-2} kg m^2, and that of wheel B is 0.2 kg m^2. Wheel A is set spinning at 600 rev min^{-1}. while wheel B is stationary. A clutch now acts to join A and B so that they must spin together.
   (a) At what speed will they rotate?
   (b) How does the rotational kinetic energy before joining compare with the kinetic energy after joining?
   (c) What torque does the clutch deliver if A makes 10 revolutions during the operation of the clutch?
8. You are given two identically looking spheres and told that one of them is hollow. Suggest a method to detect the hollow one.
9. The moment of inertia of a wheel is 1000 kg m^2. Its rotation is uniformly accelerated. At some instant of time, its angular speed is 10 rad s^{-1}. After the wheel has rotated through an angle of 100 radians, the angular velocity of the wheel becomes 100 rad s^{-1}. Calculate the torque applied to the wheel and the change in its kinetic energy.
10. A disc of radius 10 cm and mass 1 kg is rotating about its own axis. It is accelerated uniformly from rest. During the first second it rotates through 2.5 radians. Find the angle rotated during the next second. What is the magnitude of the torque acting on the disc?
11. Moment of Inertia of a spherical shell about its diameter is $\frac{MR^2}{4}$. Determine the moment of Inertia of the spherical shell about its tangent (hint : use parallel axes theorem).
12. When the radius of gyration is increased by 10%, what percentage of increase is observed in moment of Inertia.
13. A flywheel of moment of Inertia 0.3 kg m^2 is making 300 revolution per minute. What Torque must be applied on it so that it gets stopped in 20 seconds.

14. The moment of Inertia of a wheel is 0.63 kg m^2. Calculate the amount of work done in increasing its angular velocity from 40 rpm to 80 rpm.

15. Find torque of a force $7\bar{\mathbf{i}}+3\bar{\mathbf{j}}-3\bar{\mathbf{k}}$ about the origin. The force acts on a particle whose position vector is $\bar{\mathbf{i}}+\bar{\mathbf{j}}-\bar{\mathbf{k}}$.

ANSWERS TO INTEXT QUESTIONS

6.1

1. Yes, because the distance between any two points on the frame can not change.
2. No.

6.2

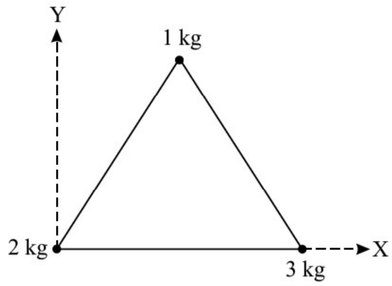
1. The coordinates of given five masses are A (–1, –1), B (–5, –1), C (6, 3), D (2, 6) and E (–3, 0) and their masses are 1 kg, 2 kg, 3 kg, 4 kg and 5 kg respectively.

   Hence, coordinates of centre of mass of the system are

$$x=\frac{-1\times 1-5\times 2+6\times 3+2\times 4-3\times 5}{1+2+3+4+5}=0$$

$$y=\frac{-1\times 1-1\times 2+1\times 3+4\times 6+0\times 5}{1+2+3+4+5}=\frac{30}{15}=2.0$$

2. Let the three particle system be as shown in the figure here. Consider axes to be as shown with 2 kg mass at the origin.



$$x=\frac{2\times 0+1\times 0.5+3\times 1}{1+2+3}=\frac{3.5}{6}m$$

$$y=\frac{2\times 0+1\times\frac{\sqrt{3}}{2}+3\times 0}{1+2+3}=\frac{\sqrt{3}}{12}m$$

   Hence, the co-ordinates of the centre of mass are $\left(\frac{3.5}{6},\frac{\sqrt{3}}{12}\right)$

3. Let the two particles be along the x-axis and let their x-coordinates be 0 and x. The coordinate of CM is

$$X=\frac{m_1\times 0+m_2\times x}{m_1+m_2}=\frac{m_2 x}{m_1+m_2},\ Y=0$$

X is also the distance of m_1 from the CM. The distance of m_2 from CM is

$$x - X = x - \frac{m_2 x}{m_1 + m_2} = \frac{m_1 x}{m_1 + m_2}$$

$$\therefore \frac{X}{x+X} = \frac{m_2}{m_1}$$

Thus, the distances from the CM are inversely proportional to their masses.

4. No.

6.3

1. Moment of inertia of the system about an axis perpendicular to the plane passing through one of the corners and perpendicular to the plane of the square,

   $= m\,r^2 + m\,(2\,r^2) + m\,r^2 = 4\,m\,r^2$

   M.I. about the axis along the side $= m\,r^2 + m\,r^2 = 2\,m\,r^2$

   Verification : Moment of inertia about the axis QP $= m\,r^2 + m\,r^2 = 2\,m\,r^2$. Now, according to the theorem of perpendicular axes, MI about SP ($2\,m\,r^2$) + MI about QP; $2\,m\,r^2$ should be equal to MI about the axis through P and perpendicular to the plane of the square ($4\,m\,r^2$). Since it is true, the results are verified.

2. M.I. of solid sphere about an axis tangential to the sphere

   $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$ according to the theorem of parallel axes.

   If radius of gyration is K, then $M\,K^2 = \frac{7}{5}\,M\,R^2$. So,

   Radius of gyration $K = R\sqrt{\frac{7}{5}}$

3. $\omega = \frac{2\pi n}{t} = \frac{2\pi \times 1}{60 \times 60} = \frac{\pi}{1800}\ rad/second$

4. To increase $|r|$ in $\bar{\boldsymbol{\tau}} = \bar{\mathbf{r}} \times \bar{\mathbf{F}}$

5. (a) Rotating the knob / tap

   (b) Applying two forces while pedalling the cycle.

   (c) Rotating the steering wheel of a car with two hands.

6. While tossing the coin, we apply force on the edge of the coin with thumb, while balancing the coin on index finger.

7. kg m^2

6.4

1. Angular momentum

$$L=\left(m\frac{d^2}{4}+m\frac{d^2}{4}\right)\omega$$

$$L=\frac{md^2\omega}{2}$$

2. Angular momentum about an axis of rotation (diameter).

$$L=I\omega= m\frac{r^2}{4}\times\omega$$

as M.I about a diameter $r=\dfrac{mr^2}{4}$

$$\therefore\ \mathrm{L}=2.0\mathrm{kg}\times\frac{(0.2)^2\,\mathrm{m}^2}{4}\times 10\,\mathrm{rad\,s^{-1}}=0.2\mathrm{kg\,m^2s^{-1}}$$

3. According to conservation of angular momentum

$$I_i\omega=\left(I_1+I_2\right)\omega_1$$

where I_1 is M.I. of the original wheel and I_2 that of the other wheel, ω is the initial angular speed and ω_1 is the common final angular speed.

$$mr^2\,\omega=\left(mr^2+\frac{m}{2}r^2\right)\omega_1$$

$$\omega=\frac{3}{2}\omega_1\Rightarrow\omega_1=\frac{2}{3}\omega$$

4. Let the present period of revolution of earth be T and earlier be T_0. According to the conservation of angular momentum.

$$\frac{2}{5}M\left(25R\right)^2\times\left(\frac{2\pi}{T_0}\right)=\frac{2}{5}MR^2\times\left(\frac{2\pi}{T}\right)$$

$$=\frac{2}{5}MR^2\times\left(\frac{2\pi}{T}\right)$$

It gives, T_0 = 6.25 T

Thus, period of revolution of earth in the past T_0 = 6.25 times the present time period.

6.5

1. Using $\left(I=\frac{2}{5}MR^2\right)$,

$$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=mgh$$

or $$\frac{1}{2}mv^2+\frac{1}{2}\times\frac{2}{5}mr^2.\frac{v^2}{r^2}=mgh$$

$$\therefore \omega = v/r$$

It gives $v=\sqrt{\frac{10}{7}gh}$

2. For a solid cylinder, $I=\frac{mR^2}{2}$

$\therefore$ Total K.E $\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=\frac{1}{2}mv^2+\frac{1}{2}\frac{mR^2}{2}.\frac{v^2}{R^2}=\frac{3}{4}mv^2$

$\because\ \omega=v/r$

Hence, fraction on translational K.E.$=\dfrac{\frac{1}{2}mv^2}{\frac{3}{4}mv^2}=\frac{2}{3}$

Proceeding as in Q.1 above: $v=\sqrt{\frac{4}{3}gh}$

3. m = 2 kg r = 10 cm = 0.1 m θ = 30°, s = 2 m
   (a) & (b) a = g sin θ

$$= 10 \times \sin 30° = 10 \times \frac{1}{2} = 5\,\text{m/s}^2$$

$$a = r \propto \ \Rightarrow\ \propto = \frac{a}{r} = \frac{5}{0.1} = 50\ \text{rad/s}^2$$

   (c) $v^2 - u^2 = 2as$
   $v^2 - 0 = 2 \times 5 \times 2 = 20$

$$v=\sqrt{20}\,\text{m/s},\quad \omega = v/r = \frac{\sqrt{2.0}}{0.1} = 10\sqrt{20}\ \text{rad/s}$$

   KE = 1/2 mv^2 + 1/2 $I\omega^2$ (where I = 2/5 mr^2)

$$= 1/\cancel{2} \times \cancel{2} \times 20 + \frac{1}{2} \times \left(\frac{2}{5} \times 2 \times (0.1)^2 \right) \times 100 \times 20$$

$$= 20 + \frac{1}{\cancel{2}} \left(\frac{\cancel{4}^2}{5} \times 0.01 \right) \times 2000$$

$$= 20 + \frac{\cancel{40}^8}{5}$$

$\therefore$ KE = 28 J

ANSWERS TO TERMINAL EXERCISE

1. No. Earth attracts all the particles. But the resultant of all these attractive forces acts as Mg at the centre of mass.
2. Yes. (1) Metal ring (2) Hallow sphere
3. $m_c = 12$; $m_{O_2} = 16$

   $x_1 = 0$ $x_2 = 1.13 \times 10^{-10}$

   $$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{16 \times 1.13 \times 10^{-10}}{28} = 0.64 \times 10^{-10} m$$
4. m = 5kg, d = 0.2 m ω = 100 rad s^{-1}

   I = mr^2 = 5 $\times$ $(0.1)^2$ = 5 $\times 10^{-2}$ Kg m^2

   KE = 1/2 $\times$ 5 $\times 10^{-2}$ $\times$ 100 $\times$ 100 = 250J

   KE = mgh $\Rightarrow$ 250 = 5 $\times$ 10 $\times$ h $\Rightarrow$ h = 5m
5. $r = 1/2m$ $\omega_0 = 2 \text{rev s}^{-1}$ $\alpha = 3 \text{rev s}^{-2}$

   (a) $\omega = \omega_0 + \alpha t$

   $= 2 + 3 \times 2 = 8 \text{rev/s} = 8 \times 2\pi \text{ rad/s}$

   $\omega = 16\pi \text{ rads}^{-1}$

   (b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 2 \times 2 + \frac{1}{\cancel{2}} \times 3 \times \cancel{4}^2 = 10 \text{ reveloution}$

   $\theta = 20\pi \text{ rad}$

   (c) $v = r\omega = \frac{1}{2} \times 16 \times \frac{22}{7} = 22 m/s$

(d) $a=\omega^2 r=(16\pi)^2\times\frac{1}{2}=1280\,m/s^2$

6. $\omega=20\,rads^{-1}\quad \omega=0\quad t=4s\quad I=0.2kgm$

$\Delta E=\frac{1}{2}I\left(\omega^2-\omega_0^2\right)=\frac{1}{2}\times 0.2\times 400=40J$

7. (a) $I_1=5\times10^{-2}\,kg\,m^2\qquad \omega_1=\frac{600\times\pi}{60}=20\pi\,rads^{-1}$

$I_2=0.2kg\,m^2\qquad \omega_{comm}=?$

$I_1\,\omega_1=\left(I_1+I_2\right)\omega_{comn}\Rightarrow\omega_{comn}=4\pi\,rads^{-1}$

(b) $E_i=\frac{1}{2}I_1\omega_1^2=\frac{1}{2}\times 5\times 10^{-2}\times 400\times\pi^2=100$

$E_f=\frac{1}{2}\left(I_1+I_2\right)\omega_{cm}^2=\frac{1}{2}\left(0.25\right)\times 16\,\pi^2$

$=0.125\times 160=20$

$E_f=\frac{E_i}{5}$

(c) $\alpha=\frac{\omega^2-\omega_0^2}{2\theta}\qquad\qquad \tau=I_1\,\alpha=49\pi J$

8. Make them to roll down on an inclined planes. Solid one moves faster.

9. I = 1000 kg m^2 $\quad\omega$ = 10 rad/s $\quad\omega_2$ = 100 rad/s

θ = 100 radian

$\omega_2^2-\omega_1^2=2\alpha\theta\Rightarrow(100)^2-(10)^2=2\alpha(100)$

$10000-100=200\alpha$

$\alpha=\frac{99\cancel{00}}{2\cancel{00}}=49.5\ rad/s^2$

$\tau=I\alpha=1000\times 49.5=49500\,Nm$

$\Delta KE=\frac{1}{2}I\left(\omega_2^2-\omega_1^2\right)=\frac{1}{2}\times 10^3\times(9900)$

49.5×10^5J

10. $r = 0.1m \quad m = 1kg \quad \omega_0 = 0 \quad \theta_1 = 2.5$

$$\theta_1 = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\alpha(1)^2 = 2.5$$

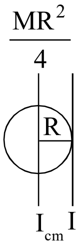
$$\alpha = 5; \quad \theta_1 = \frac{5}{2}\, rad$$

$$\theta_2 = \omega_0 + \frac{1}{2}\alpha(2)^2 = 0 + \frac{1}{2} \times 5 \times 4 = 10\, rad$$

θ in next second = $\theta_2 - \theta_1 = 10 - \frac{5}{2} = 7.5$ rad

$$\tau = I\alpha = \left(mr^2\right)\alpha = 1 \times (0.1)^2 \times 5$$

$$\tau = 5 \times 10^{-2}\, J$$

11. 

$$I = I_{cm} + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$

12. $I = MK^2 \quad \frac{I_1}{I_2} = \frac{K_1^2}{K_2^2} \quad \frac{100}{100+x} = \frac{(100)^2}{(110)^2} = \frac{100\cancel{00}}{121\cancel{00}}$

$$\frac{100}{100+x} = \frac{100}{121} \Rightarrow x = 21\%$$

13. $I = 0.3\, kg\, m^2 \quad \omega_0 = \frac{300 \times 2\pi}{60} = 10\pi \quad \omega = 0$

$$\alpha = \frac{\omega - \omega_0}{\tau} = \frac{10\pi}{20} = \frac{-\pi}{10}\, rad/s^2$$

$$\tau = I\alpha = 0.3 \times \frac{\pi}{10} = 3\pi \times 10^{-2}\, Nm$$

14. $I = 0.63\, Kgm^2 \quad \omega_1 = \frac{\overset{2}{\cancel{40}} \times 2\pi}{\underset{3}{\cancel{60}}} = \frac{4\pi}{3}\, rad/s$

$$\omega_2 = \frac{\overset{4}{\cancel{80}} \times 2\pi}{\underset{3}{\cancel{60}}} \quad = \frac{8\pi}{3}\ rad/s$$

$$\mathrm{W} = \frac{1}{2} I\left(\omega_2^2 - \omega_1^2\right) = \frac{1}{2} \times 0.63\left(\frac{64\pi^2}{9} - \frac{16\pi^2}{6}\right)$$

$$= \frac{0.63}{2} \times \frac{10}{9}(48) = 16.8J$$

15. $\overline{\mathbf{F}} = 7\overline{\mathbf{i}} + 3\overline{\mathbf{j}} - 3\overline{\mathbf{k}} \quad \overline{\mathbf{r}} = \overline{\mathbf{i}} + \overline{\mathbf{j}} - \overline{\mathbf{k}}$

$$\overline{\tau} = \overline{r} \times \overline{F} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 7 & 3 & -3 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-3+3) - \hat{\mathbf{j}}(-3+7) + \hat{\mathbf{k}}(3-7)$$

$$\overline{\boldsymbol{\tau}} = -4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

7

SIMPLE HARMONIC MOTION

INTRODUCTION

There are many different types of motions that we encounter every day, including rectilinear (straight line) motion, projectile (curved path) motion, rotatory (circular path) motion and a few other objects in a different way. These motions are defined by the path followed by the object. But some objects execute the motion which are repeated after a certain interval of time. For example, motion of the planets around the sun, the motion of the hands of a clock, the to and fro motion of a swing and to and fro motion of the pendulum of a wall clock are repetitive in nature. Such a motion is called periodic motion.

OBJECTIVES

After studying this lesson, you should be able to

- *define the periodic motion and Oscillatory motion;*
- *show that an oscillatory motion is periodic but a periodic motion may not be necessarily oscillatory;*
- *define simple harmonic motion and represent it as projection of uniform circular motion on the diameter of a circle;*
- *define the basic terms associated with of simple harmonic oscillation;*
- *derive expressions of time period of a given harmonic oscillator;*
- *derive expressions for the potential and kinetic energies of a simple harmonic oscillator;*
- *distinguish between free, damped and forced oscillations;*
- *explain the phenomenon of resonance.*

7.1 PERIODIC MOTION

The motion which repeats itself at equal intervals of time is called a periodic motion. The smallest interval of time after which the motion is repeated is called its period or time period. It is denoted by the symbol T and its S.I. unit is second. Some of the periodic motions are too fast and some other are too slow. Hence, convenient units of time are also used. For example: The period of vibrations (oscillations) of a quartz crystal is expressed in micro seconds since it is 10^{-6} seconds whereas the orbital period of planet earth is expressed in days since it is 365 days.

In the periodic motion, the motion of a swing, the motion of pendulum of a wall clock are different from the motion of the planets around the sun and the motion of the hands of a clock. Here, in the periodic motion some of the objects are moving to and fro about its mean position where as some other objects are moving around a fixed point or an axis. Hence there are two types of periodic motion: (i) Oscillatory motion, and (ii) Non–oscillatory motion. The periodic to and fro motion of the objects is defined as the oscillatory motion or harmonic motion whereas the periodic motion of the objects around a fixed point or an axis is defined as a non-oscillatory motion.

Jean Baptiste Joseph Fourier (1768 – 1830)

French Mathematician, best known for his Fourier series to analyse a complex oscillation in the form of series of sine and consine functions.

Fourier studied the mathematical theory of heat conduction. He established the partial differential equation governing heat diffusion and solved it by using infinite series of trigonometric functions.

Born as the ninth child from the second wife of a taylor, he was orphened at the age of 10. From the training as a priest, to a teacher, a revolutionary, a mathematician and an advisor to Nepolean Bonapart, his life had many shades.

He was a contemporary of Laplace, Lagrange, Biot, Poission, Malus, Delambre, Arago and Carnot. Lunar crator Fourier and his name on Eiffel tower are tributes to his contributions.

7.2 OSCILLATORY MOTION

The periodic motion of an object is to and fro about its mean position, the motion is defined as oscillatory motion or harmonic motion. It is important to note that an oscillatory motion is a periodic motion but a periodic motion is need not be an oscillatory motion. A few examples of oscillatory motion are:

1. When a child in a swing is pushed, the swing moves to and fro about its mean position and executes an oscillatory motion.
2. The pendulum of a wall clock exhibits oscillatory motion as it moves to and fro about its mean position.
3. The string of the guitar oscillates when it is strummed, moving to and fro by its mean position.
4. The motion of the bob of a simple pendulum is an oscillatory motion.
5. The voltage or current of an AC power supply oscillates alternatively going positive and negative about the mean value (zero).

Intext Questions 7.1

1. What is the difference between a periodic motion and an oscillatory motion?
2. Which of the following examples represent a periodic motion?

   (i) A bullet fired from a gun,

(ii) An electron revolving round the nucleus in an atom,

(iii) A vehicle moving with a uniform speed on a road,

(iv) A comet moving around the Sun, and

(v) Motion of an oscillating mercury column in a U-tube.

3. Give an example of (i) an oscillatory periodic motion and (ii) non-oscillatory periodic motion.

7.3 SIMPLE HARMONIC MOTION

The oscillations of a harmonic oscillator can be represented by the terms containing sine or / and cosine of an angle. Then the displacement of oscillating particle from its mean position can be represented by an equation as,

$$y = a \sin\theta \text{ (or) } y = a \cos\theta \text{ (or) } y = A \sin\theta + B \cos\theta \qquad (7.1)$$

Where a, A and B are constants.

Consider a particle of mass m, executing oscillatory motion about the origin of x-axis (E – O – E) (here the discussion is restricted to linear oscillations) under the action of a force F as shown in the Fig. 7.1. Let x be the displacement of the particle at an instant of time t. Then the displacement x of the particle from the origin is directly proportional to the force F and the direction of the force is opposite to that of the displacement. Mathematically we can express it as,

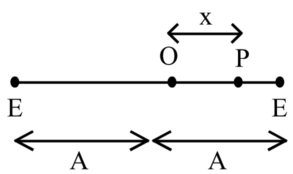


Fig. 7.1 : Particle executing oscillatory motion about the origin 'O' of the x-axis. (Linear Oscillations)

$$F \propto -x$$

$$F = -kx \qquad (7.2)$$

Where 'k' is a constant called the force constant and the negative sign represents that force and displacement are opposite in direction. Applying the Newton's second law of motion for the force acting on oscillating particle, we have,

$$F = m\,a \qquad (7.3)$$

From Eqn. (7.2) and (7.3), we have,

$$m\,a = -kx \text{ (or)}$$

$$a = -\left(\frac{k}{m}\right)x \qquad (7.4)$$

$$a \propto -x \qquad (7.5)$$

where $\left(\frac{k}{m}\right) = \omega^2$ is a constant

From the Eqn. (7.5), it is known that the acceleration of the particle executing oscillatory motion is directly proportional to its displacement from its equilibrium position and directed towards equilibrium position. Here the displacement is measured away from the mean position and force acts in such a way to bring the particle to its equilibrium position, i. e. towards the mean position. This is the simple case of an oscillatory motion and it is known as the simple harmonic motion (SHM).

A particle is said to execute simple harmonic motion if the acceleration of the particle is directly proportional to the displacement of the particle from the mean position and it is directed towards the mean position.

Substituting the second order derivative of displacement for the acceleration of the oscillating particle in the above Eqn. (7.4), we have,

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$$

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad (7.6)$$

Where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency.

Eqn. (7.6), is the differential equation of the simple harmonic oscillation.

Activity 7.1

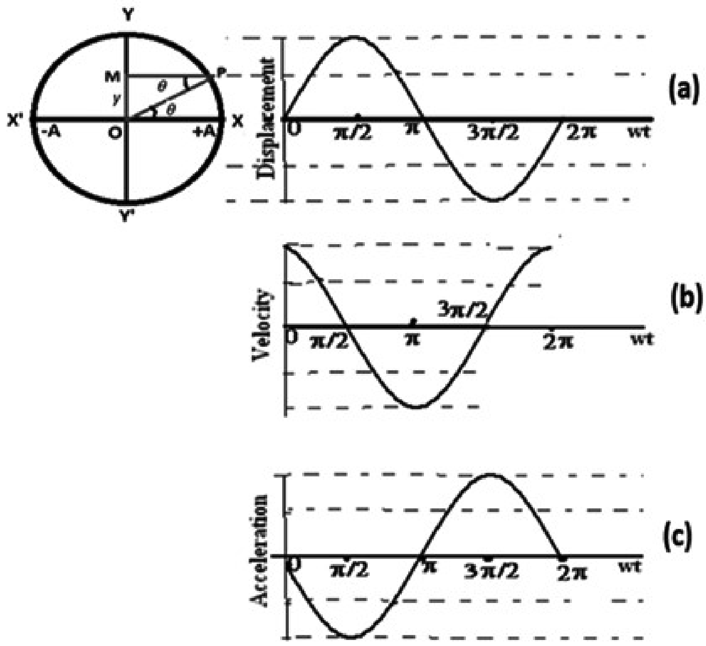
Suppose that the displacement y of a particle, executing simple harmonic motion, is represented by the equation:

$y = A \sin\theta$ (or) $y = A \cos\theta$

From your book of mathematics, obtain the values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^o, 30^o, 60^o, 90^o, 120^o, 150^o, 180^o, 210^o, 240^o, 270^o, 300^o, 330^o$ and 360^o. Then assuming that A = 2.5 cm, determine the values of y corresponding to each angle using the relation, $y = A \sin\theta$. Choose a suitable scale and plot a graph between y and θ. Similarly, using the relation, $y = A \cos\theta$, plot another graph between y and è. You will note that both graphs represent an oscillation between +A and – A. It shows that a certain type of oscillatory motion can be represented by an expression containing sine or cosine of an angle or by a combination of such expressions.

7.4 GRAPHICAL REPRESENTATION OF SHM (CIRCLE OF REFERENCE)

Consider a particle moving on the circumference of a circle of radius A with a constant velocity v. Let the position of the particle be on x-axis at time t = 0, and at an instant of time t, particle be at point P. The position vector OP specifies the position of the moving particle at an instant of time t. As the particle moving along the circumference of the circle, the projection (foot) OM of the position vector OP oscillates on the diameter YY' and thus the projection of the position vector executes simple harmonic motion. From the above Fig. 7.2 (a), in the triangle OMP,



(a) displacement

(b) Velocity

(c) Acceleration of oscillatory motion of the projection of the position vector of particle moving along the circumference of a circle of radius A.

Fig. 7.2

$$\sin\theta = \frac{OM}{OP}$$

$$OM = OP \sin\theta \qquad (7.7)$$

Where radius of the circle OP = A, projection of the position vector OM = y, and θ is the angular displacement of the rotating particle. The expression for angular velocity of a rotating particle is given by,

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega t$$

Substituting OP, OM and θ in Eqn. (7.7), we have,

$$y\ (t) = A \sin \omega t \qquad (7.8)$$

At the time t = 0, if the position vector of the particle exists at an arbitrary position (other than on the x - axis), the above Eqn. (7.8), modifies as,

$$y\ (t) = A \sin (\omega t + \phi) \qquad (7.9)$$

Where ϕ is the initial angle or initial phase which is equal to the angle between the x-axis and initial position of the position vector $\overrightarrow{OP}$ at time t = 0.

The graph plotted between the displacement (y) of the projection of the position vector OP with time 't' is obtained as a sin curve as shown in the Fig. 7.2 (a).

Velocity of Simple Harmonic Oscillator

The speed v of a simple harmonic oscillator at an instant of time t can be obtained by differentiating the displacement as,

$$v(t) = \frac{dy}{dt} = \frac{d}{dt} A \sin(\omega t + \phi)$$

$$v(t) = A\,\omega \cos(\omega t + \phi) \qquad (7.10)$$

The graph plotted between the velocity (v) of the simple harmonic oscillator (projection of the position vector) with time is obtained as a cosine curve as shown in the Fig. 7.2 (b).

Acceleration of Simple Harmonic Oscillator

The acceleration of a simple harmonic oscillator at an instant of time t can be obtained by differentiating the velocity equation as,

$$a(t) = \frac{dv}{dt} = \frac{d}{dt} A\,\omega \cos(\omega t + \phi)$$

$$a(t) = -A\,\omega^2 \sin(\omega t + \phi) \qquad (7.11)$$

The graph plotted between the acceleration (a) of simple harmonic oscillator (projection of the position vector) with time is obtained as a sine curve as shown in the Fig. 7.2 (c). Substituting the displacement from Eqn. (7.9), in the above Eqn. (7.11), we have,

$$a = -\omega^2 y \qquad (7.12)$$

$a \,\alpha\, -y$, where ω^2 is a constant

From the above Eqn. (7.12), it is clear that the acceleration 'a', of the projection of position vector of a particle moving on the circumference of a circle, is proportional to its displacement y, and opposite in the direction. Hence the motion of the projection of the position vector on the diameter of the circle is simple harmonic oscillation.

7.5 BASIC TERMS ASSOCIATED WITH SHM

Displacement : Displacement is the distance of the particle executing simple harmonic motion (Harmonic Oscillator) from its mean position at a given instant. Mathematically, the displacement of the harmonic oscillator can be expressed as,

$y(t) = A \sin\theta$ (or) $y(t) = A \cos\theta$

Where θ is the phase angle and $\theta = \omega t$, w is the angular velocity.

Hence, we can write the expression for displacement of a harmonic oscillator as,

$y(t) = A \sin \omega t$ (or) $y(t) = A \cos \omega t$ (7.13)

From the above expression, it is clear that the displacement is a function of **wt** and it can also be noticed that the displacement of a harmonic oscillator is repeated, if it is increased by an integral multiple of **2π** radians.

Amplitude : Amplitude is the maximum displacement of a harmonic oscillator from its mean position. It is denoted by either **A** (or) **a** and units are meter in S.I. units and cm in CGS units.

In the displacement equation of oscillator, **A** is the amplitude of the oscillator.

Time period (T) : The minimum path travelled by an oscillator to repeat its position is defined as one oscillation and the time taken by an oscillator to complete one oscillation is defined as the time period of that oscillator. It is denoted by **T** and it is measured in seconds.

From the above discussion an oscillator repeats its displacement when phase angle wt is increased by **2π** radians and the time required for it is known as the time period **T**. Hence, we can write,

$$\omega T = 2\pi$$

$$\text{Time period}(T) = \frac{2\pi}{\omega} \qquad (7.14)$$

Where **ω** is the angular velocity of the oscillator and it also known as the angular frequency. Angular frequency is measured in radian per second in S.I. units.

Angular frequency (ω) : Angular frequency describes the rate of change of phase angle. Since the phase angle changes from **0** to **2π** radians in one complete oscillation, the rate of change of phase angle (or) Angular frequency can be expressed as,

$$\omega = \frac{2\pi}{T} \qquad (7.15)$$

The angular frequency is expressed in the radians per seconds units.

Frequency : The number of oscillations completed by an oscillator in one second is defined as the frequency of that oscillator and it is denoted by **f** (or) **v** (or) **n**. Mathematically, frequency can also be defined as the reciprocal of time period T,

$$f = \frac{1}{T} \qquad (7.16)$$

The units of frequency are $\frac{1}{\text{sec}}$ (or) Hertz. From the above equations, we can know that,

$$\text{Angular frequency, } \omega = \frac{2\pi}{T} = 2\pi f$$

$$\left(\text{since, } \frac{1}{T} = f, \qquad \text{where f is frequency}\right)$$

$$\omega = 2\pi f \tag{7.17}$$

From the above equation it is can be understood that the angular frequency of an oscillator is 2π time to its natural frequency.

Phase of the oscillator : Phase is an angle whose sine or cosine at a given instant indicates the position and direction of motion of the oscillator. It is expressed in radians.

From the displacement equation of harmonic oscillator, we know,

$$y(t) = A \sin \omega t$$

If the angular displacement of oscillator does not coincide with the coordinate axis at time t = 0, then the angular displacement (the angle between the position vector of the oscillator and the coordinate axis at time t = 0) is considered as the initial phase (ϕ) of the oscillator and the displacement equation of the oscillator is modified as,

$$y(t) = A \sin(\omega t + \phi)$$

In the above equation, **($\omega t + \phi$)** is the phase of the oscillator and the sine or cosine of phase angle describes the position and direction of the motion of the oscillator.

Example 7.1

A tray of mass 9 kg is supported by a spring of force constant k as shown in Fig.7.3. The tray is pressed slightly downward and then released. It begins to execute SHM of period 1.0s. When a block of mass M is placed on the tray, the period increases to 2.0s. Calculate the mass of the block.

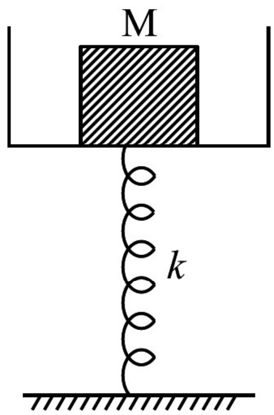


Fig. 7.3

Solution :

The angular frequency of the system is given by $\omega = \sqrt{\frac{k}{m}}$, where m is the mass of the oscillatory system.

Since $\omega = \frac{2\pi}{T}$, we have

$$\frac{4\pi^2}{T^2} = \frac{k}{m}$$

$$m = \frac{kT^2}{4\pi^2}$$

When the tray is empty, m = 9 kg and T = 1 s. Therefore,

$$9 = \frac{k(1)^2}{4\pi^2}$$

On placing the block, m = 9 + M and T = 2s. Therefore,

$$9 + M = \frac{k(2)^2}{4\pi^2}$$

From the above two equations we have

$$\frac{9 + M}{9} = 4$$

Therefore, M = 27 kg

Example 7.2

A spring of force constant 1600 N m^{-1} is mounted on a horizontal table as shown in Fig. 7.4. A mass m = 4.0 kg attached to the free end of the spring is pulled horizontally towards the right through a distance of 4.0 cm and then set free. Calculate (i) the frequency (ii) maximum acceleration and (iii) maximum speed of the mass.

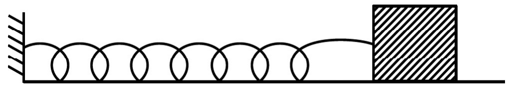


Fig. 7.4

Solution :

$$\text{we know, } \omega = \sqrt{\frac{k}{m}} \text{ and } \omega = 2\pi f$$

$$\text{Therefore, frequency } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1600}{4}} = \frac{1}{2\pi} \times 20 = 3.18 \text{ Hz}$$

maximum acceleration = a ω^2, where a is the amplitude

$$= a\left(\frac{k}{m}\right) = (0.04\,m)\left(\frac{1600\,Nm^{-1}}{4\,kg}\right) = 16\ m s^{-2}$$

$$\text{Maximum speed, } v_{max} = a\,\omega = a\sqrt{\frac{k}{m}}$$

$$= (0.04\,m)\sqrt{\left(\frac{1600\,Nm^{-1}}{4\,kg}\right)} = 0.8\,m s^{-1}$$

7.6 EXAMPLES OF SIMPLE HARMONIC MOTION

7.6.1 Horizontal oscillation of a Spring - Mass system

Consider an elastic spring of force constant placed on a smooth horizontal surface and its length along x-axis. Attach block P of mass m at one end of the spring and fix the other end of the spring at a rigid support in a wall. Let us suppose the mass of the spring is negligible in comparison to the mass of the block attached and there is no loss of energy due to air resistance and friction.

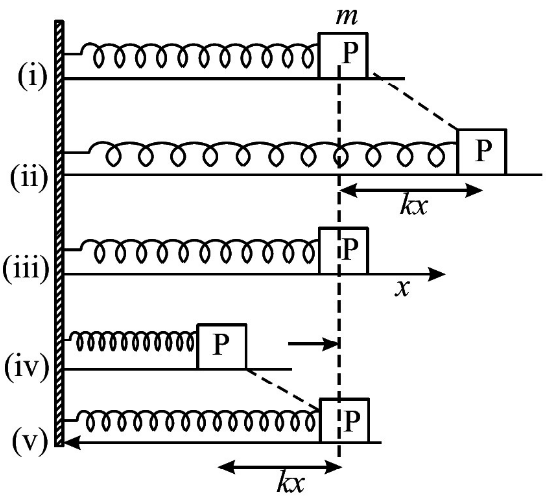


Fig. 7.5 : Horizontal oscillations of a spring-mass system.

When the block P is pulled horizontally through a small distance, the spring undergoes an extension, 'x' and it exerts a force 'kx' on the block P. This force is directed against the extension of the spring and tends to restore the block to its equilibrium position. As the block returns to its equilibrium position, it acquires a velocity 'v' and hence kinetic energy $K = \frac{1}{2}mv^2$.

Owing to inertia of motion, the block overshoots the mean position and continues moving towards the left till it arrives extreme position on left side as shown in Fig. 7.5. In this position, the block again experiences a force 'kx' which tries to bring it back to the equilibrium position. In this way, the block continues oscillating about the equilibrium position. The time period of the horizontal oscillations of the spring-mass system is given by,

$$T = \frac{2\pi}{\omega}$$

$$\text{where } \omega = \sqrt{\frac{k}{m}}$$

Hence,

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (7.18)$$

Where k is the force constant and it is defined as the force per unit extension of the spring and m is the mass of the block attached to the spring.

7.6.2 Vertical oscillations of a Spring - Mass system

Consider a spring of force constant 'k' and suspend it by fixing one of its ends in a rigid support. Attach a block of mass 'm' at the other end of the spring. Then due to the weight of the block of mass attached (mg), the spring undergoes an extension, say 'l' as shown in the Fig. 7.6. Here obviously, the force constant of the spring is,

$$k = \frac{mg}{l}$$

Now pull down the block through a small distance, say 'y' as shown in Fig. 7.6. When the block is released, a restoring force F = – ky acts on the block vertically upwards and the block is pulled to its equilibrium position. As the block returns to its equilibrium position, it continues moving upwards on account of the velocity and it overshoots the equilibrium position by a distance 'y'. Then the spring is compressed and the compressed spring now applies a restoring force F = – ky on the block. Due to this restoring force, the block is pushed downwards. Again, due to its velocity, the block overshoots the equilibrium position and thus the system continues to executes vertical oscillations. Here the time period of the vertical oscillations is,

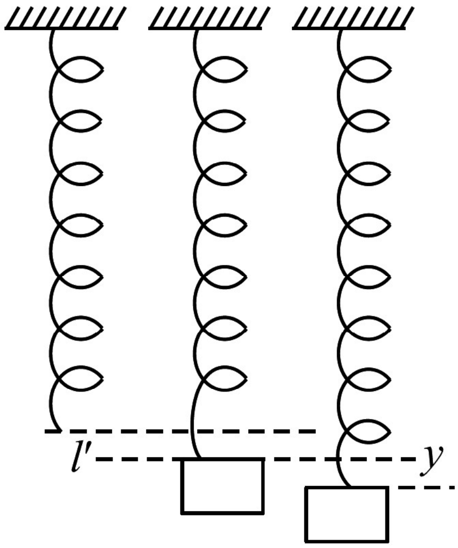
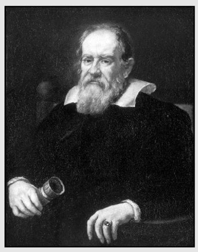


Fig. 7.6: Vertical oscillations of a spring-mass system.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad (7.19)$$

From the above Eqn. (7.19), we can notice that the acceleration due to gravity does not influence the vertical oscillations of a spring mass system.

Galileo Galilei (1564-1642)



Son of Vincenzio Galilei, a wool merchant in Pisa, Italy, Galileo is credited for initiating the age of reason and experimentation in modern science. As a child, he was interested in music, art and toy making. As a young man, he wanted to become a doctor. To pursue the study of medicine, he entered the University of Pisa. It was here that he made his first discovery - the isochronosity of a pendulum, which led Christian Huygen to construct first pendulum clock.

For lack of money, Galileo could not complete his studies, but through his efforts, he learnt and developed the subject of mechanics to a level that the Grand Duke of Tuscany appointed him professor of mathematics at the University of Pisa.

Galileo constructed and used telescope to study celestial objects. Through his observations, he became convinced that Copernican theory of heliocentric universe was correct. He published his convincing arguments in the form of a book, “A Dialogue On The Two Principal Systems of The World”, in the year 1632. The proposition being at variance with the Aristotelian theory of geocentric universe, supported by the Church, Galileo was prosecuted and had to apologize. But in 1636, he published another book "Dialogue On Two New Sciences" in which he again showed the fallacy in Aristotle's laws of motion.

Because sophisticated measuring devices were not available in Galileo's time, he had to apply his ingenuity to perform his experiments. He introduced the idea of thought-experiments, which is being used even by modern scientists, in spite of all their sophisticated devices.

7.6.3 Simple Pendulum

A simple pendulum is a small spherical bob suspended by a long spinless thread held between the two halves of a clamped split cork in retard stand as shown in the Fig. 7.7. Here the bob is considered as a point mass and the thread is taken to be inextensible. The Pendulum can oscillate freely about the point of suspension when the bob is pulled / pushed away through a small distance from its equilibrium position and released.

The distance between the point of suspension and the centre of gravity of the bob defines the length of the pendulum. The forces acting on the bob of the pendulum in the displaced position are as shown in the figure and they are:

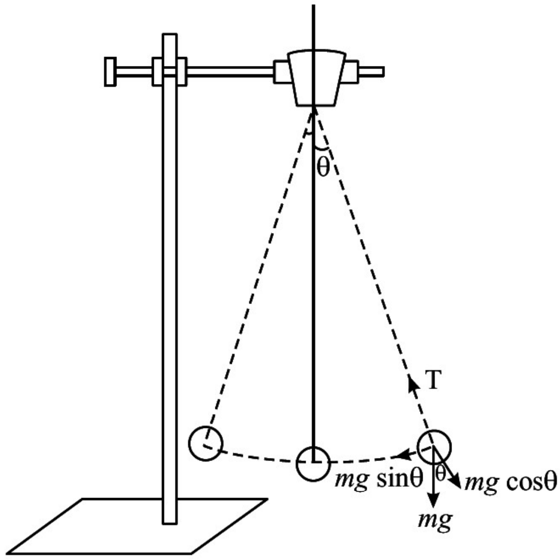


Fig. 7.7: Oscillations of a Simple Pendulum

(a) The weight of the bob, **mg** acts vertically downward.

(b) Tension in the string, **T** act upward along the string.

Here the weight of the bob can be resolved into two components as:

(i) **mg cosθ** which acts along the thread in the opposite direction to the tension and it balances the tension in the thread,

(ii) **mg sinθ** which acts in the perpendicular direction to the thread and it produces the acceleration in the bob in the direction of its equilibrium position.

Therefore, **mg sinθ** creates a restoring force in the simple pendulum and sets the oscillations. For a small displacement, say **x** of the bob, the restoring force is,

$$F = mg \sin\theta$$

For very small values 'θ', $\sin\theta \approx \theta$

Hence, we can write

$$F = mg\theta$$

From the Fig.7.7,

$$\sin\theta \approx \theta = \frac{x}{l}$$

Substituting the θ in the above equation, we have,

$$F = mg\frac{x}{l} = \frac{mg}{l}x$$

$$F = \frac{mg}{l}x \tag{7.20}$$

But the restoring force acting on the bob to bring it to its equilibrium position is given by,

$$F = kx \tag{7.21}$$

From Eqn. (7.20) and (7.21), we have,

$$k = \frac{mg}{l}$$

$$\frac{k}{m} = \frac{g}{l}$$

But $\frac{k}{m} = \omega^2$ and substituting in the above equation, we get,

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}} \tag{7.22}$$

And also, we know the relation between the angular velocity and time period as,

$$\omega = \frac{2\pi}{T}$$

Substituting in the Eqn. (7.22),

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

Time period of a simple pendulum is,

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{7.23}$$

Example 7.3

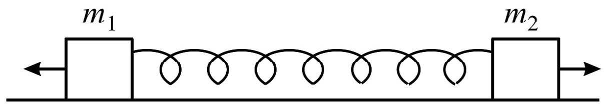


Fig. 7.8: Oscillatory system of masses attached to a spring

Fig. 7.8 shows an oscillatory system comprising two blocks of masses m_1 and m_2 joined by a massless spring of spring constant k. The blocks are pulled apart, each with a force of magnitude F and then released. Calculate the angular frequency of each mass. Assume that the blocks move on a smooth horizontal plane.

Solution :

Let x_1 and x_2 be the displacements of the blocks when pulled apart. The extension produced in the spring is $(x_1 + x_2)$. Thus, the acceleration of m_1 is $\frac{k(x_1+x_2)}{m_1}$ and acceleration of m_2 is $\frac{k(x_1+x_2)}{m_2}$. Since the same spring provides the restoring force to each mass, hence the net acceleration of the system comprising of the two masses and the massless spring equals the sum of the acceleration produced in the two masses. Thus, the acceleration of the system is

$$a = \frac{k(x_1 + x_2)}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = \frac{kx}{\mu}$$

where $x = x_1 + x_2$ is the extension of the spring and μ is the reduced mass of the system. The angular frequency of each mass of the system is therefore,

$$\omega = \sqrt{\frac{k}{\mu}}$$

Such an analysis helps us to understand the vibrations of diatomic molecules like H_2, Cl_2 and HCl etc.

Intext Questions 7.2

1. A small spherical ball of mass m is placed in contact with the surface on a smooth spherical bowl of radius r a little away from the bottom point as shown in the Fig. 7.9. Calculate the time period of oscillations of the ball.

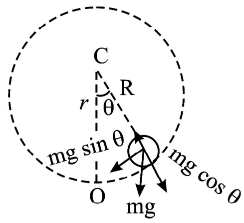


Fig. 7.9

2. A cylinder of mass m floats vertically in a liquid of density ρ. The length of the cylinder inside the liquid is l as shown in the Fig. 7.10. Obtain an expression for the time period of its oscillations.

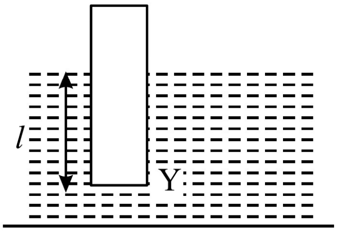


Fig. 7.10

3. Calculate the frequency of oscillation of the mass m connected to two rubber bands as shown in the Fig. 7.11. The force constant of each band is k.

Fig. 7.11

7.7 ENERGY OF THE SIMPLE HARMONIC OSCILLATOR

As we know, the displacement of the Simple harmonic oscillator can be represented by the equation,

$$x(t) = A \sin (\omega t + \phi) \tag{7.24}$$

The velocity is defined as the rate of change of the displacement. Hence the velocity of the simple harmonic oscillator is obtained by differentiating the above displacement equation with respect to time as,

$$v(t) = \frac{dx}{dt} = A\omega \cos (\omega t + \phi) \tag{7.25}$$

Due to the velocity, simple harmonic oscillator can possess the kinetic energy and it can be expressed as,

$$K = \frac{1}{2} m v^2$$

Where m is the mass of the oscillator and 'v' is the velocity of the oscillator. Substituting the Eqn. 7.25 in the above equation, we have,

$$\text{Kinetic energy, K.E.} = \frac{1}{2} m \left(A\omega \cos(\omega t + \phi)\right)^2$$

$$K = \frac{1}{2} m A^2 \omega^2 \cos^2 (\omega t + \phi) \tag{7.26}$$

$$K = \frac{1}{2} m A^2 \omega^2 (1 - \sin^2 (\omega t + \phi))$$

$$K = \frac{1}{2} m \omega^2 (A^2 - A^2 \sin^2 (\omega t + \phi))$$

Substituting Eqn. (7.24), we have

$$K = \frac{1}{2} m \omega^2 (A^2 - x^2) \tag{7.27}$$

When the simple harmonic oscillator displaces, say 'x' from its equilibrium position, a restoring force F = – kx arises in the oscillator and due to this restoring force, oscillator possesses potential energy. The potential energy of the oscillator is given by the equation,

$$U = \frac{1}{2} k x^2$$

Substituting the displacement from Eqn. (7.24), in the above equation, we have,

$$U = \frac{1}{2} k (A \sin(\omega t + \phi))^2$$

$$U = \frac{1}{2} k A^2 \sin^2(\omega t + \phi) \tag{7.28}$$

Substituting Eqn. (7.24), we have

$$U = \frac{1}{2} k x^2 \tag{7.29}$$

But we know that the angular frequency, $\omega = \sqrt{\frac{k}{m}}$ and force constant, $k = m\omega^2$.

Substituting the force constant, k in the above Eqn. (7.27), we have,

$$U = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi) \tag{7.30}$$

But the total energy of the oscillator is the sum of kinetic energy and potential energy. Hence, total energy E is given by,

$$E = K + U$$

Substituting the kinetic energy and potential energy in the above equation, we have,

$$E = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi) + \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi)$$

$$E = \frac{1}{2} m A^2 \omega^2 \left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right) \tag{7.31}$$

$$E = \frac{1}{2} m A^2 \omega^2$$

(Since, $\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi) = 1$)

$$E = \frac{1}{2} k A^2 \tag{7.32}$$

since, $k = m\omega^2$

From the above Eqn. (7.32), it can be noticed that the total energy of a harmonic oscillator is independent of time. From the Eqn. (7.26) and (7.28), the time dependence and from the Eqn. (7.27), (7.31) and (7.32), the displacement dependence of the kinetic energy, potential energy and total energy of a simple harmonic oscillator are as shown in the Fig. 7.12 (a) and 7.12 (b) respectively.

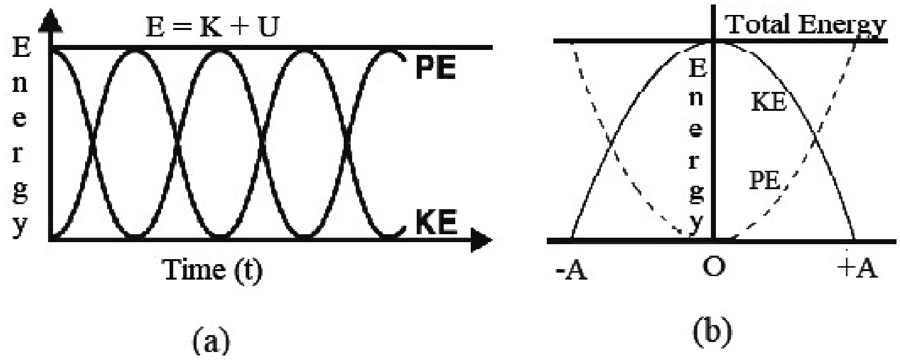


Fig. 7.12: The time and displacement dependence of the kinetic energy and potential energy of a simple harmonic oscillator.

Intext Questions 7.3

1. Is the kinetic energy of a harmonic oscillator maximum at its equilibrium position or at the maximum displacement position? Where is its acceleration maximum?
2. Why does the amplitude of a simple pendulum decrease with time? What happens to the energy of the pendulum when its amplitude decreases?

7.8 DAMPED HARMONIC OSCILLATIONS

We know that the motion of a simple harmonic oscillator, like simple pendulum, swinging in air dies out eventually. Why does it happen? This is because of the air drag and the friction at the support. Here these factors oppose the motion of the pendulum and dissipate the energy of oscillator gradually. As the energy of oscillation decreases, the amplitude of oscillation also decreases. The amplitude of oscillations of an oscillator in air (or any media) decreases continuously. Such oscillations are called damped oscillations.

The dissipating forces are generally the frictional forces. To understand the effect of such external forces on the motion of an oscillator, let us consider a system as shown in Fig.7.13.

Consider a simple harmonic oscillator comprising a metal block B suspended from a fixed support S by a spring G as shown in the Fig. 7.13 (a). Place the metal block B in a glass beaker contained water so as to keep the metal block B below 6 cm from the surface of the water and about the same distance above the bottom of the beaker. Paste a millimetre scale (vertically) on the side of the beaker just opposite the pointer attached to the metal block. Push the block a few centimetres downwards and then release it. After each oscillation, note down the uppermost and lowermost positions of the pointer on the millimetre scale and the time. Then the graph plotted, between time and the amplitude of oscillations, we obtain a graph as shown in the Fig. 7.13 (b). Here we can notice that the amplitude of the oscillations decreases gradually with time. Such oscillations are said to be damped oscillations.

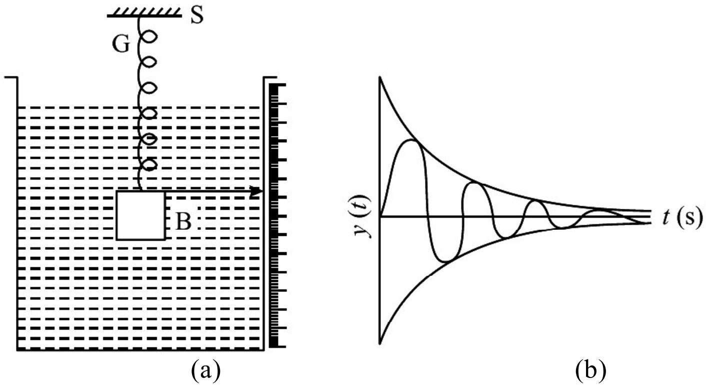


Fig. 7.13 : Damped vibrations: (a) Experimental setup; (b) Graphical representation

The damping force depends on the two factors: (i) Nature of the surrounding medium, (ii) Velocity of the oscillator. Suppose the block is suspended in the air, the magnitude of the damping will be slower whereas if the block is immersed in a liquid, the magnitude of damping will be greater and the rate of dissipation of energy is much faster. The damping force is generally proportional to velocity of the block and acts opposite to the direction of velocity. If the damping force is denoted by , then we have,

$$F_{damped} = -\,b\,v \tag{7.33}$$

Where b is a constant and it depends on the characteristics of the medium and size and shape of the block. 'v' is the velocity of the oscillator and the negative sign represents that the damping force acts in the opposite direction to the velocity of the oscillator.

When the block is set into the oscillations, it oscillates due to the restoring force arises in it and the expression can be written as,

$$F_{restoring} = -\,kx \tag{7.34}$$

Hence the total force on the oscillator is,

$$F = -\,kx - bv \tag{7.35}$$

Suppose the Newton’s law of motion applied along the direction of motion, we have

$$ma = -\,kx - bv \tag{7.36}$$

Where m is the mass of the oscillator and is the acceleration. Using the derivatives of displacement x for velocity and acceleration, we have,

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{7.37}$$

The above expression describes the motion of the oscillator under the influence of a damping force.

Activity 7.2

Take a simple harmonic oscillator comprising a metal block B suspended from a fixed support S by a spring G as shown in the Fig. 7.13 (a). Suspend the metal block B in a glass beaker and fill the beaker with water so as to keep the metal block B below 6 cm from the surface of the water and about the same distance above the bottom of the beaker. Paste a millimetre scale (vertically) on the side of the beaker and fix a pointer at the metal block to read the position of the block on the scale. Now push the block a few centimetres downwards and release it. After each oscillation, note down the uppermost and lowermost positions of the pointer on the millimetre scale and the time. Then plot a graph between time and the amplitude of oscillations. Does the graph show that the amplitude decreases with time? Such oscillations are said to be damped oscillations.

7.9 FREE AND FORCED OSCILLATIONS, RESONANCE

When a body (oscillator) is set in to the oscillations (by displaced from its equilibrium position and released), it oscillates with its natural frequency and the oscillations are called free oscillations. The frequency with which the system oscillates is known as natural frequency. These free oscillations eventually die out because of the presence of damping forces. However, an external periodic force can maintain these oscillations. These oscillations are called forced or driven oscillations and the force is known as the diving force. Suppose an external periodic force is applied to a damped oscillator, such a force can be represented as,

$$F(t) = F_o \sin\omega_p t \tag{7.38}$$

The motion of the oscillator under combined action of restoring force, damping force and driving force is given by,

$$ma = -kx - bv + F_o \sin \omega_p t \tag{7.39}$$

Substituting the first and second order derivatives of displacement for velocity and acceleration in the above equation, we have,

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_o \sin\omega_p t$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_o \sin\omega_p t \tag{7.40}$$

This is the differential equation of forced oscillations.

The above equation represents the motion of an oscillator under the action of a periodic force of frequency ω_p. The solution of the above differential equation, i. e. the displacement of the forced oscillator at an instant of time is given by,

$$x(t) = A \sin(\omega_p t + \phi) \tag{7.41}$$

Where A is the amplitude of the forced oscillation and it is a function of the frequency of the driven force ω_p and the natural frequency of the oscillator ω. By solving the above differential equation, we can obtain the expression for amplitude A as,

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_p^2) + \omega_p^2 b^2}} \tag{7.42}$$

The amplitude of oscillation is directly proportional to the amplitude of the periodic force F_o. The amplitude also depends on the difference between the applied driven force frequency ω_p and the natural frequency of oscillator ω.

From the above equation, suppose the driven force frequency ω_p is equal to the natural frequency of oscillator ω, we have,

$$A = \frac{F_o}{\omega_p b} = \text{maximum} \tag{7.43}$$

This makes it clear that the oscillator oscillates with the maximum possible amplitude and this phenomenon is known as the resonance. In the resonance, the driver (forced oscillations) and the driven (natural oscillations) reinforce each other's oscillations and hence their amplitudes become maximum. This particular case of forced oscillations when the natural frequency of the driver and the driven are equal is known as resonance.

To understand the difference between forced oscillations and resonance phenomena, let us perform the following activity:

Activity 7.3

Let us consider a set of five simple pendulums, A, B, C, D and E suspended from a horizontal rope whose ends fixed in rigid supports as shown in the Fig. 7.14. The pendulums A and C have the same lengths and the others have different lengths. Now let us set pendulum A into motion. The energy from this pendulum A gets transferred to other pendulums through the connecting rope and they start oscillating. The driving force is provided to other pendulums through the connecting rope and the frequency of this driven force is the frequency with which pendulum A oscillates. If we observe the response of pendulums B, D and E, they first start oscillating with their natural frequency and gradually change the oscillating frequency to the frequency of the pendulum A. The pendulums B, D and E are forced to oscillate with the frequency of the pendulum A. The phenomenon is called forced oscillation.

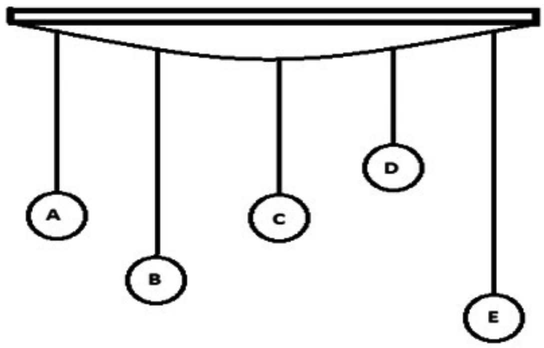


Fig. 7.14: Forced vvibrations and Resonance.

However, pendulum C oscillates with the same frequency as that of pendulum A and its amplitude gradually picks up and becomes very large. This happens because natural frequency of A and C are same since they have equal lengths. This phenomenon is known as the resonance.

Intext Questions 7.4

1. When the stem of a vibrating tuning fork is pressed against the top of a table, a loud sound is heard. Does this observation demonstrate the phenomenon of resonance or forced vibrations? Give reasons for your answer. What is the cause of the loud sound produced?
2. Why are certain musical instruments provided with hollow sound boards or sound boxes?

Mysterious happenings and Resonance

1. Tacoma Narrows Suspension Bridge, Washington, USA collapsed during a storm within six months of its opening in 1940. The wind blowing in gusts had frequency equal to the natural frequency of the bridge. So, it swayed the bridge with increasing amplitude. Ultimately a stage was reached where the structure was over stressed and it collapsed. The events of suspension bridge collapse also happened when groups of marching soldiers crossed them. That is why, now, the soldiers are ordered to break steps while crossing a bridge. The factory chimneys and cooling towers set into oscillations by the wind and sometimes get collapsed.
2. You might have heard about some singers with mysterious powers. Actually, no such power exists. When they sing, the glasses of the window panes in the auditorium are broken. They just sing the note which matches the natural frequency of the window panes.
3. You might have wondered how you catch a particular station you are interested in by operating the tuner of your radio or TV? The tuner in fact, is an electronic oscillator with a provision of changing its frequency. When the frequency of the tuner matches the frequency transmitted by the specific station, resonance occurs and the antenna catches the programme broadcasted by that station.

WHAT YOU HAVE LEARNT

- Periodic motion is a motion which repeats itself after equal intervals of time.
- Oscillatory motion is to and fro motion about a mean position.
- An oscillatory motion is normally periodic but a periodic motion may not necessarily be oscillatory.
- Simple harmonic motion is to and fro motion under the action of a restoring force, which is proportional to the displacement of the particle from its mean (equilibrium) position and is always directed towards the mean position.
- Time period is the time taken by a particle to complete one oscillation.
- Frequency is the number of oscillations completed by the oscillator in one second.

- Phase angle is the angle whose sine or cosine at the given instant indicates the position and direction of motion of the oscillating particle.

- Angular frequency is the rate of change of phase angle. Note that $\omega = \frac{2\pi}{T} = 2\pi\upsilon$, where ω is the angular frequency in rad/sec, υ is the frequency in hertz (symbol: Hz) and T is the time period in seconds.

- Equation of simple harmonic motion is, $y(t) = a \sin(\omega t + \phi)$ (or)
$$y(t) = a \cos(\omega t + \phi)$$
where y is the displacement from the mean position at an instant of time t and ϕ is the initial phase angle (at $t = 0$).

- The motion of the projection of a particle moving along the circle, on its diameter is simple harmonic.

- Time period of the oscillations of a spring mass system is $T = 2\pi\sqrt{\frac{m}{k}}$, where m is the mass attached to the spring and k is the spring constant.

- Time period of the oscillations of a Simple Pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the simple pendulum and g is the acceleration due to gravity.

- Kinetic energy of simple harmonic oscillator is $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

- Potential energy of simple harmonic oscillator is $U = \frac{1}{2}kx^2$

- Total energy of simple harmonic oscillator is $E = \frac{1}{2}kA^2$

- When an oscillatory system vibrates on its own, its vibrations are said to be free vibrations and the frequency is called as the natural frequency.

- As the energy of oscillator decreases, the amplitude of oscillations also decreases. Such oscillations are called damped oscillations.

- If, however, an oscillatory system is driven by an external system, its vibrations are said to be forced vibrations and if the frequency of the driver equals to the natural frequency of the driven, the phenomenon of resonance is said to occur.

TERMINAL EXERCISE

1. Distinguish between a periodic and an oscillatory motion.
2. What is simple harmonic motion?

3. Which of the following functions represent (i) a simple harmonic motion (ii) a periodic but not simple harmonic (iii) a non-periodic motion? Give the period of each periodic motion.

   (i) $\sin \omega t + \cos \omega t$ (ii) $1 + \omega^2 + \omega t$ (iii) $3\cos\left(\omega t - \frac{\pi}{4}\right)$

4. The time period of oscillations of mass 0.1 kg suspended from a Hooke's spring is 1 second. Calculate the time period of oscillation of mass 0.9 kg when suspended from the same spring.

5. What is phase angle? How is it related to angular frequency?

6. Why is the time period of a simple pendulum independent of the mass of the bob, when the period of a simple harmonic oscillator is $T = 2\pi\sqrt{\frac{m}{k}}$?

7. When is the magnitude of acceleration of a particle executing simple harmonic motion maximum? When is the restoring force maximum?

8. Show that the projection of a uniform circular motion on the diameter of the circle is a simple harmonic motion. Obtain an expression for the time period of a simple harmonic oscillator in terms of mass and force constant.

9. Obtain expressions for the instantaneous kinetic energy, potential energy and the total energy of a simple harmonic oscillator.

10. Show graphically how the potential energy U, the kinetic energy K and the total energy E of a simple harmonic oscillator vary with the displacement from equilibrium position.

11. The displacement of a moving particle from a fixed point at any instant is given by $x = a \cos \omega t + b \sin \omega t$. Is the motion of the particle simple harmonic? If your answer is no, explain why? If your answer is yes, calculate the amplitude of vibration and the phase angle.

12. A simple pendulum oscillates with amplitude 0.04 m. If its time period is 10 s, calculate the maximum velocity.

13. Imagine a ball dropped in a frictionless tunnel cut across the earth through its center. Obtain an expression for its time period in terms of radius of the earth and the acceleration due to gravity.

14. The below figure shows a block of mass m = 2 kg connected to two springs, each of force constant $k = 400\ N\ m^{-1}$. The block is displaced by 0.05 m from equilibrium position and then released. Calculate

    (a) The angular frequency ω of the block,

    (b) Its maximum speed,

    (c) Its maximum acceleration and

    (d) Total energy dissipated against damping when it comes to rest.

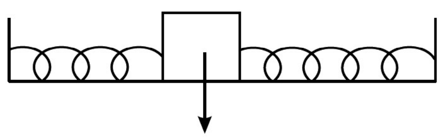


Fig. 7.15

ANSWERS TO INTEXT QUESTIONS

7.1

1. A motion which repeats itself after some fixed interval of time is a periodic motion. A to and fro motion on the same path is an oscillatory motion. A periodic motion may or may not be oscillatory but oscillation motion is parodic.
2. (ii), (iv), (v)
3. (i) To and fro motion of a pendulum.

   (ii) Motion of a planet in its orbit.

7.2

1. Return force on the ball when displaced a distance x from the equilibrium position is

$$F = mg\sin\theta = mg\,\theta = mg\frac{x}{r}$$

$$\text{But, } F = -kx$$

Comparing the above Eqns., we have

$$k = \frac{mg}{r}\ (\text{or})\ \frac{k}{m} = \frac{g}{r}$$

But $\frac{k}{m} = \omega^2$ where ω is the angular frequency.

$$\omega^2 = \frac{g}{r}$$

$$\omega = \sqrt{\frac{g}{r}}$$

$$\omega = \frac{2\pi}{T}\ (\text{or})$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g}}$$

2. On being pushed down through a distance , the cylinder experiences an upthrust

$$F = -\rho g v$$

Where ρ is the density of the liquid, is the acceleration due to gravity and V (= Al) is the volume of the cylinder

$$F = -\rho g A\, l$$

Comparing the above equation with F = – kl, we have force constant, $k = \rho g A$

Most of the liquid, $m = \rho\ V = \rho\ A\ l$

Therefore,

$$\omega^2 = \frac{k}{m} = \frac{\rho g A}{\rho\, A\, l} = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

3. $$\omega^2 = \frac{k}{m} \quad \text{and}$$

$$\text{Frequency, } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Note that when the mass is displaced, only one of the bands exerts the restoring force.

7.3

1. K.E is maximum at mean position or equilibrium position; acceleration is maximum when displacement is maximum.
2. As the pendulum oscillates it does work against the viscous resistance of air and friction at the support from which it is suspended. This work done is dissipated as heat. As a consequence, the amplitude decreases.

7.4

1. When an oscillatory system called the driver applies is periodic of force on another oscillatory system called the driven and the second system is forced to oscillate with the frequency of the first, the phenomenon is known as forced vibrations. In the particular case of forced vibrations in which the frequency of the driver equals the frequency of the driven system, the phenomenon is known as resonance.

   The table top is forced to vibrate not with its natural frequency but with the frequency of the tuning fork. Therefore, this observation demonstrates forced vibrations. Since a large area is set into vibrations, the intensity of the sound increases.

2. The sound board or box is forced to vibrate with the frequency of the note produced by the instrument. Since a large area is set into vibrations, the intensity of the note produced increases and its duration decreases.

ANSWERS TO TERMINAL EXERCISE

4. 3 s

11. $A = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}\left(\frac{a}{b}\right)$

12. $\frac{2}{\pi} \times 10^{-3}\, \text{m s}^{-1}$

14. (a) 14.14 s^{-1} (b) 0.6 m s^{-1} (c) 0.3 m s^{-2} (d) 0.5 J

8

GRAVITATION

INTRODUCTION

Have you ever thought why a ball thrown upward always comes back to the ground? Or a coin tossed in air falls back on the ground. Since the beginning of time, people have been curious about this phenomenon. The answer was provided in the 17^{th} century by Sir Isaac Newton. He proposed that the gravitational force is responsible for bodies being attracted to the earth. He also said that it is the same force which keeps the moon in its orbit around the earth and planets bound to the Sun. It is a universal force, that is, it is present everywhere in the universe. In fact, it is this force that keeps the whole universe together. In this lesson you will learn Newton's law of gravitation. We shall also study the acceleration caused in objects due to the pull of the earth. This acceleration is known as the acceleration due to gravity (g) and which is not a constant on the earth. You will learn the factors due to which the acceleration due to gravity varies. In this lesson, you will also study Kepler's laws of planetary motion and orbits of artificial satellites of various kinds. Finally, we shall recall some of the key programmes and achievements of India in the field of space research.

OBJECTIVES

After studying this lesson, you should be able to

- *state the law of gravitation;*
- *define the Gravitation constant (G) and determine the value of G;*
- *explain the Gravitational Field, Field strength and Gravitational Potential energy;*
- *calculate the value of acceleration due to gravity of a heavenly bodies;*
- *analyse the variation in the value of the acceleration due to gravity with height, depth and latitude;*
- *identify the force responsible for planetary motion and state Kepler's laws of planetary motion;*
- *calculate the orbital velocity and the escape velocity;*
- *explain how an artificial satellite is launched;*
- *distinguish between polar and equatorial satellites;*
- *state conditions for a satellite to be a geostationary satellite;*
- *calculate the height of a geostationary satellite and list their applications;*
- *state the achievements of India in the field of satellite technology.*

8.1 LAW OF GRAVITATION

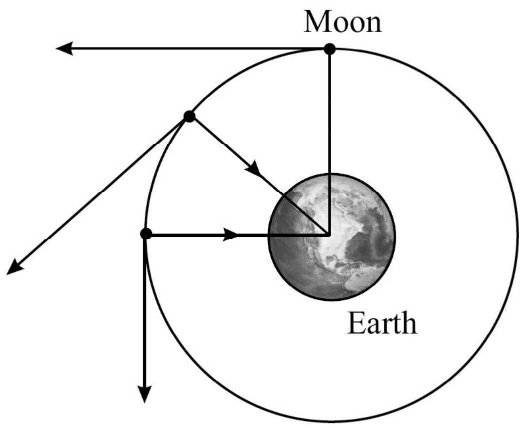


Fig. 8.1 : At each point on its orbit, the moon would have flown off along a tangent but the attraction of the earth keeps it in its orbit.

It is said that Newton was sitting under a tree when an apple fell on the ground. This set him thinking: since all apples and other objects fall to the ground, there must be some force from the earth acting on them. He asked himself: Could it be the same force which keeps the moon in orbit around the earth? Newton argued that at every point in its orbit, the moon would have flown along a tangent as shown in the Fig. 8.1, but it is held back to the orbit by some force. Could this continuous 'fall' be due to the same force which forces apples to fall to the ground? He had deduced from Kepler's laws that the force between the Sun and planets varies as $1/r^2$ where r is the distance between the sun and planet. Using this result, he was able to show that it is the same force that keeps the moon in its orbit around the earth. Then he generalised the idea to formulate the universal law of gravitation as:

"Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them".

Thus, if m_1 and m_2 are the masses of the two bodies and r is the distance between them, the magnitude of the force F is given by,

$$\vec{F} \,\alpha\, \frac{m_1 m_2}{r^2}$$

$$\vec{F} = -G\frac{m_1 m_2}{r^2} \qquad (8.1)$$

Here '–' sign indicates that the force is attractive.

The constant of proportionality, G is called the universal gravitational constant. Its value remains the same between any two objects everywhere in the universe. This means that if the force between two particles is F on the earth, the force between these particles kept at the same distance anywhere in the universe would be the same.

One of the extremely important characteristics of the gravitational force is that it is always attractive. It is also one of the fundamental forces of nature.

Remember that the attraction is mutual, that is, body of mass m_1 attracts the body of mass m_2 and vice versa and also, the force is along the line joining the two bodies.

Knowing that the force is a vector quantity, does Eqn. (8.1) need modification? The answer to this question is that the equation should reflect both magnitude and the direction of the force. As stated, the gravitational force acts along the line joining the two particles.

That is, m_2 and m_1 attracts each other with a force which is along the line joining the two bodies as shown in the Fig. 8.2. If the force of attraction exerted by m_1 on m_2 is denoted by F_{12} and the distance between them is denoted by r_{12}, then the vector form of the law of gravitation is,

$$\vec{F}_{12} = G\frac{m_1 m_2}{(\mathbf{r}_{12})^2}\hat{\mathbf{r}}_{12} \qquad (8.2)$$

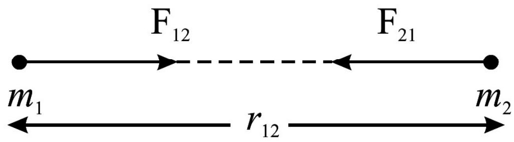


Fig. 8.2 : The masses m_1 and m_2 are placed at a distance r_{12} from each other. The mass m_1 attracts m_2 with a force F_{12} and the mass m_2 attracts m_1 with a force F_{21}.

Here $\hat{r}_{12}$ is the unit vector from mass m_1 to m_2. In a similar way, we can write the force exerted by m_2 on m_1 which is denoted by $\vec{F}_{21}$ as,

$$\vec{F}_{21} = -G\frac{m_2 m_1}{(\mathbf{r}_{21})^2}\hat{\mathbf{r}}_{21} \qquad (8.3)$$

Remember that $\hat{\mathbf{r}}_{12}$ and $\hat{\mathbf{r}}_{21}$, have unit magnitude. However, the directions of these vectors are opposite to each other.

Here the negative sign represents that the direction of the unit vector $\hat{\mathbf{r}}_{21}$, is opposite to that of the direction of unit vector $\hat{\mathbf{r}}_{12}$ and as they have unit magnitude, $\hat{\mathbf{r}}_{12} = -\hat{\mathbf{r}}_{21}$. Hence, from the Eqns. 8.2 and 8.3, we can know,

$$\vec{F}_{12} = -\vec{F}_{21} \qquad (8.4)$$

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude and opposite in direction and form a pair of forces of action and reaction in accordance with Newton's third law of motion. Unless specified, in this lesson we would use only the magnitude of the gravitational force. The value of the universal gravitational constant G is so small that it could not be determined by Newton or his contemporary experimentalists. It was determined by Henry Cavendish for the first time about 100 years later. Its accepted value today is 6.67×10^{-11} Nm^2 kg^{-2}. It is because of the smallness of G that the gravitational force due to ordinary objects is not felt by us.

Example 8.1

Kepler's third law states (we shall discuss this in greater details later) that if 'r' is the mean distance of a planet from the Sun, and T is its orbital period, then $\frac{r^3}{T^2}$ = constant. Show that the force acting on a planet is inversely proportional to the square of the distance.

Solution :

Assume for simplicity that the orbit of a planet is circular. (In reality, the orbits are nearly circular.) Then the centripetal force acting on the planets is, $\vec{F} = \frac{mv^2}{r}$

Where v is the orbital velocity. Since $\vec{v} = r\omega = \frac{2\pi r}{T}$, where T is the period, we can rewrite the above equation as

$$\vec{F} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 mr}{T^2}$$

But $T^2 \propto r^3$ or $T^2 = Kr^3$ (Kepler's third law), where K is a constant of proportionality. Hence

$$\vec{F} = \frac{4\pi^2 mr}{Kr^3} = \frac{4\pi^2}{K} \times \frac{m}{r^2} = \frac{4\pi^2 m}{K} \cdot \frac{1}{r^2}$$

$$\vec{F} \propto \frac{1}{r^2} \qquad (\because \frac{4\pi^2 m}{K} \text{ is constant for a planet})$$

8.2 THE GRAVITATIONAL CONSTANT G

Experimentally the value of the gravitational constant G was determined by English scientist, Henry Cavendish in 1798. The experimental setup used is schematically shown in Fig. 8.3.

Two small spheres of each mass 'm' are attached to the ends of a rod 'PQ' and the rod was suspended by a thin wire. When the rod becomes twisted, the torsion of the wire begins to exert a torsional force and it is proportional to the angle of rotation of the rod. The schematic diagram of the apparatus is shown in Fig. 8.3.

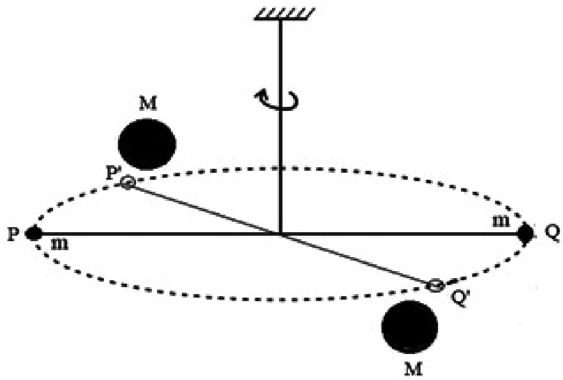


Fig. 8.3 : Schematic drawing of Cavendish's experiment.

When two large spheres of each of mass 'M' are brought near the smaller spheres attached at the end of the rod 'PQ', since all masses attract, the large spheres exerted a gravitational force upon the smaller spheres. Due to this gravitational force, the rod is twisted and torsional force arises in the wire. Once the torsional force balanced the gravitational force, the rod 'PQ' come to rest. Suppose, 'F' is the gravitational force of attraction between the masses 'M' then, 'm' we have,

$$\vec{F} = G\frac{Mm}{d^2} \qquad (8.5)$$

Where the 'd' is the separation between the centres of the larger sphere and its neighbouring smaller sphere.

But this force acts on the two smaller spheres in the opposite direction and constitutes a restoring torque on the rod 'PQ'. Suppose 'L' is the length of the rod, the restoring torque is,

$$\text{Restoring torque} = L \times F = G\frac{Mm}{d^2}L \qquad (8.6)$$

Suppose 'θ' is the angle of twist and τ is the restoring torque per unit angle of twist, then the

$$\text{restoring torque on the rod PQ} = \tau\theta \qquad (8.7)$$

From Eqns. (8.6) and (8.7), we have,

$$G\frac{Mm}{d^2}L = \tau\theta \qquad (8.8)$$

Suppose, the masses 'M', 'm' and length of the rod 'L' are known and the rod is suspended by a wire of known restoring torque per unit angle, the angle of rotation 'θ' and distance between the centres of the nearest masses 'd' can be measured experimentally. This enables one to calculate 'G' from this Eqn. (8.8). Since Cavendish's experiment, the measurement of 'G' has been refined and found the value of 'G' as 6.67×10^{-11} Nm^2 kg^{-2}.

8.3 GRAVITATIONAL FIELD AND FIELD STRENGTH

Gravitational field: A region around a mass in which other masses experience gravitational force of attraction is called the gravitational field. The gravitational field strength $\vec{E}$ at a point in the field is the force per unit mass experienced by a test mass placed at that point.

If a test mass m is placed at a distance r from a mass M producing the field, then the test mass experiences a force F is

$$\vec{F} = -G\frac{mM}{\mathbf{r}^2}\hat{\mathbf{r}} \qquad (8.9)$$

Where $\hat{\mathbf{r}}$ is the unit vector.

From the definition, the gravitational field strength at point P due to mass M is

$$\vec{E} = \frac{\vec{F}}{m} \qquad (8.10)$$

From Eqns. (8.9) and (8.10), we have,

$$\vec{E} = -G\frac{M}{\mathbf{r}^2}\hat{\mathbf{r}} \qquad (8.11)$$

The negative sign indicates that the field is acting towards the centre of gravity of the mass M. Eqn. (8.11) represents expression for the gravitational field strength or intensity.

8.4 GRAVITATIONAL POTENTIAL ENERGY (U)

The gravitational potential energy at a point in a gravitational field of a body is defined as the amount of work done in moving another body of unit mass from infinity to that point.

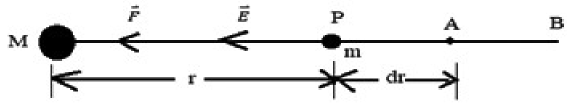


Fig. 8.4 : A point mass is placed at point P located at distance from the mass M and its gravitational field.

Consider the amount of work done is dW, in moving a body of mass m through a distance dr in the gravitational field produced by body of mass M. Then, the gravitational potential energy is given by,

$$dU = dW = \text{Force} \times \text{displacement}$$

$$dU = -F \cdot dr \qquad (8.12)$$

But in moving the mass m from infinity to a point P which is at r distance from the mass M is,

$$U = -\int_{\infty}^{r} F.dr \qquad (8.13)$$

But here the force is the gravitational force acting between the two bodies of masses M and m. Hence, from Eqn. (8.1), we can write

$$U = -\int_{\infty}^{r} -G\frac{Mm}{r^2}dr$$

$$U = GMm\int_{\infty}^{r} \frac{1}{r^2}dr$$

$$U = -G\frac{Mm}{r} \qquad (8.14)$$

The above Eqn. (8.14) represents the expression for the gravitational potential energy stored in the mass 'm' when it is kept in the gravitational field of mass 'M'.

Intext Questions 8.1

1. The period of revolution of the moon around the earth is 27.3 days. Remember that this is the period with respect to the fixed stars (the period of revolution with respect to the moving earth is about 29.5 days; it is this period that is used to fix the duration of a month in some calendars). The radius of moon's orbit is 3.84×10^8 m (60 times the earth's radius). Calculate the centripetal acceleration of the moon and show that

it is very close to the value given by 9.8 ms^{-2} divided by 3600, to take account of the variation of the gravity as $\frac{1}{r^2}$.

2. From Eqn. (8.1), deduce dimensions of G.
3. Using Eqn. (8.1), show that G may be defined as the magnitude of force between two masses of 1 kg each separated by a distance of 1 m.
4. The magnitude of force between two masses placed at a certain distance is F. What happens to F if (i) the distance is doubled without any change in masses,

   (ii) the distance remains the same but each mass is doubled,

   (iii) the distance is doubled and each mass is also doubled?
5. Two bodies having masses 50 kg and 60 kg are separated by a distance of 1 m. Calculate the gravitational force between them.

8.5 ACCELERATION DUE TO GRAVITY

From Newton's second law of motion, we know that a force F exerted on a body of mass 'm', produces an acceleration 'a' in the body according to the relation,

$$\vec{\mathbf{F}} = m\vec{a} \qquad (8.15)$$

Earth exerts a force on a body lying on or near its surface due to gravitational force and also this force produces an acceleration in the body. The acceleration produced by the force of gravity is called the acceleration due to gravity. It is denoted by the symbol 'g'. Hence, from the Eqn. (8.15), the gravitational force of earth acting on a body of mass 'm' is,

$$\vec{\mathbf{F}} = mg \qquad (8.16)$$

According to Eqn. (8.1), the magnitude of the gravitational force between the earth of mass 'M' and a body of mass 'm' is given by,

$$\vec{F} = G\frac{mM}{R^2} \qquad (8.17)$$

where R is the radius of the earth. From Eqns. (8.16) and (8.17), we get

$$mg = G\frac{mM}{R^2}$$

(or)

$$g = G\frac{M}{R^2} \qquad (8.18)$$

Remember that the force due to gravity on a body is directed towards the centre of the earth. It is the direction that we call vertical. Fig. 8.5 shows vertical directions at different places on the earth. The direction perpendicular to the vertical is called the horizontal direction.

Once we know the mass (M) and the radius (R) of the earth, or of any other celestial body such as a planet, the value of 'g' at its surface can be calculated using Eqn. (8.18). On the surface of the earth, the value of 'g' was found as 9.8 ms^{-2}.

Given the mass and the radius of a satellite or a planet, we can use Eqn. (8.18) to find the acceleration due to the gravitational attraction of that satellite or planet.

Before proceeding further, let us look at Eqn. (8.18) again. The acceleration due to gravity produced in a body is independent of its mass. This means that a heavy ball and a light ball will fall with the same velocity. If we drop these balls from a certain height at the same time, both would reach the ground simultaneously.

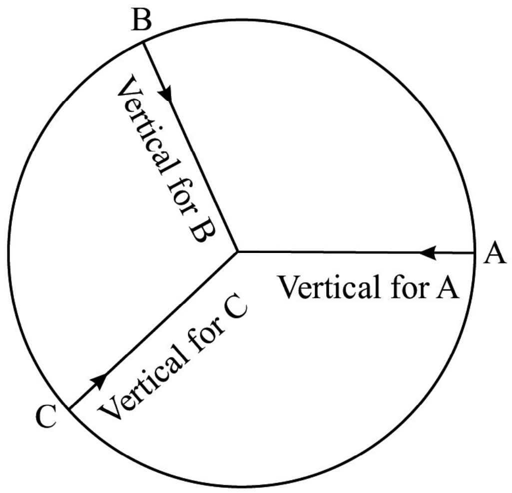


Fig. 8.5 : The vertical direction at any place is the direction towards the centre of earth at that point

Activity 8.1

Take a piece of paper and a small pebble (small stone). Drop them simultaneously from a certain height. Observe the path followed by the two bodies and note the times at which they touch the ground. Then take two pebbles, one heavier than the other. Release them simultaneously from a height and observe the time at which they touch the ground.

Under the influence of gravity, a body falls vertically downwards towards the earth. For small heights above the surface of the earth, the acceleration due to gravity does not change much. Therefore, the equations of motion for initial and final velocities and the distance covered in time ‘t’ are given by

$$v = u + gt \qquad (8.19\ [a])$$

$$s = ut + \frac{1}{2}gt^2 \qquad (8.19\ [b])$$

$$v^2 = u^2 + 2gs \qquad (8.19\ [c])$$

It is important to remember that 'g' is always directed vertically downwards, no matter what the direction of motion of the body is. A body falling with an acceleration equal to 'g' is said to be in free-fall.

From Eqn. (8.19 [b]), it is clear that if a body begins to fall from rest, it would fall (travel) a distance, say s = h, in time ‘t’ is given by

$$h = \frac{1}{2}gt^2 \qquad (8.19\ [d])$$

Here, u = 0, body starting from rest.

So, a simple experiment like dropping a heavy coin from a height and measuring its time of fall with the help of an accurate stop watch could give us the value of 'g'. If we measure the time taken by a five-rupee coin to fall through a distance of 1 meter, we will find that the average time of fall for several trials is 0.45 seconds. From this data, the value of 'g' can be calculated from Eqn. (8.19 [d]). However, in the laboratory we would determine by an 'g' indirect method, using a simple pendulum.

You must be wondering as to why we take radius of the earth as the distance between the earth and a body on its surface while calculating the force of gravity on that body. When we consider two discreet bodies or mass points, the separation between them is just the distance between them. But when we calculate gravitational force between extended bodies, what distance do we take into account? To resolve this problem, the concept of centre of gravity of a body is introduced. This is a point such that, as far as the gravitational effect is concerned, we may replace the whole body by just this point and the effect would be the same. For geometrically regular bodies of uniform density, such as spheres, cylinders, rectangles, the geometrical center is also the centre of gravity. That is why we choose the center of the earth to measure distances to other bodies. For irregular bodies, there is no easy way to locate their centres of gravity.

Where is the center of gravity of metallic ring located? It should lie at the center of the ring. But this point is outside the mass of the body. It means that the centre of gravity of a body may lie outside it. Where is our own centre of gravity located? Assuming that we have a regular shape, it would be at the centre of our body, somewhere beneath the navel.

Later on in this course, we would also learn about the centre of mass of a body. This is a point at which the whole mass of the body can be assumed to be concentrated. In a uniform gravitational field, like the one near the earth, the centre of gravity coincides with the centre of mass.

The use of centre of gravity, or the center of mass, makes our calculations extremely simple. Just imagine the number of calculations we would have to do if we have to calculate the forces between individual particles a body is made of and then finding the resultant of all these forces.

You should remember that 'G' and 'g' represent different physical quantities. 'G' is the universal constant of gravitation which remains the same everywhere, while 'g' is the acceleration due to gravity, which may change from place to place, as we shall see in the next section.

Intext Questions 8.2

1. The mass of the earth is 5.97×10^{24} kg and its mean radius is 6.37×10^{6} m. Calculate the value of 'g' at the surface of the earth.

2. Careful measurements show that the radius of the earth at the equator is 6378 km while at the poles it is 6357 km. Compare values of 'g' at the poles and at the equator.

3. A particle is thrown up. What is the direction of 'g' when (i) the particle is going up, (ii) when it is at the top of its journey, (iii) when it is coming down, and (iv) when it has come back to the ground?

4. The mass of the moon is 7.3×10^{22} kg and its radius is 1.74×10^{6} m. Calculate the gravitational acceleration at its surface.

8.6 VARIATION IN THE VALUE OF 'g'

8.6.1 Variation of 'g' with height

From the Eqn. (8.18), the acceleration due to gravity is inversely proportional to the square of the distance which suggests that the magnitude of 'g' decreases as square of the distance from the centre of the earth increases. So, at a distance from the surface, that is, at a distance '2R' from the centre of the earth, the value of becomes 1/4th of the value of 'g' at the surface. However, if the distance 'h' above the surface of the earth, called altitude, (is small compared with the radius of the earth) the value of 'g', denoted by 'g_h' is given by

$$g_h = G\frac{M}{(h+R)^2}$$

$$g_h = G\frac{M}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$g_h = G\frac{M}{R^2}\frac{1}{\left(1+\frac{h}{R}\right)^2}$$

From Eqn. (8.18), the acceleration due to gravity on the surface of the earth, $g = G\frac{M}{R^2}$. Substituting in the above equation, we have,

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g}{g_h} = \left(1 + \frac{h}{R}\right)^2$$

$$\frac{g}{g_h} = 1 + \frac{2h}{R} + \left(\frac{h}{R}\right)^2$$

Since $\frac{h}{R}$ is a small quantity, $\left(\frac{h}{R}\right)^2$ will be a still smaller quantity. So, it can be neglected in comparison to $\frac{h}{R}$. Thus

$$\frac{g}{g_h} = 1 + \frac{2h}{R}$$

$$g_h = \frac{g}{\left(1 + \frac{2h}{R}\right)} \qquad (8.20)$$

$$g_h = g\left(1 + \frac{2h}{R}\right)^{-1}$$

Expanding the RHS term of the above equation and neglecting the higher order terms, since they have very small values, we have

$$g_h = g\left(1 - \frac{2h}{R}\right) \qquad (8.21)$$

From the Eqn. (8.21), it is known that at small hights 'h' above the earth, the value of acceleration due to gravity decreases by a factor of $\left(1 - \frac{2h}{R}\right)$ and becomes zero at a height of the half of the radius of the earth, i.e., $\frac{R}{2}$.

Example 8.2

Modern aircrafts fly at heights upward of 10 km. Let us calculate the value of g at an altitude of 10 km. Take the radius of the earth as 6400 km and the value of g on the surface of the earth as 9.8 ms^{-2}.

Solution :

From the Eqn. (8.20), we have

$$g_h = \frac{g}{\left(1 + \frac{2h}{R}\right)}$$

$$g_h = \frac{9.8\,ms^{-2}}{\left(1+\frac{2(10\,km)}{(6400\,km)}\right)} = \frac{9.8\,ms^{-2}}{1.003} = 9.77\,ms^{-2}$$

8.6.2 Variation of 'g' with depth

Consider a point 'P' at a depth 'd' inside the earth from its surface as shown in the Fig. 8.6. Let us assume that the earth is a sphere of uniform density 'ρ'. The distance of the point 'P' from the center of the earth is r = (R – d). Draw a sphere of radius (R – d). The point mass placed at 'P' will experience gravitational force from the following two ways:

1. Particles in the shell of thickness 'd',

and

2. Particles in the sphere of radius 'r'.

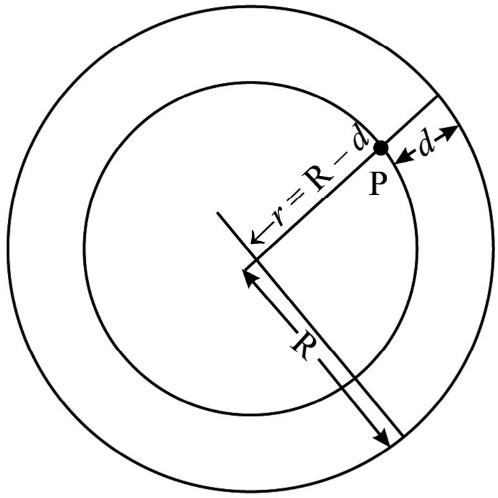


Fig. 8.6 : A point at depth d is at a distance r = (R – d) from the centre of the earth

From the Fig. 8.6, it can be observed that the point 'P' is located on the outer surface of the sphere of radius 'r' and inner surface of the shell of radius 'r' and thickness 'd'. It can be shown that the forces on point mass at 'P' due to all the particles in the shell cancel each other. That is, the net force on the point mass at 'P' due to the matter in the shell is zero. Therefore, to calculate the acceleration due to gravity at point 'P', we should consider only the mass of the sphere of radius r = (R – d). The mass of the sphere of radius r = (R – d) is,

Mass of the sphere of radius 'r' is = Volume × density

$$M' = \left(\frac{4}{3}\pi\,(R-d)^3\right)\rho \qquad (8.22)$$

The acceleration due to gravity experienced by a point mass placed at 'P' is, therefore,

$$g_d = G\frac{M'}{(R-d)^2} \qquad (8.23)$$

From Eqn. (8.22) and Eqn. (8.23), we have,

$$g_d = G\frac{1}{(R-d)^2}\left(\frac{4}{3}\pi(R-d)^3\right)\rho$$

$$g_d = \frac{4\pi G}{3}(R-d)\rho \tag{8.24}$$

Note that as 'd' increases, (R – d) decreases. This means that the value of 'g' decreases as we go below the earth. At d = R, that is, at the centre of the earth, the acceleration due to gravity will vanish. Also note that (R – d) = r is the distance from the centre of the earth. Therefore, acceleration due to gravity is linearly proportional to 'r'. The variation of 'g' from the centre of the earth to distances far from the earth's surface is shown in Fig. 8.7.

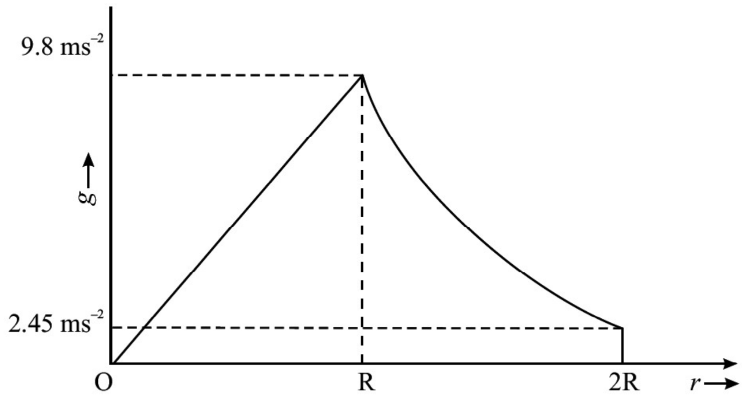


Fig. 8.7 : Variation of g with distance from the centre of the earth.

From the Eqn. (8.24), we can express 'g_d' in terms of the value at the surface by realizing that at d = 0, we get the surface value:

$$g = \frac{4\pi G}{3}\rho R \tag{8.25}$$

From Eqns. (8.24) and (8.25), we can obtain the relation between g_d and g as,

$$g_d = g\frac{(R-d)}{R} = g\left(1-\frac{d}{R}\right),\ 0 \le d \le R \tag{8.26}$$

On the basis of Eqns. (8.21) and (8.26), we can conclude that 'g' decreases with both height as well as depth as show in the Fig. 8.7.

8.6.3 Variation of 'g' with Latitude

We know that the earth rotates about its axis. Due to this, every particle on the earth's surface executes circular motion. In the absence of gravity, all these particles would be flying off the earth along the tangents to their circular orbits. Gravity plays an important role in keeping us tied to the earth's surface. We also know that to keep a particle in circular motion, it must be supplied centripetal force. A small part of the gravity force is used in supplying this centripetal force. As a result, the force of attraction of the earth on objects on its surface is slightly reduced. The maximum effect of the rotation of the earth is felt at the equator. At poles, the effect vanishes completely. We now quote the formula for variation in 'g' with latitude without derivation. If 'g_λ' denotes the value accelaration due to gravity 'g' at latitude angle 'λ' and 'g' is the value at the equator, then

$$g_\lambda = g - R\omega^2\cos^2\lambda \tag{8.27}$$

where 'ω' is the angular velocity of the earth and 'R' is its radius. We can easily see that at the poles, λ = 90 degrees, and hence g_λ = g.

Example 8.3

Let us calculate the value of g at the poles.

Solution :

The radius of the earth at poles = 6357 km = 6.357×10^6 m

The mass of the earth = 5.97×10^{24} kg

We know the relation, $g = G\dfrac{M}{R^2}$

$$g_{\text{Poles}} = \frac{\left(6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}\right) \times \left(5.97 \times 10^{24}\ \text{kg}\right)}{\left(6.357 \times 10^6\ \text{m}\right)^2} = 9.853\,\text{ms}^{-2}$$

Example 8.4

Now let us calculate the value of g at $\lambda = 60^\circ$, where radius of earth is 6371 km.

Solution :

The period of rotation of the earth, T = 24 hours = $24 \times 60 \times$ seconds

Frequency of the earth's rotation = 1/ T

Angular frequency of the earth, $\omega = \dfrac{2\pi}{T} = \dfrac{2 \times 3.14}{24 \times 60 \times 60} = 7.27 \times 10^{-5}\ \text{rad}\,\text{s}^{-1}$

g_λ (at latitude 60 degrees) = $9.853 - (6.37 \times 10^6)(7.27 \times 10^{-5})^2 \cos^2 60$

g_λ (at latitude 60 degrees) = $9.853 - 0.008 = 9.844\ \text{ms}^{-2}$

Internal Structure of the Earth

Refer to Fig. 8.8, you will note that most of the mass of the earth is concentrated in its core. The top surface layer is very light. For very small depths, there is hardly any decrease in the mass to be taken into account for calculating g, while there is a decrease in the radius. So, the value of g increases up to a certain depth and then starts decreasing. It means that assumption about earth being a uniform sphere is not correct.

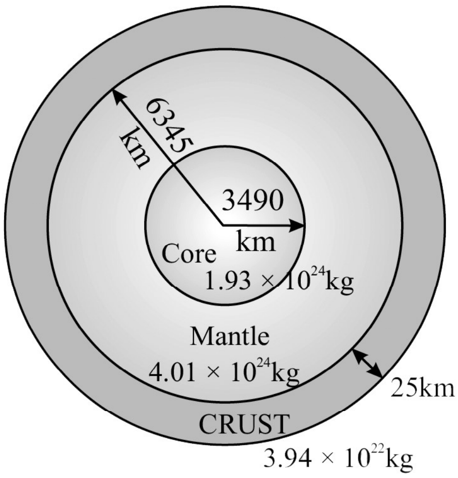


Fig. 8.8: Structure of the earth (not to scale). Three prominent layers of the earth are shown along with their estimated masses.

Intext Questions 8.3

1. At what height must we go so that the value of 'g' becomes half of what it is at the surface of the earth?
2. At what depth would the value of 'g' be 80% of what it is on the surface of the earth?
3. The latitude of Delhi is approximately 30 degrees north. Calculate the difference between the values of g at Delhi and at the poles.
4. A satellite orbits the earth at an altitude of 1000 km. Calculate the acceleration due to gravity acting on the satellite (i) using Eqn. (8.21) and (ii) using the relation 'g' is proportional to $1/r^2$, where 'r' is the distance from the centre of the earth. Which method do you consider better for this case and why?

8.7 WEIGHT AND MASS

The force with which a body is pulled towards the earth is called its weight. If 'm' is the mass of the body, then its weight 'W' is given by

$$W = mg \tag{8.28}$$

Since weight is a force, its unit is newton. If your mass is 50 kg, your weight would be 50 kg × 9.8 ms^{-2} = 490 N.

Since 'g' varies from place to place, weight of a body also changes from place to place. The weight is maximum at the poles and minimum at the equator. This is because the radius of the earth is minimum at the poles and maximum at the equator. The weight decreases when we go to higher altitudes or inside the earth. The mass of a body, however, does not change. Mass is an intrinsic property of a body. Therefore, the mass of a body is constant wherever the body may be situated.

Note: In everyday life we often use mass and weight interchangeably. Spring balances, though they measure weight, are marked in kg (and not in N).

Activity 8.2

Calculate the weight of an object of mass 50 kg at distances of 2R, 3R, 4R, 5R and 6R from the centre of the earth. Plot a graph showing the weight against distance. Show on the same graph how the mass of the object varies with distance. Try the following questions to consolidate your ideas on mass and weight.

Intext Questions 8.4

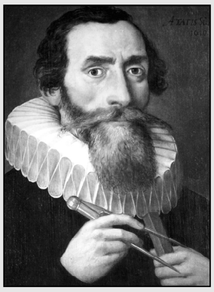
1. Suppose you land on the moon. In what way would your weight and mass be affected?
2. Compare your weight at Mars with that on the earth? What happens to your mass? Take the mass of Mars = 6×10^{23} kg and its radius as 4.3×10^6 m.

3. You must have seen two types of balances for weighing objects. In one case there are two pans. In one pan, we place the object to be weighed and in the other we place weights. The other type is a spring balance. Here the object to be weighed is suspended from the hook at the end of a spring and reading is taken on a scale. Suppose you weigh a bag of potatoes with both the balances and they give the same value. Now you take them to the moon. Would there be any change in the measurements made by the two balances?

8.8 KEPLER'S LAWS OF PLANETARY MOTION

In ancient times it was believed that all heavenly bodies move around the earth. Greek astronomers lent great support to this notion. So strong was the faith in the earth-centred universe that all evidences showing that planets revolved around the sun were ignored. However, Polish Astronomer Copernicus in the 15th century proposed that all the planets revolved around the sun. In the 16th century, Galileo, based on his astronomical observations, supported Copernicus. Another European astronomer, Tycho Brahe, collected a lot of observations on the motion of planets. Based on these observations, his assistant Kepler formulated **laws of planetary motion**.

Johannes Kepler



German by birth, Johannes Kepler, started his career in astronomy as an assistant to Tycho Brahe. Tycho religiously collected the data of the positions of various planets on the daily basis for more than 20 years. On his death, the data was passed on to Kepler who spent 16 years to analyse the data. On the basis of his analysis, Kepler arrived at the three laws of planetary motion.

He is considered as the founder of geometrical optics as he was the first person to describe the working of a telescope through its ray diagram.

For his assertion that the earth revolved around the Sun, Galileo came into conflict with the church because the Christian authorities believed that the earth was at the centre of the universe. Although he was silenced, Galileo kept recording his observations quietly, which were made public after his death. Interestingly, Galileo was freed from that blame recently by the present Pope.

Kepler formulated three laws which govern the motion of planets. These are:

1. **Law of orbits :** All the planets move in elliptical orbits with sun situated at one of the foci. (An ellipse has two foci.)

   An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points is a constant. The two other points are known as the foci of the ellipse. It is characteristic of an ellipse that the sum of the distances of any planet from two foci is constant. During the motion of planet around the sun in an elliptical orbit, the point at which the planet is close to the sun is known as perihelion and the point at which the planet is farther from the sun is known as aphelion.

2. **Law of areas :** The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

As the orbit is not circular, the planet's kinetic energy is not constant in its path. It has more kinetic energy near the perihelion, and less kinetic energy near the aphelion implies more speed at the perihelion and less speed at the aphelion.

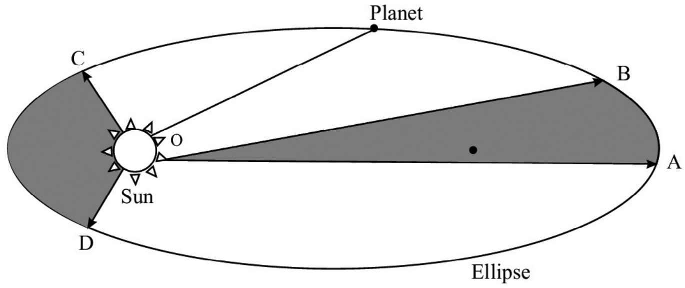


Fig. 8.9: The path of a planet is an ellipse with the Sun at one of its foci. If the time taken by the planet to move from point A to B is the same as from point C to D, then according to the second law of Kepler, the areas AOB and COD are equal.

3. **Law of periods :** The square of the time period of revolution of a planet around the sun is proportional to the cube of its average distance from the Sun.

If we denote the time period by 'T' and the average distance from the planet to the sun as 'r', the law of periods can be expressed mathematically as, $T^2 \alpha\ r^3$. The shorter the orbit of the planet around the sun, the shorter the time taken to complete one revolution.

Let us look at the third law a little more carefully. You may recall that Newton used this law to deduce that the force acting between the Sun and the planets varied as $1/r^2$. Moreover, if T_1 and T_2 are the orbital periods of two planets and r_1 and r_2 are their mean distances from the Sun, then the third law implies that

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \tag{8.29}$$

The constant of proportionality cancels out when we divide the relation for one planet by the relation for the second planet. This is a very important relation. For example, it can be used to get T_2, if we know T_1, r_1 and r_2.

Example 8.5

Calculate the orbital period of planet mercury, if its distance from the Sun is 57.9×10^9 m. You are given that the distance of the earth from the Sun is 1.5×10^{11} m.

Solution :

We know that the orbital period of the earth is 365.25 days. So, $T_1 = 365.25$ days and $r_1 = 1.5 \times 10^{11}$ m. We are told that $r^2 = 57.9 \times 10^9$ m for mercury. Therefore, the orbital period of mercury is given by T^2

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

On substituting the values of various quantities, we get

$$T_2 = \sqrt{\frac{T_1^2 r_2^3}{r_1^3}} = \sqrt{\frac{(365.25)^2 \times (57.9 \times 10^9)^3}{(1.5 \times 10^{11})^3}} = 87.6 \text{ days}$$

In the same manner you can find the orbital periods of other planets. The data is given below. You can also check your results with numbers in Table 8.1.

Table - 8.1: Some data about the planets of solar system

Name of the planet	Mean distance from the Sun (In terms of the distance of earth)	Radius ($\times 10^3$ km)	Mass (Earth Masses)
Mercury	0.387	2.44	0.53
Venus	0.72	6.05	0.815
Earth	1.0	6.38	1.00
Mars	1.52	3.39	0.107
Jupiter	5.2	71.4	317.8
Saturn	9.54	60.00	95.16
Uranus	19.2	25.4	14.50
Neptune	30.1	24.3	17.20

Kepler's laws can be applied to any system where the force binding the system is gravitational in nature. For example, they can be applied to Jupiter and its satellites. They also apply to the Earth and its satellites, such as the Moon and artificial satellites.

Example 8.6

A satellite has an orbital period equal to one day. (Such satellites are called geosynchronous satellites.) Calculate its height from the earth's surface, given that the distance of the moon from the earth is 60 R_E (R_E is the radius of the earth), and its orbital period is 27.3 days. [This orbital period of the moon is with respect to the fixed stars. With respect to the earth, which itself is in orbit round the Sun, the orbital period of the moon is about 29.5 day.]

Solution :

A geostationary satellite has a period T_2 equal to 1 day. For moon T_1 = 27.3 days and r_1 = 60 R_E, T_2 = 1 day. Using the following relations, we can get

$$r_2 = \left[\frac{r_1^3 T_2^2}{T_1^2}\right]^{1/3} = \left[\frac{(60 R_E)^3 (1 \text{day})^2}{(27.3 \text{ day})^2}\right]^{1/3} = 6.6 R_E.$$

Remember that the distance of the satellite is taken from the centre of the earth. To find its height from the surface of the earth, we must subtract R_E from 6.6 R_E. The required distance from the earth's surface is 5.6 R_E. If you want to get this distance in km, multiply 5.6 by the radius of the earth in km.

8.9 ORBITAL VELOCITY OF PLANETS

We have so far talked of orbital periods of planets. If the orbital period of a planet is 'T' and its average distance from the Sun is 'r', then it covers a distance 2πr in time T. Its orbital velocity is, therefore,

$$v_o = \frac{2\pi r}{T} \qquad (8.30)$$

The average speed of a planet while orbiting about the sun is known as the orbital velocity of the planet. There is another way also to calculate the orbital velocity. While revolving in the orbit, planet experiences a centripetal force which is, $\frac{mv_0^2}{r}$ where m is the mass of the planet and r is the radius of the orbit (or average distance between the planet and sun). This force must be supplied by the force of gravitation between the Sun and the planet. If M is the mass of the Sun, then the gravitational force on the planet is $G\frac{mM}{r^2}$. Equating the two forces, we get,

$$\frac{mv_o^2}{r} = G\frac{mM}{r^2}$$

$$v_o^2 = G\frac{M}{r}$$

$$v_o = \sqrt{\frac{GM}{r}} \qquad (8.31)$$

Notice that the mass of the planet does not affect orbital velocity of planet. The orbital velocity depends only on the distance from the Sun. Note also that if you substitute orbital velocity v_o from Eqn. (8.30) in Eqn. (8.31), we get the third law of Kepler.

Intext Questions 8.5

1. Many planetary systems have been discovered in our Galaxy. Would Kepler's laws be applicable to them?
2. Two artificial satellites are orbiting the earth at distances of 1000 km and 2000 km from the surface of the earth. Which one of them has the longer period? If the time period of the former is 90 min, find the time period of the latter.
3. A new small planet, named Sedna, has been discovered recently in the solar system. It is orbiting the Sun at a distance of 86 AU. An AU is the distance between the

Sun and the earth. It is equal to 1.5×10^{11} m. Calculate its orbital time period in years.

4. Obtain an expression for the orbital velocity of a satellite orbiting the earth.
5. Using Eqns. (8.30) and (8.31), obtain Kepler's third law

8.10 ESCAPE VELOCITY

We now know that a ball thrown upwards always comes back due to the force of gravity. If we throw it with greater force, it goes a little higher but again comes back. If you have a friend with great physical power, ask him to throw the ball upwards. The ball may go higher than what you had managed, but it still comes back. You may then ask: Is it possible for an object to escape the pull of the earth? The answer is 'yes'. The object must acquire what is called the escape velocity. It is defined as the minimum velocity required by an object to escape the gravitational pull of the earth.

If a body of mass 'm' is projected with the escaped velocity (v_e), the energy possessed by that body is $\frac{1}{2}mv_e^2$ and this should be either equal or greater than the gravitational potential $G\frac{Mm}{R}$ of the earth. Hence,

$$\frac{1}{2}mv_e^2 = G\frac{Mm}{R}$$

$$v_e^2 = \frac{2GM}{R}$$

$$v_e = \sqrt{\frac{2GM}{R}} \qquad (8.32)$$

where M is the mass of the earth and R is its radius. For calculating escape velocity from any other planet or heavenly body, mass and radius of that heavenly body will have to be substituted in the above expression. It is not that the force of gravity ceases to act when an object is launched with escape velocity. The force does act. Both the velocity of the object as well as the force of gravity acting on it decrease as the object goes up. It so happens that the force becomes zero before the velocity becomes zero. Hence the object escapes the pull of gravity.

From the relation between 'g' and 'G', we know, $g = GM/R^2$. Substituting $GM = gR^2$ in the Eqn. (8.32), we have,

$$v_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} \qquad (8.33)$$

Eqn. (8.33) is the expression for the escape velocity. Substituting the acceleration due to gravity $g = 9.8\ ms^{-2}$ and radius of the earth $R = 6.4 \times 10^6$ m, we get the value of the escape velocity $v_e = 11.2\ km\ s^{-1}$.

Intext Questions 8.6

1. The mass of the earth is 5.97×10^{24} kg and its radius is 6371 km. Calculate the escape velocity from the earth.
2. Suppose the earth shrunk suddenly to one-fourth its radius without any change in its mass. What would be the escape velocity then?
3. An imaginary planet X has mass eight times that of the earth and radius twice that of the earth. What would be the escape velocity from this planet in terms of the escape velocity from the earth?

8.11 ARTIFICIAL SATELLITES

A cricket match is played in Sydney in Australia but we can watch it live in India. A game of Tennis played in America is enjoyed in India. Have you ever wondered what makes it possible? All this is made possible by artificial satellites orbiting the earth. You may now ask: How is an artificial satellite put in an orbit?

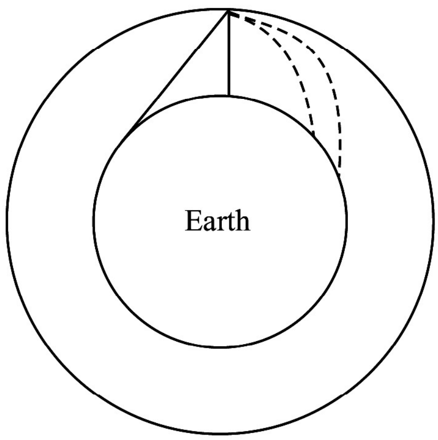


Fig. 8.10: A projectile to orbit the earth.

We have already studied the motion of a projectile. If we project a body at an angle to the horizontal, it follows a parabolic path. Now imagine launching bodies with increasing force. What happens is shown in Fig. 8.10. Projectiles travel larger and larger distances before falling back to the earth. If the projectile body is launched with a suitable force, it can rotate in an orbit around the earth. Objects that move around a planet is called a satellite. Moon is a natural satellite of earth. Man made satellite that revolve around the planets are called artificial satellites. The velocity with which a satellite revolve around the earth in an orbit is called as the orbital velocity, v_o of the satellite.

In order to obtain the expression for the orbital velocity, consider a satellite in a circular orbit of radius (R + h) where R is the radius of the earth and h is the height of the orbit from the earth surface. If m is the mass of the satellite and v_o is its speed which is known as its orbital velocity, the centripetal force required for this orbit is,

$$\vec{F}_{centripetal} = \frac{mv_o^2}{(R+h)} \tag{8.34}$$

Which is directed towards the centre of the earth. This centripetal force is provided by the gravitational force, which is

$$\vec{F}_{gravitational} = \frac{GmM}{(R+h)^2} \tag{8.35}$$

Where M is the mass of the earth. When equating the Eqns. (8.34) and (8.35) we get,

$$\frac{m v_o^2}{(R+h)} = \frac{GmM}{(R+h)^2}$$

$$v_o^2 = \frac{GM}{(R+h)} \tag{8.36}$$

From the relation between g and G, we know that GM = gR^2. Substituting in the above equation, we get,

$$v_o^2 = \frac{gR^2}{(R+h)}$$

$$v_o = \sqrt{\frac{gR^2}{(R+h)}} \tag{8.37}$$

If the satellite is very close to the earth, then $(R+h) \approx R$. Then,

$$v_o = \sqrt{gR} \tag{8.38}$$

Substituting the acceleration due to gravity, g = 9.8 ms^{-2} and radius of the earth, R = 6.4 × 10^6 m, we get the value of the orbital velocity v_o = 8 km s^{-1}.

In order to put a satellite in orbit, it is first lifted to a height of about 200 km to minimize loss of energy due to friction in the atmosphere of the earth. Then it is given a horizontal push with a velocity of about 8 km s^{-1}.

The orbit of an artificial satellite also obeys Kepler's laws because the controlling force is gravitational force between the satellite and the earth. The orbit is elliptic in nature and its plane always passes through the center of the earth. Remember that the orbital velocity of an artificial satellite has to be less than the escape velocity; otherwise, it will break free of the gravitational field of the earth and will not orbit around the earth. From the expressions for the orbital velocity (Eqn. 8.38) of a satellite close to the earth and the escape velocity (Eqn. 8.33) from the earth, we can write,

$$v_e = \sqrt{2}v_o \qquad (or) \qquad v_o = \frac{v_e}{\sqrt{2}} \tag{8.39}$$

Artificial satellites have generally two types of orbits (Fig. 8.11) depending on the purpose for which the satellite is launched. Satellites used for tasks such as remote sensing have polar orbits. The altitude of these orbits is about 800 km. If the orbit is at a height of less than about 300 km, the satellite loses energy because of friction caused by the particles of the atmosphere. As a result, it moves to a lower height where the density is high. There it gets burnt. The time period of polar satellites is around 100 minutes. It is possible to make a polar satellite sun-synchronous, so that it arrives at the same latitude at the same time every day. During repeated crossing, the satellite can scan the whole earth as it spins about its axis (Fig. 8.11). Such satellites are used for collecting data for weather prediction, monitoring floods, crops, bushfires, etc.

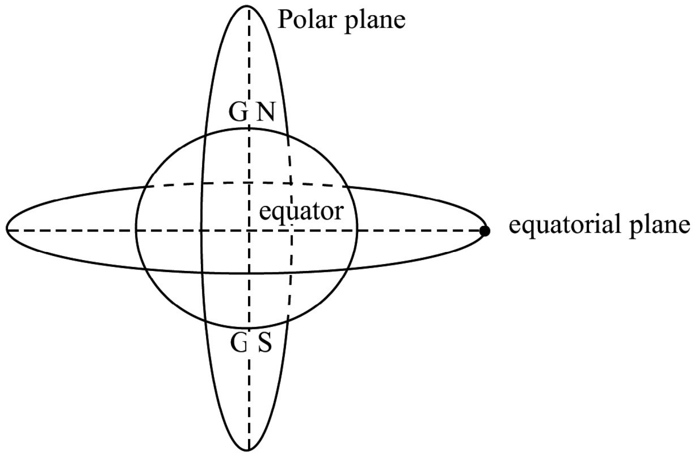


Fig. 8.11 : Equatorial and Polar orbits

Satellites used for communications are put in equatorial orbits at high altitudes. Most of these satellites are geo-synchronous, the ones which have the same orbital period as the period of rotation of the earth, equal to 24 hours. Their height is fixed at around 36000 km. Since their orbital period matches that of the earth, they appear to be hovering above the same spot on the earth. A combination of such satellites covers the entire globe, and signals can be sent from any place on the globe to any other place. Since a geo-synchronous satellite observes the same spot on the earth all the time, it can also be used for monitoring any peculiar happening that takes a long time to develop, such as severe storms and hurricanes.

8.12 APPLICATIONS OF SATELLITES

Artificial satellites have been very useful to mankind. Following are some of their applications:

1. **Weather Forecasting :** The satellites collect all kinds of data which is useful in forecasting long term and short-term weather. The weather chart that you see every day on the television or in newspapers is made from the data sent by these satellites. For a country like India, where so much depends on timely rains, the satellite data is used to watch the onset and progress of monsoon. Apart from weather, satellites can watch unhealthy trends in crops over large areas, can warn us of possible floods, onset and spread of forest fire, etc.
2. **Navigation :** A few satellites together can pinpoint the position of a place on the earth with great accuracy. This is of great help in locating our own position if we have forgotten our way and are lost. Satellites have been used to prepare detailed maps of large chunks of land, which would otherwise take a lot of time and energy.
3. **Telecommunication :** We have already mentioned about the transmission of television programmes from anywhere on the globe to everywhere became possible with satellites. Apart from television signals, telephone and radio signals are also transmitted. The

communication revolution brought about by artificial satellites has made the world a small place, which is sometimes called a global village.

4. **Scientific Research :** Satellites can be used to send scientific instruments in space to observe the earth, the moon, comets, planets, the Sun, stars and galaxies. You must have heard of Hubble Space Telescope and Chandra X-Ray Telescope. The advantage of having a telescope in space is that light from distant objects does not have to go through the atmosphere. So, there is hardly any reduction in its intensity. For this reason, the pictures taken by Hubble Space Telescope are of much superior quality than those taken by terrestrial telescopes. Recently, a group of European scientists have observed an earth like planet outside our solar system at a distance of 20 light years.

5. **Monitoring Military Activities :** Artificial satellites are used to keep an eye on the enemy troop movement. Almost all countries that can afford cost of these satellites have them.

Vikram Ambalal Sarabhai

Born in a family of industrialists at Ahmedabad, Gujarat, India. Vikram Sarabhai grew to inspire a whole generation of scientists in India. His initial work on time variation of cosmic rays brought him laurels in scientific fraternity. A founder of Physical Research Laboratory, Ahmedabad and a pioneer of space research in India, he was the first to realise the dividends that space research can bring in the fields of communication, education, metrology, remote sensing and geodesy, etc.

8.13 INDIAN SPACE RESEARCH ORGANIZATION

India is a very large and populous country. Much of the population lives in rural areas and depends heavily on rains, particularly the monsoons. So, weather forecast is an important task that the government has to perform. It has also to meet the communication needs of a vast population. Then much of our area remains unexplored for minerals, oil and gas. Satellite technology offers a cost-effective solution for all these problems. With this in view, the Government of India set up the Indian Space Research Organization (ISRO) in 1969 under the dynamic leadership of Dr. Vikram Sarabhai. Dr. Sarabhai had a vision for using satellites for educating the nation. ISRO has pursued a very vigorous programme to develop space systems for communication, television broadcasting, meteorological services, remote sensing and scientific research. It has also developed and successfully launched vehicles for polar satellites (PSLV) (Fig. 8.12) and geo-synchronous satellites (GSLV) (Fig. 8.13). In fact, it has launched satellites for other countries like Germany, Belgium and Korea. and has joined the exclusive club of five countries. Its scientific programme includes studies of

(a) Climate, environment and global change,

(b) Upper atmosphere,

(c) Astronomy and astrophysics, and

(d) Indian Ocean.

Recently, ISRO launched an exclusive educational satellite Edusat, first of its kind in the world. It is being used to educate both young and adult students living in remote places. It is now making preparation for a mission to the moon.

Fig. 8.12 : PSLV

Fig. 8.13 : GSLV

Intext Questions 8.7

1. Some science writers believe that some day human beings will establish colonies on the Mars. Suppose people living this desire to put in orbit a Mars synchronous satellite. The rotation period of Mars is 24.6 hours. The mass and radius of Mars are 6.4×10^{23} kg and 3400 km, respectively. What would be the height of the satellite from the surface of Mars?

2. List the advantages of having a telescope in space.

WHAT YOU HAVE LEARNT

- The force of gravitation exists between any two particles in the universe. It varies as the product of their masses and inversely as the square of the distance between them.
- The gravitational constant, G is a universal constant.
- The force of gravitation of the earth attracts all bodies towards it.
- A region around a mass in which other masses experience gravitational force of attraction is called the gravitational field.
- The force experienced by a unit mass at a point in the gravitational field is defined as the gravitational filed strength at that point.

- The amount of work done in moving a mass from infinity to a certain point in the gravitational field is defined as the gravitational potential energy at that point.
- The acceleration due to gravity near the surface of the earth is 9.8 ms^{-2}. It varies on the surface of the earth because the shape of the earth is not perfectly spherical.
- The acceleration due to gravity varies with height, depth and latitude.
- The weight of a body is the force of gravity acting on it.
- Kepler's first law states that the orbit of a planet is elliptic with sun at one of its foci.
- Kepler's second law states that the line joining the planet with the Sun sweeps equal areas in equal intervals of time.
- Kepler's third law states that the square of the orbital period of a planet is proportional to the cube of its mean distance from the Sun.
- The velocity with which a planet revolves in the orbit around the Sun is known as the orbital velocity of the planet and the orbital velocity of the planet depends only on the distance from the Sun and does not depend on the mass of the planet.
- A body can escape the gravitational field of the earth if it can acquire a velocity equal to or greater than the escape velocity.
- The minimum velocity required for a satellite to revolve around the Earth is defined as the orbital velocity, which depends on its distance from the Earth.

TERMINAL EXERCISE

1. You have learnt that the gravitational attraction is mutual. If that is so, does an apple also attract the earth? If yes, then why does the earth not move in response?
2. We set up an experiment on earth to measure the force of gravitation between two particles placed at a certain distance apart. Suppose the force is of magnitude F. We take the same set up to the moon and perform the experiment again. What would be the magnitude of the force between the two particles there?
3. Suppose the earth expands to twice its size without any change in its mass. What would be your weight if your present weight were 500 N?
4. Suppose the earth loses its gravity suddenly. What would happen to life on this plant?
5. Refer to Fig. 8.8 which shows the structure of the earth. Calculate the values of g at the bottom of the crust (depth 25 km) and at the bottom of the mantle (depth 2855 km).
6. Derive an expression for the mass of the earth, given the orbital period of the moon and the radius of its orbit.
7. Suppose your weight is 500 N on the earth. Calculate your weight on the moon. What would be your mass on the moon?

8. A polar satellite is placed at a height of 800 km from earth's surface. Calculate its orbital period and orbital velocity.
9. State the Law of gravitation and explain how the value of Gravitation constant G can be determined by Cavendish experiment?
10. Define the gravitational field and obtain the expression for the gravitational field strength.
11. Define the gravitational potential energy and obtain the expression for it.
12. State the Kepler's laws of planetary motion.

ANSWERS TO INTEXT QUESTIONS

8.1

1. Time period of the Moon, T = 27.3 days = 27.3 × 24 × 3600 seconds

   Radius of the moon's orbit, $R = 3.84 \times 10^8$ m

   Orbital speed of the moon, $v = \frac{2\pi R}{T}$

   Centripetal force, $F = m\frac{v^2}{R}$

   Comparing the above expression with F = ma, we have,

   Centripetal acceleration, $a = \frac{v^2}{R}$

$$= \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times 3.84 \times 10^8}{(27.3 \times 24 \times 3600)^2} = 0.00272 \text{ms}^{-2}$$

   If we calculate centripetal acceleration on dividing 'g' by 3600, we get,

$$\frac{9.8}{3600}\text{ms}^{-2} = 0.0027222\text{ms}^{-2}$$

   Hence, the centripetal acceleration of the moon is very close to the value given by 9.8 ms^{-2} divided by 3600.

2. From Eqn. (8.1), we have,

$$F = G\frac{m_1 m_2}{r^2}$$

For 'G' we can write,

$$G = \frac{Fr^2}{m_1 m_2} = \frac{(\text{Force}) \times (\text{distance})^2}{(\text{mass}) \times (\text{mass})} = \frac{Nm^2}{kg^2}$$

6. $F = G\frac{m_1 m_2}{r^2}$

If $m_1 = m_2 = 1$ kg and r = 1m, then

$$F = G$$

i.e. the Gravitational constant 'G' is equal to the force between the two masses of each 1 kg separated at 1 meter distance.

7. (i) As $F \alpha \frac{1}{r^2}$, if 'r' is doubled, the force becomes one -fourth.

(ii) As $F \alpha m_1 m_2$, if the masses are doubled, force becomes 4 times.

(iii) As $F \alpha \frac{m_1 m_2}{r^2}$, if the distance is doubled and each mass is also doubled, force remains unchanged.

8. $F = G\frac{m_1 m_2}{r^2}$

$m_1 = 50$ kg, $m_2 = 60$ kg, r = 1 m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$F = \left(6.67 \times 10^{-11} Nm^2 kg^{-2}\right)\frac{(50kg)\times(60kg)}{(1)^2} = 2\times10^{-7} N$$

8.2

1. $g = \frac{GM}{R^2}$

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M = 5.97 \times 10^{24}$ kg, $R = 6.371 \times 10^6$ m

$$g = \frac{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\times(5.97\times10^{24}\,\mathrm{kg})}{\left(6.371\times10^{6}\,\mathrm{m}\right)^2} = 9.81\mathrm{ms^{-2}}$$

2. $g = \dfrac{GM}{R^2}$

$$g_{equator} = \frac{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\times(5.97\times10^{24}\,\mathrm{kg})}{\left(6.378\times10^{6}\,\mathrm{m}\right)^2} = 9.79\mathrm{ms^{-2}}$$

$$g_{pole} = \frac{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\times(5.97\times10^{24}\,\mathrm{kg})}{\left(6.357\times10^{6}\,\mathrm{m}\right)^2} = 9.81\mathrm{ms^{-2}}$$

3. The value of 'g' is always vertically downwards.

4. $$g_{moon} = \frac{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\times(7.3\times10^{22}\,\mathrm{kg})}{\left(1.74\times10^{6}\,\mathrm{m}\right)^2} = 1.61\mathrm{ms^{-2}}$$

8.3

1. $g_{surface} = \dfrac{GM}{R^2}$, where R is the radius of the earth

$g_{hieght} = \dfrac{GM}{r^2}$, where r is the hieght from the centre of the earth

$$\frac{g_{surface}}{g_{height}} = \frac{r^2}{R^2}$$

when $g_{height} = \dfrac{g_{surface}}{2}$, we have $\dfrac{g_{surface}}{g_{height}} = 2$

Hence, $$\frac{r^2}{R^2} = 2$$

$$r = \sqrt{2}R = 1.414R$$

But the hieght from the surface where the 'g' is half of its surface value is,

$$h = r - R = 1.414\ R - R = 0.414R$$

2. Inside the earth, 'g' varies as, $g_d = g\frac{(R-d)}{R}$

$$\frac{g_d}{g} = \frac{(R-d)}{R}$$

$$\text{If} g_d = 80\%\ \text{of}\ g,\quad \text{then}\ \frac{g_d}{g} = 80\%\ (\text{or})\ \frac{g_d}{g} = \frac{80}{100} = 0.80$$

$$\text{then,}\ \frac{(R-d)}{R} = 0.80$$

$$d = 0.2R$$

3. Variation of 'g' with latitude is $g_\lambda = g - R\omega^2\cos^2\lambda$

$$g_{poles} = 9.853\ ms^{-2}$$

radius of earth, $R = 6.37 \times 10^6$ m

angular velocity of earth, $\omega = 7.27 \times 10^{-5}$ rad s^{-1}

$g_{Delhi} = 9.853\ ms^{-2} - (6.37 \times 10^6\ m) \times (7.27 \times 10^{-5}\ rad\ s^{-1})^2\ \cos^2 30$

$g_{Delhi} = 9.853\ ms^{-2} - 0.025\ ms^{-2}$

$g_{Delhi} = 9.828\ ms^{-2}$

4. From the Eqn (8.21), $$g_h = g\left(1 - \frac{2h}{R}\right)$$

$$g_h = 9.81 ms^{-2}\left(1 - \frac{2 \times 1000 km}{6371 km}\right) = 7.47\ ms^{-2}$$

$$\text{from the relation,}\ g = \frac{GM}{(R+h)^2}$$

$$g = \frac{\left(6.67 \times 10^{-11}\ Nm^2kg^{-2}\right)(5.97 \times 10^{24}\ kg)}{(6371 \times 10^3 m + 1000 \times 10^3 m)^2} = 7.33\ ms^{-2}$$

8.4

1. On the moon the value of 'g' is only 1/6th that of the earth. So, your weight on moon will become 1/6th of your weight on the earth. However, the mass remains constant.

2. Mass of Mars = 6 × 10^{23} kg, Radius as 4.3 × 10^{6} m.

$$g_{\text{Mars}} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\right)\left(6\times10^{23}\,\text{kg}\right)}{\left(4.3\times10^{6}\,\text{m}\right)^2} = 2.16\text{ms}^{-2}$$

$$\frac{\text{Weight on Mars}}{\text{Weight on Earth}} = \frac{\text{Mass}\times g_{\text{Mars}}}{\text{Mass}\times g_{\text{Earth}}} = \frac{g_{\text{Mars}}}{g_{\text{Earth}}} = \frac{2.16}{9.81} = 0.22$$

So, your weight will become roughly 1/4 th that on the earth and mass remains constant.

3. Balance with two pans actually compare masses because 'g' acts on both the pans and gets cancelled. The other type of balance, spring balance, measures weight. The balance with two pans gives the same reading on the moon as on the earth. But the spring balance gives weight as 1/6th that on the earth since 'g'on the moon is 1/6 of the earth.

8.5

1. Yes. Wherever the force between bodies is gravitational, Kepler's laws will be applicable.

2. According to Kepler's third law,

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$r_1 = R + h_1 = 6371\text{ km} + 1000\text{ km} = 7371\text{ km}$

$r_2 = R + h_2 = 6371\text{ km} + 2000\text{ km} = 8371\text{ km}$

$$T_1 = 90\text{ min}$$

$$T_2^2 = \left(\frac{r_2}{r_1}\right)^3 T_1^2$$

$$T_2^2 = \left(\frac{8371}{7371}\right)^3 (90)^2 = 108.9\,\text{min}$$

3. According to the Kepler's third law,

$$\frac{T_{earth}^2}{T_{sedna}^2} = \frac{r_{earth}^3}{r_{sedna}^3}$$

$T_{earth} = 1$ yr, $r_{earth} = 1$ AU and $r_{sedna} = 86$ AU

$$T_{sedna}^2 = \left(\frac{r_{sedna}}{r_{earth}}\right)^3 T_{earth}^2$$

$$T_{sedna} = \left(\frac{r_{sedna}}{r_{earth}}\right)^{\frac{3}{2}} T_{earth} = \left(\frac{86AU}{1AU}\right)^{\frac{3}{2}} 1yr = 797.5yr$$

4. The centripetal force required for a satellite to orbit around the earth is,

$$F_{centripetal} = \frac{mv_o^2}{r}$$

This centripetal force is provided by the gravitational force, which is

$$F_{gravitational} = \frac{GmM}{r^2}$$

Where M is the mass of the earth.

While orbiting a satellite around the earth, the above two forces are balanced, hence, we have,

$$\frac{mv_o^2}{r} = \frac{GmM}{r^2}$$

$$v_o^2 = \frac{GM}{r}$$

From the relation between g and G, we know that $GM = gR^2$. Substituting in the above equation, we get,

$$v_o^2 = \frac{gR^2}{r}$$

$$v_o = \sqrt{\frac{gR^2}{(R+h)}}$$

Here the radius of the orbit of the satelite is $r = (R + h)$, where R is the radius of earth and h is the height of the satellite from surface of the earth. If the satellite is very close to the earth, then $(R+h) \approx R$. Then,

$$v_o = \sqrt{gR}$$

5. From Eqn (8.30),the velocity of planet orbiting in a orbit of radius 'r' is,

$$v_o = \frac{2\pi r}{T}$$

From Eqn (8.31), the orbital velocity of planet is, $v_o = \sqrt{\frac{GM}{r}}$

Form the above equations,we have,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

As all terms in the bracket of RHS are constants,we have,

$$T^2 \; \alpha \; r^3$$

8.6

1. $$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{\frac{2\left(6.67\times10^{-11}\,Nm^2kg^{-2}\right)(5.97\times10^{24}\,kg)}{(6.371\times10^6\,m)}} = 11.3\times10^3\,ms^{-1}$$

$$v_e = 11.3 \text{ km s}^{-1}$$

2. $$v_e \, \alpha \sqrt{\frac{1}{R}}$$

if R becomes 1/4th, then

$$v_e \, \alpha \sqrt{\frac{1}{R/4}} \Rightarrow v_e \, \alpha \sqrt{\frac{4}{R}} \Rightarrow v_e \, \alpha \, 2\sqrt{\frac{1}{R}}$$

Escape velocity becomes double.

3. Escape velocit on earth, $v_e = \sqrt{\frac{2GM_e}{R_e}}$

Escape velocit on palnet X, $v_X = \sqrt{\frac{2GM_X}{R_X}}$

$$\frac{v_X}{v_e} = \sqrt{\frac{M_X R_e}{M_e R_X}}$$

Here, $M_X = 8\ M_e$ and $R_X = 2R_e$

$$\frac{v_X}{v_e} = \sqrt{\frac{(8M_e)R_e}{M_e(2R_e)}} = \sqrt{4} = 2$$

$$v_X = 2v_e$$

The escape velocity on imaginary planet X is double that on the earth.

8.7

1. Velocity of satellite orbiting in a orbit of radius 'r' is, $v_o = \frac{2\pi r}{T}$

Orbital velocity of a satellite is, $v_o = \sqrt{\frac{GM}{r}}$

from the above equations, we have,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \left(\frac{GM}{4\pi^2}\right) T^2$$

$$r^3 = \frac{\left(6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}\right) \times \left(6.4 \times 10^{23}\,\text{kg}\right) \times (24.6 \times 60 \times 60\text{s})^2}{4 \times (3.14)^2}$$

$$r^3 = 8370 \times 10^{18}\ \text{m}^3$$

$$r = 20300\ \text{km}$$

But 'r' is the distance from the center of the planet. Hence r = R + h, where R is the radius and h is the height from the surface.

$$R + h = 20300\ \text{km}$$

$$3400\ \text{km} + h = 20300\ \text{km}$$

$$h = 20300\ \text{km} - 3400\ \text{km} = 16{,}900\ \text{km}$$

2. (a) Images are clearer
   (b) X - ray telescopy etc. also works.

ANSWERS TO TERMINAL EXERCISE

3. 125 N
5. $g = 5.5\ \text{ms}^{-2}$
7. $\text{Weight} = \frac{500}{6}\text{N}$, mass 50 kg on moon as well as on earth
8. $T = 1\frac{1}{2}\text{hours}, \quad v = 7.47\text{kms}^{-1}$

9

MECHANICAL PROPERTIES OF SOLIDS

INTRODUCTION

In the previous lessons you have studied the effect of force on a body to produce displacement. The force applied on an object may also change its shape or size. For example, when a suitable force is applied on a spring, you will find that its shape as well as size changes. But when you remove the force, it will regain original position. Now apply a force on some objects like lump of putty or molten wax. Do they regain their original position after the force has been removed? They do not regain their original shape and size. Thus, some objects regain their original shape and size whereas others do not. Such a behaviour of objects depends on a property of matter called elasticity.

The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed is known as elasticity and the deformation caused in the body is known as elastic deformation. However, if force is applied on some bodies, they have no gross tendency to regain their previous shape and they get permanently deformed. Such substances are called plastic and this property is called plasticity. Putty, molten wax and mud are close to ideal plastics.

The elastic property of materials plays an important role in engineering design. For example, while designing a building, knowledge of elastic properties of materials like steel, concrete etc. is essential and also it is true in the design of bridges, automobiles, ropeways etc. We use this property to find the strength of beams for construction of buildings and bridges. It is used to help us determine the strength of cables to support the weight of bodies such as in cable cars, cranes, lifts etc. In this lesson you will learn about nature of changes and the manner in which these can be described.

OBJECTIVES

After studying this lesson, you should be able to

- *distinguish between three states of matter on the basis of molecular theory;*
- *distinguish between elastic and plastic bodies;*
- *distinguish between stress and pressure;*
- *define the longitudinal, Normal and Shearing Stress;*
- *define the strain and different types of strains;*
- *study stress-strain curve for an elastic solid;*
- *understand the Hook's Law;*

- *define Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio;*
- *define the Elastic potential energy and derive the expression for it;*
- *apply the elastic behaviour of the materials to design safe and stable structures.*

9.1 ELASTICITY

You would have noticed that when an external force is applied on an object, its shape or size (or both) change, i.e. deformation takes place. The extent of deformation depends on the material and shape of the body and the external force. When the deforming forces are withdrawn, the body tries to regain its original shape and size.

You may compare this with a spring loaded with a mass or a force applied on the string of a bow or pressing of a rubber ball. If you apply a force on the string of the bow to pull it as shown in the Fig. 9.1, you will observe that its shape changes. But on releasing the string, the bow regains its original shape and size. The property of matter to regain its original shape and size after removal of the deforming forces is called elasticity.

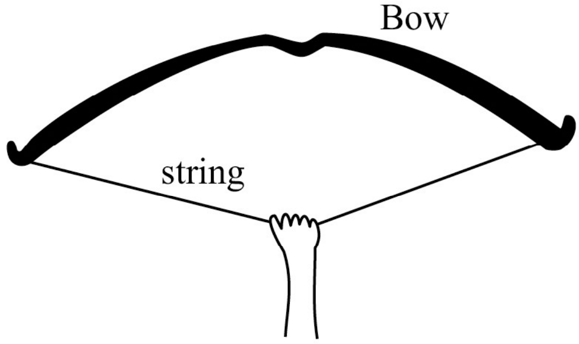


Fig. 9.1: Force applied on the string of a bow changes its shape

9.2 ELASTIC AND PLASTIC BODIES

A body which regains its original state completely on removal of the deforming force is called perfectly elastic. On the other hand, if it completely retains its modified form even on removing the deforming force, i.e. shows no tendency to recover the deformation, it is said to be perfectly plastic. However, in practice the behaviour of all bodies is in between these two limits. There exists no perfectly elastic or perfectly plastic body in nature. The nearest approach to a perfectly elastic body is quartz fiber and to the perfectly plastic is ordinary putty. Here it can be added that the object which opposes the deformation more is more elastic. No doubt elastic deformations are very important in science and technology, but plastic deformations are also important in mechanical processes. You might have seen the processes such as stamping, bending and hammering of metal pieces. These are possible only due to plastic deformations. The phenomenon of elasticity can be explained in terms of inter-molecular forces.

9.3 MOLECULAR THEORY OF MATTER: INTER-MOLECULAR FORCES

We know that matter is made up of atoms and molecules. The forces which act between them are responsible for the structure of matter. The interaction forces between molecules are known as inter-molecular forces.

The variation of inter molecular forces with inter molecular separation is shown in Fig. 9.2. When the separation is large, the force between two molecules is attractive and

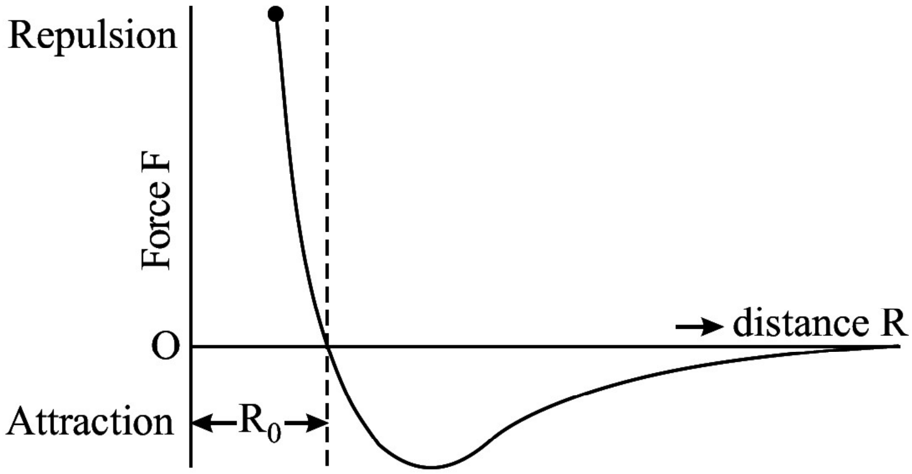


Fig. 9.2: Graph between inter – molecular force and inter molecular separation.

weak. As the separation decreases, the net force of attraction increases up to a particular value and beyond this, the force becomes repulsive. At a distance $R = R_0$ the net force between the molecules is zero. This separation is called equilibrium separation. Thus, if inter-molecular separation $R > R_0$ there will be an attractive force between molecules. When $R < R_0$ a repulsive force will act between them.

In solids, molecules are very close to each other at their equilibrium separation (10^{-10} m). Due to high intermolecular forces, they are almost fixed at their positions. You may now appreciate why a solid has a definite shape.

In liquids, the average separation between the molecules is somewhat larger (10^{-8} m). The attractive force is weak and the molecules are comparatively free to move inside the whole mass of the liquid. You can understand now why a liquid does not have fixed shape. It takes the shape of the vessel in which it is filled.

In gases, the intermolecular separation is significantly larger and the molecular force is very weak (almost negligible). Molecules of a gas are almost free to move inside a container. That is why gases do not have fixed shape and size.

9.4 MOLECULAR THEORY OF ELASTICITY

You are aware that a solid is composed of a large number of atoms arranged in a definite order. Each atom is acted upon by forces due to neighbouring atoms. Due to inter-atomic forces, solid takes such a shape that each atom remains in a stable equilibrium. When the body is deformed, the atoms are displaced from their original positions and the inter–atomic distances change. If in deformation, the separation increases beyond their equilibrium separation (i.e., $R > R_0$), strong attractive forces are developed. However, if inter–atomic separation decreases (i.e., $R < R_0$), strong repulsive forces develop. These forces, called restoring forces, drive atoms to their original positions. The behaviour of atoms in a solid can be compared to a system in which balls are connected with springs.

Now, let us learn how forces are applied to deform a body.

Ancient Indian view about Atom

Kanada was the first expounder of the atomic concept in the world. He lived around 6th century B.C. He resided at Prabhasa (near Allahabad).

According to him, everything in the universe is made up of Parmanu or Atom. They are eternal and indestructible. Atoms combine to form different molecules. If two atoms combine to form a molecule, it is called duyanuka and a triatomic molecule is called triyanuka. He was the author of "Vaisesika Sutra".

The size of atom was also estimated. In the biography of Buddha (Lalitavistara), the estimate of atomic size is recorded to be of the order 10^{-10}m, which is very close to the modern estimate of atomic size.

9.5 STRESS

When an external force or system of forces is applied on a body, it undergoes a change in the shape or size according to nature of the forces. We have explained that in the process of deformation, internal restoring force is developed due to molecular displacements from their equilibrium positions. The internal restoring force opposes the deforming force. "The internal restoring force acting per unit area of cross-section of a deformed body is called stress".

In equilibrium, the restoring force is equal in magnitude and opposite in direction to the external deforming force. Hence, stress is measured by the external force per unit area of cross-section when equilibrium is attained. If the magnitude of deforming force is 'F' and it acts on area 'A', we can write

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} = \frac{\text{deforming force (F)}}{\text{area (A)}}$$

$$\text{Stress} = \frac{\text{F}}{\text{A}} \tag{9.1}$$

The unit of stress is Nm^{-2}. The stress may be longitudinal, normal or shearing. Let us study them one by one.

9.5.1 Longitudinal Stress

If the deforming forces are along the length of the body, we call the stress produced as longitudinal stress, as shown in its two forms in Fig 9.3 (a) and Fig 9.3 (b).

There are two ways in which a solid may change its length when deforming forces act on it. These are shown in Fig. 9.3 (a) and 9.3 (b). In Fig. 9.3 (a), a cylinder is stretched by two equal forces applied normal to its cross-

Fig. 9.3: (a) Tensile stress; (b) Compressive stress

sectional area. The restoring force per unit area in this case is called tensile stress. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as compressive stress. Tensile or compressive stress can also be known as longitudinal stress.

9.5.2 Normal Stress

If the deforming forces are applied uniformly and normally all over the surface of the body so that the change in its volume occurs without change in shape as shown in the Fig. 9.4, we call the stress produced as normal stress. You may produce normal stress by applying force uniformly over the entire surface of the body. Deforming force per unit area normal to the surface is called pressure while restoring force developed inside the body per unit area normal to the surface is known as stress.

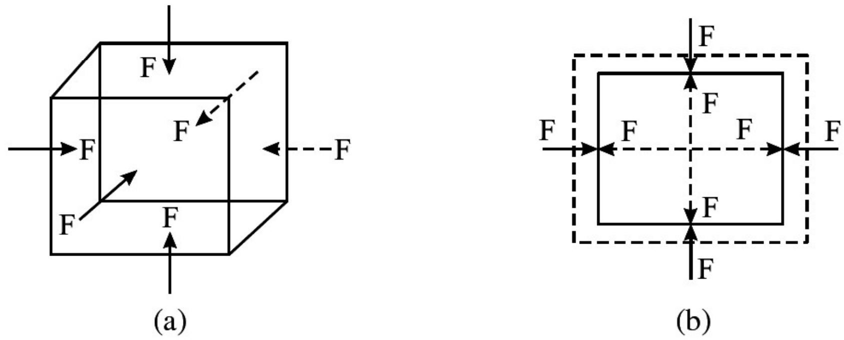


Fig. 9.4: Normal stress

9.5.3 Shearing Stress

If the deforming forces act tangentially or parallel to the surface as shown in the Fig 9.5 (a) so that shape of the body changes without change in volume, the stress is called shearing stress. An example of shearing stress is shown in Fig 9.5 (b) in which a book is pushed side ways. Its opposite face is held fixed by the force of friction.

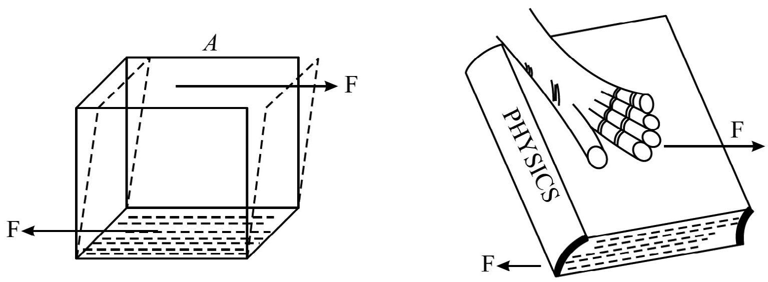


Fig. 9.5: (a) Shearing stress; (b) Pushing a book side ways.

9.6 STRAIN

Deforming forces produce changes in the dimensions of the body. In general, the strain is defined as the change in dimension (e.g. length, shape or volume) per unit dimension of the body. As the strain is ratio of two similar quantities, it is a dimensionless quantity.

Depending on the kind of stress applied, strains are of three types: (i) linear strain, (ii) volume (bulk) strain, and (iii) shearing strain.

9.6.1 Linear Strain

If on application of a longitudinal deforming force, the length 'l' of a body change by 'Δl' (Fig. 9.6), then

$$\text{linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l} \qquad (9.2)$$

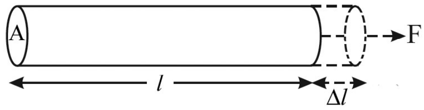


Fig. 9.6 : Linear strain

9.6.2 Volume Strain

If on application of a uniform pressure 'ΔP', the volume 'V' of the body changes by 'ΔV' (Fig. 9.7) without change of shape of the body, then

$$\text{volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V} \qquad (9.3)$$

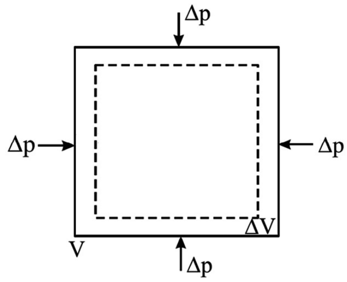


Fig. 9.7 : Volume strain

9.6.3 Shearing Strain

When the deforming forces are tangential (Fig. 9.8), the shearing strain is given by the angle θ through which a line perpendicular to the fixed plane is turned due to deformation. This angle θ is usually very small, then we can write

$$\theta = \frac{\Delta x}{y} \qquad (9.4)$$

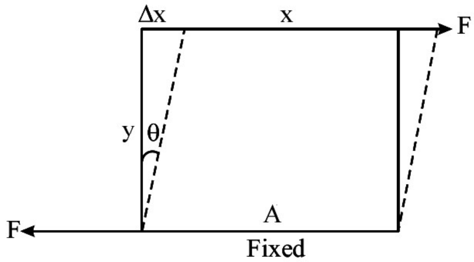


Fig. 9.8 : Shearing strain

9.7 STRESS-STRAIN CURVE

9.7.1 Stress-strain Curve for a Metallic Wire

The relation between the stress and the strain for a given metallic wire (of uniform cross–section) under tensile stress is found experimentally stretching it by an applied force. The applied force is gradually increased in steps and the change in length (strain) is noted. A graph is plotted between the stress (applied force per unit area) and the strain produced. A typical graph for a metallic wire is shown in Fig. 9.9. These stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, the some regions and points that are particular importance can be found on the stress–strain curve and let us study these.

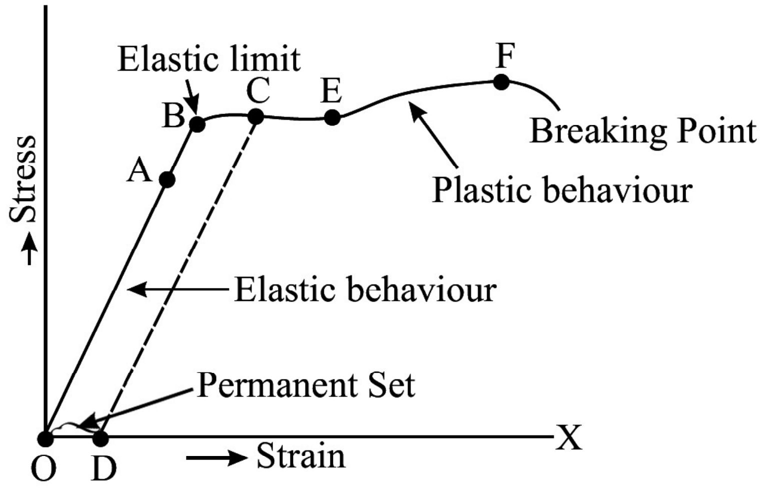


Fig. 9.9 : Stress-strain curve for a steel wire

1. **Region of Proportionality (OA) :** In the stress–strain curve, the straight-line OA indicates that in this region, stress is linearly proportional to strain and the body behaves like a perfectly elastic body. In this region, Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed.

2. **Elastic Limit (AB) :** If we increase the strain a little beyond A, the stress is not linearly proportional to strain. However, the wire still remains elastic, i.e. after removing the deforming force (load), it regains its original state. The maximum value of strain for which a body (metallic wire) shows elastic property is called elastic limit or yielding point and the corresponding stress is known as yield strength of the material. Beyond the elastic limit (Point B), a body behaves like a plastic body.

3. **Point C :** When the wire is stretched beyond the elastic limit B, the strain increases more rapidly even for a small change in the load and the body becomes plastic. This is shown in the graph by BE region. It means that even if the deforming load is removed, the wire will not recover its original length at some point C between B and E and the wire follows dotted line CD. In this case, even when the stress is zero, the strain is not zero. The left–over strain on zero load strain is known as a permanent set. The deformation is said to be plastic deformation. After point E on the curve, no extension is recoverable.

4. **Breaking point F :** Beyond point E, strain increases very rapidly and near point F, the length of the wire increases continuously even without increasing of load. The wire breaks at point F. This is called the breaking point or fracture point and the corresponding stress is known as breaking stress.

The stress corresponding to breaking point F is called breaking stress or tensile strength. Within the elastic limit, the maximum stress which an object can be subjected to is called working stress and the ratio between working stress and breaking stress is called factor of safety. In U.K, it is taken 10, in USA it is 5. We have adopted UK norms. If large deformation takes place between the elastic limit and the breaking point, the material is called ductile. If it breaks soon after the elastic limit is crossed, it is called brittle e.g. glass.

9.7.2 Stress-Strain Curve for Rubber

When we stretch a rubber cord to a few times its natural length, it returns to its original length after removal of the forces. That is, the elastic region is large and there is no well-defined plastic flow region. Substances having large strain are called elastomers. This property arises from their molecular arrangements. The stress-strain curve for rubber is distinctly different from that of a metallic wire. There are two important things to note from Fig. 9.10. Firstly, you can observe that there is no region of proportionality. Secondly, when

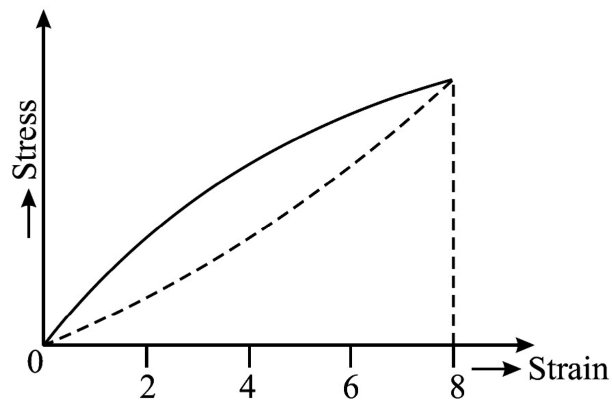


Fig. 9.10 : Stress – strain curve for rubber

the deforming force is gradually reduced, the original curve is not retraced, although the sample finally acquires its natural length. The work done by the material in returning to its original shape is less than the work done by the deforming force. This difference of energy is absorbed by the material and appears as heat. (You can feel it by touching the rubber band with your lips.) This phenomenon is called **elastic hysteresis**.

Elastic hysteresis has an important application in shock absorbers. A part of energy transferred by the deforming force is retained in a shock absorber and only a small part of it is transmitted to the body to which the shock absorber is attached.

9.7.3 Steel is more Elastic than Rubber

A body is said to be more elastic if on applying a large deforming force on it, the strain produced in the body is small. If you take two identical rubber and steel wires and apply equal deforming forces on both of them, you will see that the extension produced in the steel wire is smaller than the extension produced in the rubber wire. But to produce same strain in the two wires, significantly higher stress is required in the steel wire than in rubber wire. Large amount of stress needed for deformation of steel indicates that magnitude of internal restoring force produced in steel is higher than that in rubber. Thus, steel is more elastic than rubber.

Example 9.1

A load of 100 kg is suspended by a wire of length 1.0 m and cross – sectional area 0.10 cm^2. The wire is stretched by 0.20 cm. Calculate the (i) tensile stress, and (ii) strain in the wire. Given, g = 9.80 ms^{-2}.

Solution :

(i) $$\text{Tensile stress} = \frac{F}{A} = \frac{Mg}{A} = \frac{(100\,\text{kg})\,(9.80\,\text{ms}^{-2})}{(0.10\times10^{-4}\,\text{m}^2)} = 9.8\times10^{7}\ \text{Nm}^{-2}$$

(ii) $$\text{Tensile strain} = \frac{\Delta l}{l} = \frac{0.20\times10^{-2}\,\text{m}}{1.0\,\text{m}} = 0.20\times10^{-2}$$

Example 9.2

Calculate the maximum length of a steel wire that can be suspended without breaking under its own weight, if its breaking stress = 4.0 × 10^8 Nm^{-2}, density = 7.9 × 10^3 kg m^{-3} and g = 9.80 ms^{-2}.

Solution :

The weight of the wire W = $Al\rho g$, where A is area of cross section of the wire, l is the maximum length and ρ is the density of the wire. Therefore, the breaking stress developed in the wire due to its own weight $\frac{W}{A} = \rho l g$.

It was told that breaking stress is 4.0 × 10^8 Nm^{-2}. Hence

$$l = \frac{4.0 \times 10^8\ \text{Nm}^{-2}}{(7.9 \times 10^3\ \text{kg m}^{-3})(9.80\ \text{ms}^{-2})} = 0.05 \times 10^5\ \text{m} = 5 \times 10^3\ \text{m}$$

Now it is time to take a break and check your understanding.

Intext Questions 9.1

1. What will be the nature of inter-atomic forces when deforming force applied on an object (i) increases, (ii) decreases the inter-atomic separation?
2. If we clamp a rod rigidly at one end and a force is applied normally to its cross section at the other end, name the type of stress and strain?
3. The ratio of stress to strain remains constant for small deformation of a metal wire. For large deformations what will be the changes in this ratio?
4. Under what conditions, a stress is known as breaking stress?
5. If mass of 4 kg is attached to the end of a vertical wire of length 4 m with a diameter 0.64 mm, the extension is 0.60 mm. Calculate the tensile stress and strain?

9.8 HOOKE'S LAW

In 1678, Robert Hooke obtained the stress-strain curve experimentally for a number of solid substances and established a law of elasticity known as Hooke's law. According to this law: Within elastic limit, stress is directly proportional to corresponding strain.

$$\text{stress} \ \alpha \ \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant (E)} \qquad (9.5)$$

This constant of proportionality E is a measure of elasticity of the substance and is called modulus of elasticity. As strain is a dimensionless quantity, the modulus of elasticity has the same dimensions (or units) as stress. Its value is independent of the stress and strain but depends on the nature of the material. To see this, you may like to do the following activity.

Activity 9.1

Arrange a steel spring with its top fixed with a rigid support on a wall and a metre scale along its side, as shown in the Fig. 9.11. Add 100 gm load at a time on the bottom of the hanger in steps. It means that while putting each 100 gm load, you are increasing the stretching force by 1 N. Measure the extension. Take the reading up to 500 gm and note the extension each time.

Plot a graph between load and extension. What is the shape of the graph? Does it obey Hooke's law? The graph should be a straight line indicating that the ratio (load / extension) is constant. Repeat this activity with rubber and other materials.

You should know that the materials which obey Hooke's law are used in spring balances or as force measurer, as shown in the Fig. 9.11. You would have seen that when some object is placed on the pan, the length of the spring increases. This increase in length shown by the pointer on the scale can be treated as a measure of the increase in force (i.e., load applied).

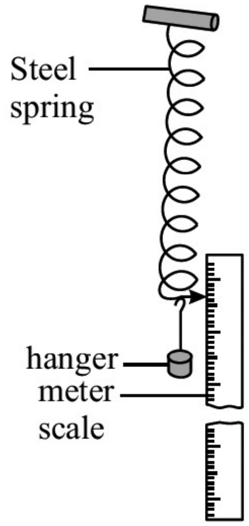


Fig. 9.11 : Hooke's law apparatus

Robert Hooke (1635 – 1703)

Robert Hooke, experimental genius of seventeenth century, was a contemporary of Sir Isaac Newton. He had varied interests and contributed in the fields of physics, astronomy, chemistry, biology, geology, paleontology, architecture and naval technology. Among other accomplishments he has to his credit the invention of a universal joint, an early proto type of the respirator, the iris diaphragm, anchor escapement and balancing spring for clocks. As chief surveyor, he helped rebuild London after the great fire of 1666. He formulated Hooke's law of eleasticity and correct theory of combustion. He is also credited to invent or improve meteorological instruments such as barometer, anemometer and hygrometer.

9.9 MODULI OF ELASTICITY

In previous sections, you have learnt that there are three kinds of strain. It is therefore clear that there should be three moduli of elasticity corresponding to these strains. These are Young's modulus, Bulk Modulus and Rigidity Modulus corresponding to linear strain, volume strain and shearing strain, respectively. We now study these one by one.

9.9.1 Young's Modulus

The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus for the material of the body. Suppose that when a wire of length L and area of cross-section A is stretched by a force of magnitude F, the change in its length is equal to ΔL. Then

$$\text{Longitudinal stress} = \frac{F}{A}$$

and $$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

Hence, $$\text{Young's modulus } Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L} \qquad (9.6)$$

If the wire of radius 'r' is suspended vertically with a rigid support and a mass 'M' hangs at its lower end, then $A = \pi r^2$ and $F = mg$.

$$\therefore \qquad Y = \frac{MgL}{\pi r^2 \Delta L} \qquad (9.7)$$

The SI unit of Y is Nm^{-2}. The values of Young's modulus for a few typical substances is given in Table 9.1. Note that steel is most elastic.

Table - 9.1 : Young's modulus of some typical materials

Name of the substance	**Y (10^9 Nm^{-2})**
Aluminium	70
Copper	120
Iron	190
Steel	200
Glass	65
Bone	9
Polystyrene	3

9.9.2 Bulk Modulus

The ratio of normal stress to the volume strain is called bulk modulus of the material of the body. If due to increase in pressure 'P', volume 'V' of the body decreases by without 'ΔV' change in shape, then

$$\text{Normal stress} = \Delta P$$

$$\text{Volume strain} = \frac{\Delta V}{V}$$

$$\text{Bulk modulus } B = \frac{\Delta P}{\Delta V / V} = V\frac{\Delta P}{\Delta V} \qquad (9.8)$$

The reciprocal of bulk modulus of a substance is called compressibility :

$$k = \frac{1}{B} = \frac{1}{V}\frac{\Delta V}{\Delta P} \qquad (9.9)$$

Gasses being most compressible, these are least elastic while solids are most elastic or least compressible i.e. $B_{solid} > B_{liquid} > B_{gas}$.

9.9.3 Rigidity Modulus or Shear Modulus

The ratio of the shearing stress to shearing strain is called modulus of rigidity of the material of the body. If a tangential force F acts on an area A and θ is the shearing strain, the modulus of rigidity

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \qquad (9.10)$$

You should know that both solid and fluids have bulk modulus. However, fluids do not have Young's modulus and shear modulus because a liquid can not sustain a tensile or shearing stress.

Example 9.3

Calculate the force required to increase the length of a wire of steel of cross–sectional area 0.1 cm^2 by 50%. Given $Y = 2 \times 10^{11}$ Nm^{-2}.

Solution :

Increase in the length of wire = 50%. If ΔL is the increase and L is the normal

length of wire then $\frac{\Delta L}{L} = \frac{1}{2}$

$$\therefore \qquad Y = \frac{F \times L}{A \times \Delta L}$$

$$F = \frac{Y \times A \times \Delta L}{L} = \frac{(2 \times 10^{11}\ Nm^{-2})(0.1 \times 10^{-4}\ m^2) \times 1}{2} = 0.1 \times 10^6\ N$$

Example 9.4

When a solid rubber ball is taken from the surface to the bottom of a lake, the reduction in its volume is 0.0012 %. The depth of lake is 360 m, the density of lake water is 10^3 kg m^{-3} and acceleration due to gravity at the place is 10 ms^{-2}. Calculate the bulk modulus of rubber.

Solution :

Increase of pressure on the ball $P = h\rho g = 360\ m \times 10^3\ kg\ m^{-3} \times 10\ ms^{-2}$

$$= 3.6 \times 10^6\ Nm^{-2}$$

$$\text{Volume strain} = \frac{\Delta V}{V} = \frac{0.0012}{100} = 1.2 \times 10^{-5}$$

$$\text{Bulk Modulus } B = \frac{PV}{\Delta V} = \frac{3.6 \times 10^6}{1.2 \times 10^{-5}} = 3.0 \times 10^{11}\ Nm^{-2}$$

9.9.4 Poisson's Ratio

You may have noticed that when a rubber tube is stretched along its length, there is a contraction in its diameter (Fig. 9.12). (This is also true for a wire but may not be easily visible.) While the length increases in the direction of forces, a contraction occurs in the perpendicular direction. The strain perpendicular to the applied force is called lateral strain.

Poisson pointed out that within elastic limit, lateral strain is directly proportional to longitudinal strain i.e. the ratio of lateral strain to longitudinal strain is constant for a material body and is known as Poisson's ratio. It is denoted by a Greek letter σ (sigma). If α and β are the longitudinal strain and lateral strain respectively, then Poisson's ratio

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\beta}{\alpha} \qquad (9.11)$$

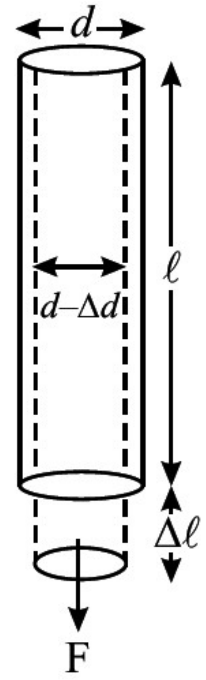


Fig. 9.12 : A stretched rubber tube

If a wire (rod or tube) of length l and diameter d is elongated by applying a stretching force by an amount Δl and its diameter decreases by Δd, then

$$\text{longitudinal strain} \qquad \alpha = \frac{\Delta l}{l}$$

$$\text{lateral strain} \qquad \beta = \frac{\Delta d}{d}$$

$$\text{and} \qquad \text{Possion ratio } \sigma = \frac{\Delta d / d}{\Delta l / l} = \frac{l}{d}\frac{\Delta d}{\Delta l} \qquad (9.12)$$

Since Poisson's ratio is a ratio of two strains, it is a pure number. The value of Poisson's ratio depends only on the nature of material and for most of the substances, it lies between 0.2 and 0.4. When a body under tension suffers no change in volume, i.e. the body is perfectly incompressible, the value of Poisson's ratio is maximum i.e. 0.5. Theoretically, the limiting values of Poisson's ratio are –1 and 0.5.

9.10 ELASTIC POTENTIAL ENERGY IN A STRETCHED WIRE

When a wire is stretched under a tensile stress, work is done against the inter-atomic forces. This work is stored in the wire in the form of elastic potential energy.

To obtain the expression for this elastic potential energy, consider a wire of length L and cross-sectional area of A is subjected to a deforming force F along its length. Let 'l' be the elongation of the wire. Then, from Eq. 9.6, we have

$$\text{Deforming force } F = YA\frac{l}{L} \qquad (9.13)$$

Here Y is the Young's modulus of the material of the wire. Now consider dW is the work done for an infinitesimal elongation dl of the wire. Then,

$$dW = F\, dl$$

Substituting Eqn. 9.13 in the above equation, we have

$$dW = YA\frac{l}{L}dl$$

Therefore, the total work done during the elongation of the wire from 0 to l can be obtained by integrating the above equation.

$$W = \int dW = \int_0^1 YA\frac{l}{L}dl$$

$$W = \frac{YA}{L}\int_0^1 l\,dl = \frac{YA}{2L}\left[l^2\right]_0^l$$

$$W = \frac{1}{2}\frac{YA}{L}l^2$$

We can rewrite the above equation as,

$$W = \frac{1}{2}Y\left(\frac{l}{L}\right)^2 (AL)$$

As $\frac{l}{L}$ is the longitudinal strain, (AL) is the volume of the wire and Young's modulus $= \frac{\text{stress}}{\text{strain}}$, we have

$$W = \frac{1}{2} \times \left(\frac{\text{stress}}{\text{strain}}\right) \times (\text{Strain})^2 \times \text{Volume of the wire}$$

$$W = \frac{1}{2} \times (\text{Stress}) \times (\text{Strain}) \times \text{Volume of the wire} \qquad (9.14)$$

This work is stored in the wire as energy and this energy is known as the elastic potential energy. Hence the elastic potential energy per unit volume is

$$U = \frac{1}{2} \times (\text{Stress}) \times (\text{Strain})$$

Activity 9.2

Take two identical wires, A and B, made of the same material. Wire A is subjected to repetitive deforming forces over several days, while wire B remains untouched. After this period, when similar vibrations are applied to both wires, it is observed that the vibrations in wire A dissipate sooner, whereas wire B continues to vibrate for a longer duration. This indicates that wire B possesses greater elasticity, while wire A has undergone **elastic fatigue**.

Elastic fatigue occurs when a body experiences repeated strains or deforming forces, even within its elastic limits. In such situations, the body loses its inherent elasticity.

Some other facts about elasticity

1. If we add some suitable impurity to a metal, its elastic properties are modified. For example, if carbon is added to iron or potassium is added to gold, their elasticity increases.
2. The increase in temperature decreases elasticity of materials. For example, carbon, which is highly elastic at ordinary temperature, becomes plastic when heated by a current through it. Similarly, plastic becomes highly elastic when cooled in liquid air.
3. The value of modulus of elasticity is independent of the magnitude of stress and strain. It depends only on the nature of the material of the body.

Example 9.5

A Metal cube of side 20 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity, if top of the cube is displaced by 0.01 cm. with respect to bottom.

Solution :

Shearing stress = 10^4 Nm^{-2}, Δx = 0.01 cm and y = 20 cm

$$\therefore \quad \text{Shearing strain} = \frac{\Delta x}{y} = \frac{0.01\text{ cm}}{20\,\text{cm}} = 0.0005$$

$$\text{Hence.} \quad \text{Modulus of rigidity } \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{10^4\text{ Nm}^{-2}}{0.0005} = 2\times10^7\text{ Nm}^{-2}$$

Example 9.6

A 10 kg mass is attached to one end of a copper wire of length 5 m long and 1 mm in diameter. Calculate the extension and lateral strain, if Poisson's ratio is 0.25. Given Young's modulus of the wire = 11 × 10^{10} Nm^{-2}.

Solution :

Here length of the wire L = 5 m, radius of the wire $r = \frac{d}{2} = 0.5\times10^{-3}\text{m}$,

Y = 11 × 10^{10} Nm^{-2}, F = mg = (10 kg × 9.8 ms^{-2}) = (10 × 9.8) N and σ = 0.25

Expansion produced in the wire

$$\Delta L = \frac{F.L}{A.Y} = \frac{F.L}{(\pi r^2).Y} = \frac{(10\times9.8\text{ N})\times(5\text{m})}{3.14\times(0.5\times10^{-3}\text{m})^2\times(11\times10^{10}\text{ Nm}^{-2})}$$

$$= \frac{490}{8.63\times10^4}\text{m} = 5.6\times10^{-3}\text{m}$$

$$\text{longitudinal strain } (\alpha) = \frac{\Delta L}{L} = \frac{5.6 \times 10^{-3}\,\text{m}}{5\,\text{m}} = 1.12 \times 10^{-2}$$

$$\text{poissions' ratio } (\sigma) = \frac{\text{lateral strain } (\beta)}{\text{longitudinal strain } (\alpha)}$$

$$\text{lateral strain } (\beta) = \sigma \times \alpha = 0.125 \times 1.12 \times 10^{-2} = 0.14 \times 10^{-3}$$

Now take a break to check your progress.

Intext Questions 9.2

1. Is the unit of longitudinal stress same as that of Young's modulus of elasticity? Give reason for your answer.
2. Solids are more elastic than liquids and gases. Justify.
3. The length of a wire is cut to half. What will be the effect on the increase in its length under a given load?
4. Two wires are made of the same metal. The length of the first wire is half that of the second and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of increase in their lengths?
5. A wire increases by 10^{-3} of its length when a stress of 1×10^{8} Nm^{-2} is applied to it. Calculate Young's modulus of material of the wire.

9.11 APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

Elastic behaviour of materials plays an important role in our day-to-day life. To design safe and stable structures, the stress due to the maximum load does not exceed the breaking stress. Pillars and beams are important parts of our structures. While constructing pillars and beams for a bridge, the load of traffic that it can withstand should be adequately measured before construction. A uniform beam clamped at one end and loaded at the other is called a Cantilever (Fig. 9.13). A cantilever of length l, breadth b and thickness d undergoes a depression δ at its free end when it is loaded by a weight of mass M :

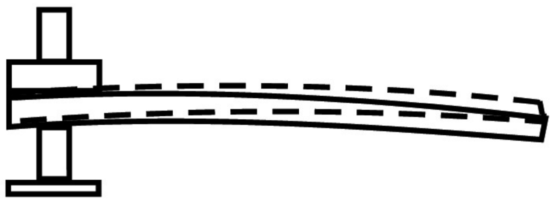


Fig. 9.13 : Cantilever

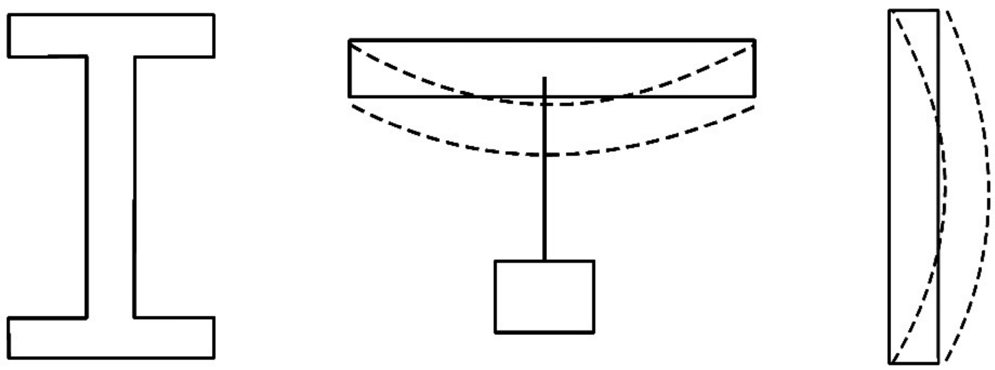
$$\delta = \frac{4\,Mgl^3}{Ybd^3} \tag{9.16}$$

From the above equation, to maintain the depression of a cantilever beam with in the elastic limit, it should be constructed with a high Young's modulus material and adequate dimensions should be fixed depending on the load. The hanging bridge of Laxman Jhula in Rishkesh and Vidyasagar Sethu in Kolkata are supported on cantilevers.

It is now easy to understand as to why the cross-section of rails are kept I-shaped [Fig. 9.14 (a)]. From the Eqn. 9.16, all the other factors remaining same, $\delta \; \alpha \; d^{-3}$. Therefore, by increasing thickness, we can decrease depression under the same load more effectively. This considerably saves the material without sacrificing strength.

For a beam clamped at both ends and loaded in the middle [Fig. 9.14 (b)], the sag in the middle is given by

$$\delta = \frac{Mgl^3}{4bd^3Y} \tag{9.17}$$



(a)
Cross section of rails

(b)
Beam clamped at both ends and loaded in the middle

(c)
A thin bar and how it can buckle.

Fig. 9.14

Thus, for a given load, we will select a material with a large Young's modulus (Y) and again a large thickness to keep δ small. However, a deep beam may have a tendency to buckle [Fig. 9.14 (c)]. To avoid this, a large load bearing surface is provided. In the form **I** – shaped cross-section, both these requirements are fulfilled.

In cranes, we use a thick metal rope to lift and move heavy loads from one place to another. To lift a load of 10 metric tons with a steel rope of yield strength 300 mega pascal, it can be shown, that the minimum area of cross section required will be 10 cm or so. A single wire of this radius will practically be a rigid rod. That is why ropes are always made of a large number of turns of thin wires braided together. This provides ease in manufacturing, flexibility and strength.

Do you know that the maximum height of a mountain on earth can be ~ 10 km? The answer to this question can also be provided by considering the elastic properties of rocks. A mountain base is not under uniform compression and this provides some shearing stress to the rocks under which they can flow. The bottom of the mountain experiences a force due to weight of the top part of the mountain in the vertical direction and sides are free. Hence there is a tangential shear of the order of hρg. The elastic limit for a typical rock is about 3×10^8 Nm^{-2} and its density is 3×10^3. Therefore, and $h_{max}\, \rho g = 3 \times 10^8$ Nm^{-2} solving, we have h_{max} = 10 km.

WHAT YOU HAVE LEARNT

- A force which causes deformation in a body is called deforming force.
- The property of matter to restore its original shape and size after withdrawal of deforming force is called elasticity.
- On deformation, internal restoring force is produced in a body and enables it to regain its original shape and size after removal of deforming force.
- The body which gains completely its original state on the removal of the deforming forces is called perfectly elastic.
- If a body completely retains its modified form after withdrawal of deforming force, it is said to be perfectly plastic.
- In normal state, the net inter-atomic force on an atom is zero. If the separation between the atoms becomes more than the separation in normal state, the interatomic forces become attractive. However, for smaller separation, these forces become repulsive.
- The stress equals the internal restoring force per unit area. Its units is Nm^{-2}.
- The strain equals the change in dimension (e.g. length, volume or shape) per unit dimension. Strain has no unit.
- The maximum value of stress up to which a body shows elastic property is called its elastic limit. A body beyond the elastic limit behaves like a plastic body.
- Hooke's law states that within elastic limit, stress developed in a body is directly proportional to strain.
- Young's modulus is the ratio of longitudinal stress to longitudinal strain.
- Bulk modulus is the ratio of normal stress to volume strain.
- Rigidity modulus is the ratio of the shearing stress to shearing strain.
- Poisson's ratio is the ratio of lateral strain to longitudinal strain.
- The energy stored in a stretched wire against the work done to the inter atomic forces during stretching is known as the elastic potential energy.

TERMINAL EXERCISE

1. Define the term elasticity. Give examples of elastic and plastic objects.
2. Explain the terms stress, strain and Hooke's Law.
3. Explain elastic properties of matter on the basis of inter-molecular forces.
4. Define Young's modulus, Bulk modulus and Rigidity modulus.

5. Discuss the behaviour of a metallic wire under increasing load with the help of stress–strain graph.
6. Why steel is more elastic than rubber.
7. Why poission's ratio has no units.
8. In the three states of matter i.e., solid, liquid and gas, which is more elastic and why?
9. A metallic wire 4m in length and 1mm in diameter is stretched by putting a mass 4 kg. Determine the elongation produced. Given that the Young's modulus of elasticity for the material of the wire is 13.78×10^{10} Nm^{-2}.
10. A sphere contracts in volume by 0.02% when taken to the bottom of sea 1 km deep. Calculate the bulk modulus of the material of the sphere. You may take density of sea water as 1000 kg m^{-3} and $g = 9.8$ ms^{-2}.
11. How much force is required to have an increase of 0.2% in the length of a metallic wire of radius 0.2 mm. Given $Y = 9 \times 10^{10}$ Nm^{-2}.
12. What are shearing stress, shearing strain and modulus of rigidity?
13. The upper face of the cube of side 10 cm is displaced 2 mm parallel to itself when a tangential force of 5×10^5 N is applied on it, keeping lower face fixed. Find out the strain.
14. Property of elasticity is of vital importance in our lives. How does the plasticity help us?
15. A wire of length L and radius r is clamped rigidly at one end. When the other end of wire is pulled by a force F, its length increases by x. Another wire of the same material of length 2L and radius 2r, when pulled by a force 2F, what will be the increase in its length.
16. Define the Elastic potential energy. Obtain the expression for elastic potential energy.

ANSWERS TO INTEXT QUESTIONS

9.1

1. (i) If $R > R_o$, the nature of force is attractive and (ii) if $R < R_o$, it is repulsive.
2. Longitudinal stress and linear strain.
3. The ratio will decrease.
4. The stress corresponding to breaking point is known as breaking stress.
5. 0.12×10^{10} Nm^{-2}

9.2

1. Both have same units since strain has no unit.
2. As compressibility of liquids and gases is more than solids, the bulk modulus is reciprocal of compressibility. Therefore, solids are more elastic than liquid and gases.
3. Half
4. 1 : 8
5. 1×10^{11} Nm^{-2}

ANSWERS TO TERMINAL EXERCISE

9. 0.15 m
10. 4.9×10^{-10} Nm^{-2}
11. 22.7 N
13. 2×10^{-2}
15. x

10 MECHANICAL PROPERTIES OF FLUIDS

INTRODUCTION

In the previous lesson, you have learnt that interatomic forces in solids are responsible for determining the elastic properties of solids. Does the same hold for liquids and gases? (These are collectively called fluids because of their nature to flow in suitable conditions). Have you ever visited the site of a dam on a river in your area / state/ region? If so, you would have noticed that as we go deeper, the thickness of the walls increases. Did you think of the underlying physical principle? Similarly, can you believe that you can lift a car, truck or an elephant by your own body weight standing on one platform of a hydraulic lift? Have you seen a car on the platform of a hydraulic jack at a service centre? How easily is it lifted? You might have also seen that mosquitoes can sit or walk on still water, but we cannot do so. You can explain all these observations on the basis of properties of liquids like hydrostatic pressure, Pascal's law and surface tension. You will learn about these in this chapter.

Have you experienced that you can walk faster on land than under water? If you pour water and honey in separate funnels, you will observe that water comes out more easily than honey. In this lesson, we will learn the properties of liquids which cause this difference in their flow.

You may have experienced that when the opening of soft plastic or rubber water pipe is pressed, the stream of water falls at larger distance. Do you know how a cricketer swings the ball? How does an aeroplane take off? These interesting observations can be explained on the basis of Bernoulli's principle. You will learn about it in this lesson.

OBJECTIVES

After studying this lesson, you should be able to

- *calculate the hydrostatic pressure at a certain depth inside a liquid;*
- *describe buoyancy and Archimedes Principle;*
- *state Pascal's law and explain the functioning of hydrostatic press, hydraulic lift and hydraulic brakes;*
- *explain surface tension and surface energy;*
- *derive an expression for the rise of water in a capillary tube;*
- *differentiate between streamline and turbulent motion of fluids;*
- *define critical velocity of flow of a liquid and calculate Reynold's number;*

- *define viscosity and explain some daily life phenomena based on viscosity of a liquid;*
- *state Bernoulli's Principle and apply it to some daily life experiences.*

10.1 HYDROSTATIC PRESSURE

While pinning papers, you must have experienced that it is easier to work with a sharp tipped pin than a flatter one. If area is large, you will have to apply greater force. Thus, we can say that for the same force, effect is greater for the smaller area. This effect of force on unit area is called pressure. Refer to Fig. 10.1. It shows the shape of the side wall of a dam. Note that it is thicker at the base. Do we use similar shape for the walls of our house. No, the walls of rooms are of uniform thickness. Do you know the basic physical characteristic which makes us to introduce this change?

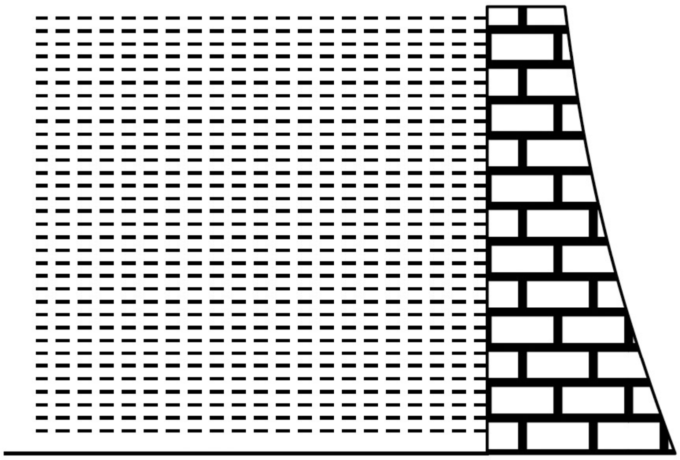


Fig. 10.1 : The structure of side wall of a dam

From the previous chapter, you may recall that solids develop shearing stress when deformed by an external force, because the magnitude of inter-atomic forces is very large. But fluids do not have shearing stress and when an object is submerged in a fluid, the force due to the fluid acts normal to the surface of the object Fig. 10.2. Also, the fluid exerts a force on the container normal to its walls at all points. The normal force or thrust per unit area exerted by a fluid is called pressure. We denote it by P and we have

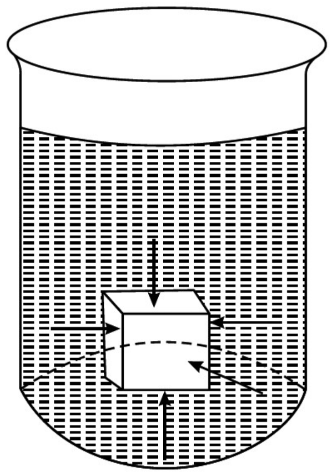


Fig. 10.2 : Force exerted by a fluid on a submerged object

$$\text{Pressure (P)} = \frac{\text{Thrust}}{\text{Area}} \qquad (10.1)$$

The pressure exerted by a fluid at rest is known as hydrostatic pressure. Pressure is a scalar quantity. The SI Unit of pressure is Nm^{-2} and is also called pascal (Pa) in the honour of French scientist Blaise Pascal who carried out pioneering studies on fluid pressure. We have another common unit of pressure i.e., the atmosphere (atm), which is defined as the pressure exerted by the atmosphere at sea level and one atmosphere pressure is equals to 1.013×10^5 Pa.

10.1.1 Hydrostatic Pressure at a point in side a liquid

Consider a liquid in a container and an imaginary cylinder of cross-sectional area A and height h, as shown in Fig. 10.3. Let the pressure exerted by the liquid on the bottom and top faces of the cylinder be P_1, and P_2, respectively. Therefore, the upward force exerted by the liquid on the bottom of the cylinder is P_1A and the downward force on the top of the cylinder is P_2A.

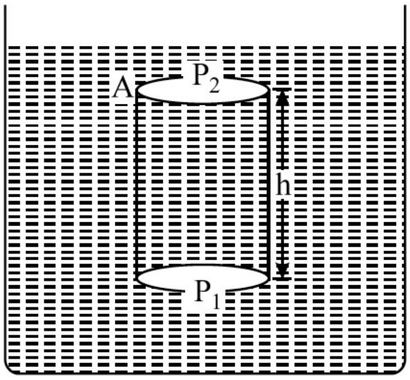


Fig. 10.3 : An imaginary cylinder of height h in a liquid.

Therefore, the net force in upward direction is $(P_1A - P_2A)$.

Now mass of the liquid in cylinder = density × volume of the cylinder = ρAh

where ρ is the density of the liquid.

$\therefore$ Weight of the liquid in the cylinder = $(\rho h\, A)g$

Since the cylinder is in equilibrium, the resultant force acting on it must be equal to zero, i.e.

$$P_1A - P_2A - \rho gh\, A = 0$$

$$P_1 - P_2 = \rho\, g\, h \qquad (10.2)$$

So, the pressure P at the bottom of a column of liquid of height h is given by

$$P = \rho\, g\, h \qquad (10.3)$$

That is, hydrostatic pressure due to a fluid increase linearly with depth. It is for this reason that the thickness of the wall of a dam has to be increased with increase in the depth of the dam.

If we consider the upper face of the cylinder to be at the open surface of the liquid, as shown in Fig. 10.4, then P_2 will have to be replaced by P_{atm} (Atmospheric pressure). If we denote P_1 by P, the absolute pressure at a depth below the surface will be

$$P - P_{atm} = \rho\, g\, h$$

$$P = P_{atm} + \rho\, g\, h \qquad (10.4)$$

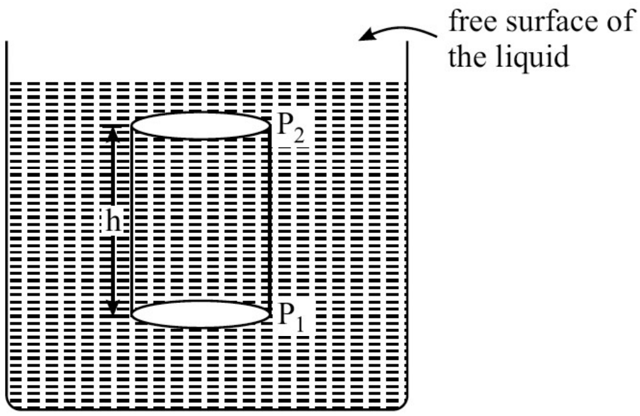


Fig. 10.4 : Cylinder in a liquid with one face at the surface of the liquid

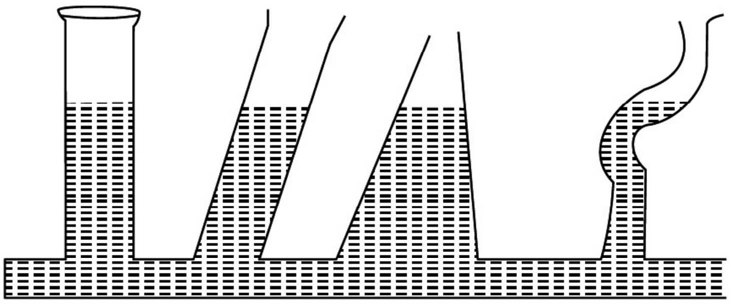


Fig. 10.5 : Pressure does not depend upon shape of the vessel

Note that the expression given in Eqn. 10.4 does not show any term having area of the cylinder. It means that pressure in a liquid at a given depth is equal, irrespective of the shape of the vessel and it can be observed in the following Fig 10.5.

Example 10.1

A cemented wall of thickness one metre can withstand a side pressure of 10^5 Nm^{-2}. What should be the thickness of the side wall at the bottom of a water dam of depth 100 m. Take the density of water = 10^3 kg m^{-3} and g = 9.8 ms^{-2}.

Solution :

The pressure on the side wall of the dam at its bottom is given by

$$P = \rho\ g\ h$$

$$P = 10^3\ kg\ m^{-3} \times 9.8\ ms^{-2} \times 100\ m$$

$$P = 9.8 \times 10^5\ Nm^{-2}$$

Using unitary method, we can calculate the thickness of the wall, which will withstand pressure of 9.8×10^5 Nm^{-2}. Therefore, thickness of the wall

$$t = \frac{9.8 \times 10^5\ Nm^{-2}}{10^5\ Nm^{-2}} = 9.8\,m$$

10.1.2 Atmospheric Pressure

We know that the earth is surrounded by an atmosphere up to a height of about 200 km. The pressure exerted by the atmosphere is known as the atmospheric pressure. A German Scientist O.V. Guericke performed an experiment to demonstrate the force exerted on bodies due to the atmospheric pressure. He took two hollow hemispheres made of copper, having diameter 20 inches and tightly joined them with each other. These could easily be separated when air was inside. When air between them was exhausted with an air pump, 8 horses were required to pull the hemispheres apart.

Torricelli used the formula for hydrostatic pressure to determine the magnitude of atmospheric pressure. He took a tube of about 1 m long filled with mercury of density 13600 kg m^{-3} and placed it vertically inverted in a mercury tub as shown is Fig. 10.6. This device is known as 'mercury barometer'. He observed that the column of 76 cm of mercury above the free surface remained filled in the tube. In equi1ibrium, atmospheric pressure equals the pressure exerted by the mercury column. Therefore,

$$P_{atm} = \rho\ g\ h = 13600\ kg\ m^{-3} \times 9.8\ ms^{-2} \times 0.76\ m$$

$$P_{atm} = 1.01 \times 10^5\ Nm^{-2}$$

$$P_{atm} = 1.01 \times 10^5\ Pa$$

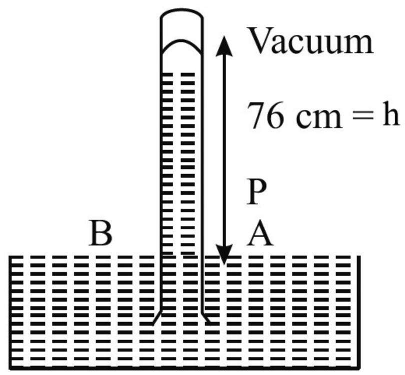
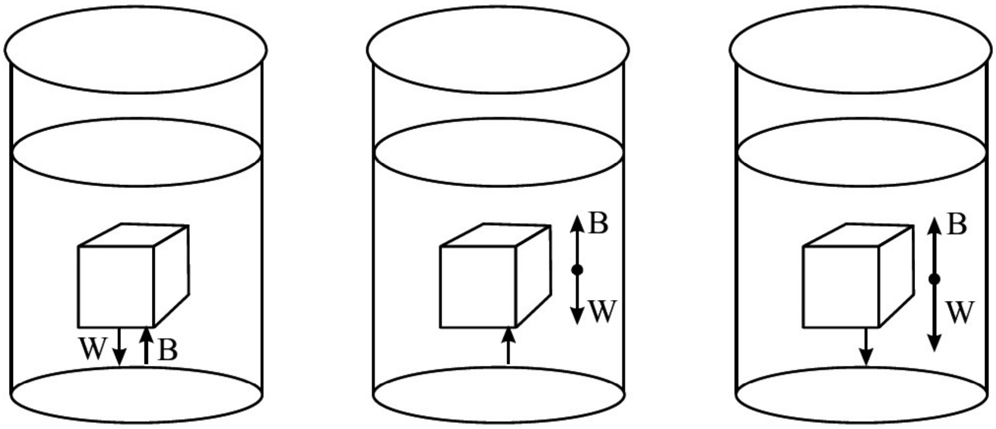


Fig. 10.6 : Torricelli's Barometer

There are some other units to express the pressure and the most common way is in terms of cm or mm of mercury (Hg) column. The pressure equivalent of 1 mm of Hg column is called a torr. This unit was named after Torricelli. 1 torr = 133 Pa. The unit torr is used in medicine and physiology. In meteorology, a common unit is the bar and millibar. 1 bar = 10^5 Pa.

10.2 BUOYANCY

It is a common experience that lifting an object in water is easier than lifting it in air. It is because of the difference in the upward forces exerted by these fluids on these objects. The upward force, which acts on an object when submerged in a fluid, is known as **buoyant force**. The nature of buoyant force that acts on objects placed inside a fluid was discovered by Archimedes. Based on his observations, he enunciated a law now known as Archimedes principle. It states that when an object is submerged partially or fully in a fluid, the magnitude of the buoyant force on it is always equal to the weight of the fluid displaced by the object. The different conditions of an object under buoyant force are shown in Fig. 10.7.



(a) : The magnitude of buoyant force B on the object is exactly equal to its weight in equilibrium.

(b) : A totally submerged object of density less than that of the fluid experiences a net upward force.

(c) : A totally submerged object denser than that the fluid sinks.

Fig : 10.7

Another example of buoyant force is provided by the motion of hot air balloon shown in Fig. 10.8. Since hot air has less density than the cold air, a net upward buoyant force on the balloon makes it to float.

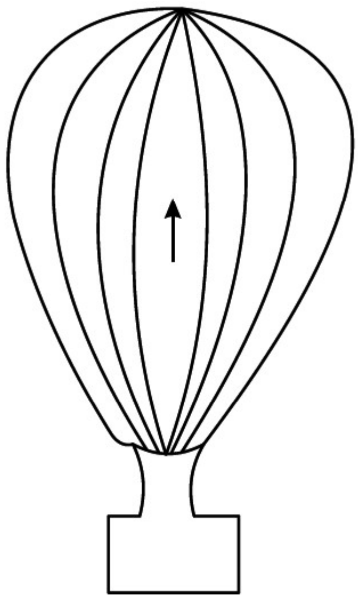
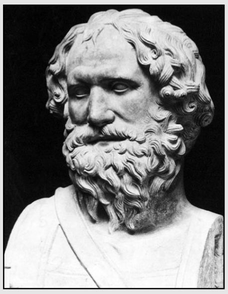


Fig. 10.8 : Hot air balloon floating in air

Floating objects

You must have observed a piece of wood floating on the surface of water. Can you identify the forces acting on it when it is in equilibrium? Obviously, one of the forces is due to gravitational force, which pulls it downwards. However, the displaced water exerts buoyant force which acts upwards. These forces balance each other in equilibrium state and object is then said to be floating on water. It means that a floating body displaces the fluid equal to its own weight.

Archimedes (287-212 BC)



A Greek physicist, engineer and mathematician was perhaps the greatest scientist of his time. He is well known for discovering the nature of buoyant forces acting on objects. The Archimedes screw is used even today. It is an inclined rotating coiled tube used originally to lift water from the hold of ships. He also invented the catapult and devised the system of levers and pulleys.

Once Archimedes was asked by king Hieron of his native city Syracuse to determine whether his crown was made up of pure gold or alloyed with other metals without damaging the crown. While taking bath, he got a solution, noting a partial loss of weight when submerging his arm and legs in water. He was so excited about his discovery that he ran undressed through the streets of city shouting "Eureka, Eureka", meaning I have found it.

10.3 PASCAL'S LAW

While travelling by a bus, you must have observed that the driver stops the bus by applying a little force on the brakes by his foot. Have you seen the hydraulic jack or lift which can lift a car or truck up to a desired height? For this purpose, you may visit a motor workshop. Packing of cotton bales is also done with the help of hydraulic press which works on the same principle.

These devices are based on Pascal's law, which states that when pressure is applied at any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container. This law is also known as the law of transmission of liquid pressure.

10.3.1 Applications of Pascal's Law

(A) Hydraulic Press/Balance/Jack/Lift : It is a simple device based on Pascal's law and is used to lift heavy loads by applying a small force. The basic arrangement is shown in Fig. 10.9. Let a force F_1 be applied to the smaller piston of area A_1. On the other side, the piston of large area A_2 is attached to a platform where heavy load may be placed. The pressure on the smaller piston is transmitted to the larger piston through the liquid filled in-between the two pistons. Since the pressure is same on both the sides, we have

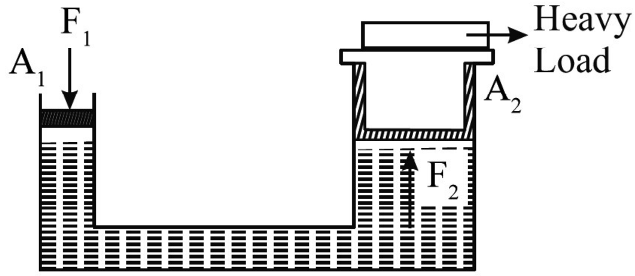


Fig. 10.9 : Hydraulic lift

Pressure on the smaller piston, $P = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F_1}{A_1}$

According to Pascal's law, the same pressure is transmitted to the larger cylinder of area A_2. Hence the force acting on the larger piston,

$$F_2 = \text{Pressure} \times \text{Area} = \frac{F_1}{A_1} \times A_2 \qquad (10.5)$$

It is clear from Eqn. 10.5 that force $F_2 > F_1$ by an amount equal to the ratio $\left(\frac{A_2}{A_1}\right)$ with slight modifications, the same arrangement is used in hydraulic press, hydraulic balance, and hydraulic Jack, etc.

(B) Hydraulic Jack or Car Lifts : At automobile service stations, you would see that cars, buses and trucks are raised to the desired heights so that a mechanic can work under them Fig. 10.10. This is done by applying pressure, which is transmitted through a liquid to a large surface to produce sufficient force needed to lift the car.

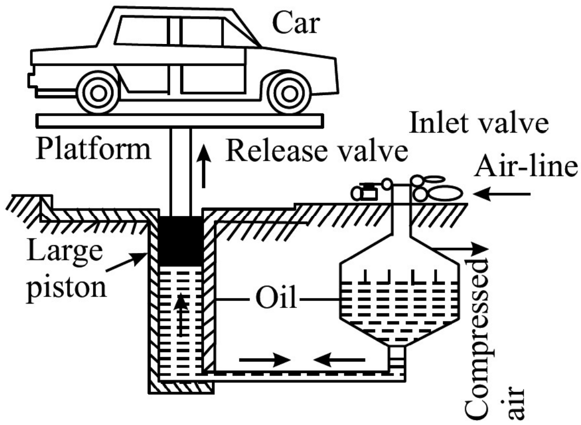
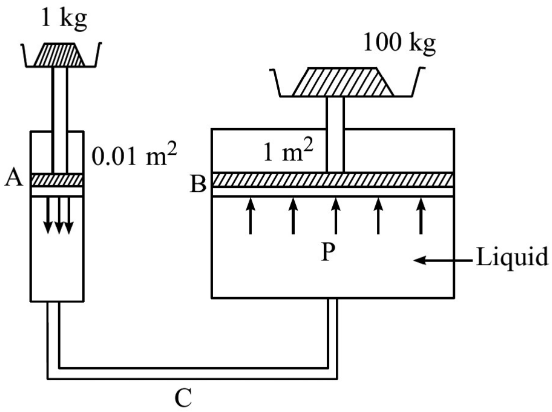
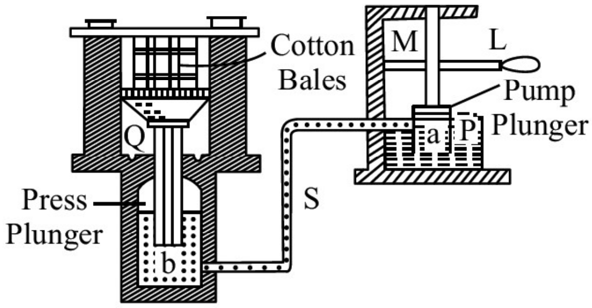


Fig. 10.10 : Hydraulic jack

(C) Hydraulic Brakes : While traveling in a bus or a car, we see how driver applies a little force by his foot on the brake paddle to stop the vehicle. The pressure so applied gets transmitted through the brake oil to the piston of slave cylinders, which, in turn, pushes the brake shoes against the brake drum in all four wheels, simultaneously. The wheels stop rotating at the same time and the vehicle comes to stop instantaneously.



(a) Hydraulic balance



(b) Hydraulic press

Fig. 10.11

Intext Questions 10.1

1. Why are the shoes used for skiing on snow made big in size?
2. Calculate the pressure at the bottom of an ocean at a depth of 1500 m. Take the density of sea water 1.024×10^3 kg m^{-3}, atmospheric pressure 1.01×10^5 Pa and $g = 9.80$ ms^{-2}.
3. An elephant of weight 5000 kg-wt is standing on the bigger piston of area 10 m^2 of a hydraulic lift. Can a boy of weight 25 kg-wt standing on the smaller piston of area 0.05 m^2 balance or lift the elephant?
4. If a pointed needle is pressed against your skin, you are hurt but if the same force is applied by a rod on your skin nothing may happen. Why?
5. A body of 50 kg is put on the smaller piston of area 0.1 m^2 of a big hydraulic lift. Calculate the maximum weight that can be balanced on the bigger piston of area 10 m^2 of this hydraulic lift.

10.4 SURFACE TENSION

It is common experience that in the absence of external forces, drops of liquid are always spherical in shape. If you drop small amount of mercury from a small height, it spreads in small spherical globules. The water drops falling from a tap or shower are also spherical. Do you know why it is so? You may have enjoyed the soap bubble game in your childhood. But you cannot make pure water bubbles with same case? All the above experiences are due to a characteristic property of liquids, which we call surface tension. To appreciate this, we would like you to do the following activity.

Activity 10.1

1. Prepare a soap solution.
2. Add a small amount of glycerine to it.
3. Take a narrow hard plastic or glass tube.
4. Dip its one end in the soap solution so that some solution enters into it.
5. Take it out and blow air at the other end with your mouth.
6. Large soap bubble will be formed.
7. Give a jerk to the tube to detach the bubble which then floats in the air.

To understand as to how surface tension arises, let us refresh our knowledge of intermolecular forces. In the previous lesson, you have studied the variation of intermolecular forces with distance between the centres of molecules/atoms.

The intermolecular forces are of two types: cohesive and adhesive. Cohesive forces characterise attraction between the molecules of the same substance, whereas force of adhesion is the attractive force between the molecules of two different substances. It is the force of adhesion which makes it possible for us to write on this paper. Gum, Fevicol etc. show strong adhesion.

We hope that now you can explain why water wets glass while mercury does not.

Activity 10.2

To show adhesive forces between glass and water molecule.

1. Take a clean sheet of glass.
2. Put a few drops of water on it.
3. Hold water containing side downward.
4. Observe the water drops.

The Adhesive forces between glass and water molecules keep the water drops sticking on the glass sheet, as shown in Fig. 10.12.

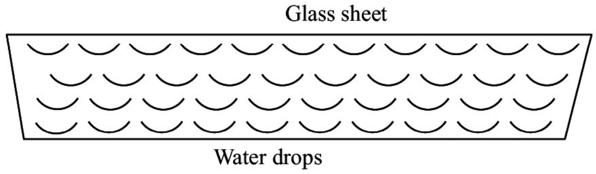


Fig. 10.12 : Water drops remain stuck to the glass sheet

10.4.1 Surface Energy

The surface layer of a liquid in a container exhibits a property different from the rest of the liquid. In Fig. 10.13, molecules are shown at different heights in a liquid. A molecule, say P, well inside liquid is attracted by the other molecules from all sides. However, it is not the case for molecules at the surface.

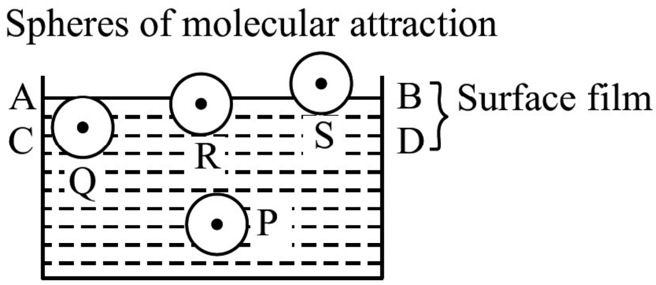


Fig. 10.13 : Resultant force acting on P and Q is zero but molecules R and S experience a net vertically downward force.

Molecules S and R, which lie on the surface layer, experience a net resultant force downward because the number of molecules in the upper half of the sphere of influence are less than those in the lower half. Hence, all the molecules on the surface of the liquid or at the liquid – air interface experience a net downward force because only lower half side of these molecules are surrounded by the liquid molecules. Therefore, if any liquid molecule is brought to the surface layer, work has to be done against the net downward force, which increases their potential energy. This means that the surface layer possesses an additional energy, which is termed as surface energy.

For a system to be in equilibrium, its potential energy must be minimum. Therefore, the area of surface must be minimum. That is why free surface of a liquid at rest tends to attain minimum surface area. This produces a tension in the surface, called **surface tension**.

Surface tension is a property of the liquid surface due to which it has the tendency to decrease its surface area. As a result, the surface of a liquid acts like a stretched membrane. You can visualise its existence easily by placing a needle gently on water surface and see it float.

Let us now understand this physically. Consider an imaginary line AB drawn at the surface of a liquid at rest, as shown in Fig. 10.14. The surface on either side of this line exerts a pulling force on the surface on the other side.

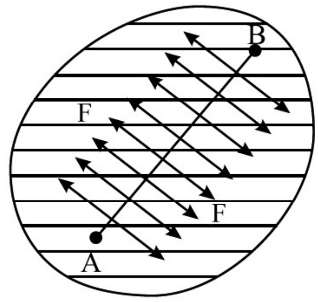


Fig. 10.14 : Direction of surface tension on a liquid surface

The surface tension of a liquid can be defined as the force per unit length in the plane of liquid surface:

$$T = \frac{F}{L} \qquad (10.6)$$

where surface tension is denoted by T, and F is the magnitude of total force acting in a direction normal to the imaginary line of length L, Fig. 10.14 and tangential to the liquid surface. SI unit of surface tension is Nm^{-1} and its dimensions are $[MT^{-2}]$.

Let us take a rectangular frame, as shown in Fig. 10.15 having a sliding wire on one of its arms. Dip the frame in a soap solution and take out. A soap film will be formed on the frame and have two surfaces. Both the surfaces are in contact with the sliding wire, So, we can say that surface tension acts on the wire due to both these surfaces.

Let T be the surface tension of the soap solution and L be the length of the wire. The force exerted by each surface on the wire will be equal to T × L. Therefore, the total force F on the wire is 2TL.

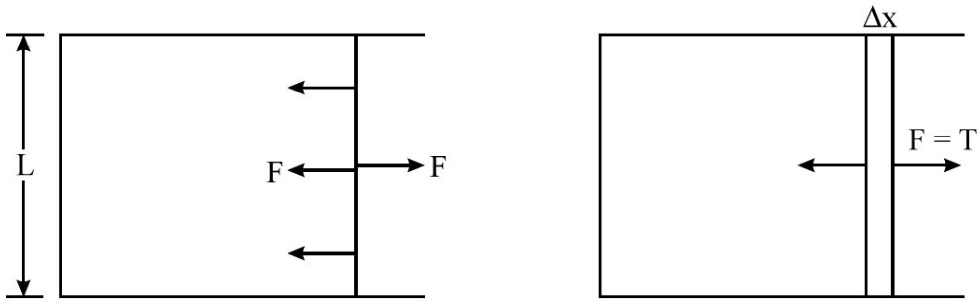


Fig. 10.15 : A Film in equilibrium

Suppose that the surfaces tend to contract say, by Δx. To keep the wire in equilibrium we will have to apply an external uniform force equal to F. If we increase the surface area of the film by pulling the wire with a constant speed through a distance Δx, as shown in Fig. 10.15, the work done on the film is given by

$$W = F \times \Delta x = 2LT \times \Delta x$$

$$W = T\ 2\ (L \times \Delta x)$$

where $2\ (L \times \Delta x)$ is the total increase in the area of both the surfaces of the film. Let us denote it by A. Then, the expression for work done on the film simplifies to

$$W = T \times A$$

This work done by the external force is stored as the potential energy of the new surface and is called as surface energy. By rearranging terms, we get the required expression for surface tension :

$$T = \frac{W}{A} \qquad (10.7)$$

Thus, we see that surface tension of a liquid is equal to the work done in increasing the surface area of its free surface by one unit. We can also say that surface tension is equal to the surface energy per unit area.

We may now conclude that surface tension

- is a property of the surface layer of the liquid or the interface between a liquid and any other substance like air,
- tends to reduce the surface area of the free surface of the liquid,
- acts perpendicular to any line at the free surface of the liquid and is tangential to its meniscus,
- has genesis in intermolecular forces which depend on temperature and
- decreases with temperature.

A simple experiment described below demonstrates the property of surface tension of liquid surfaces.

Activity 10.3

Take a thin circular frame of wire and dip it in a soap solution. You will find that a soap film is formed on it. Now take a small circular loop of cotton thread and put it gently on the soap film. The loop stays on the film in an irregular shape as shown in Fig. 10.16 (a). Now take a needle and touch its tip to the soap film inside the loop. What do you observe?

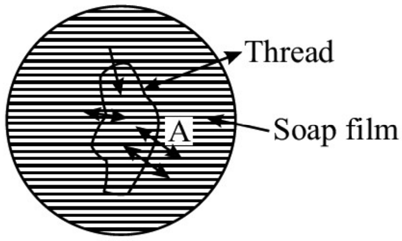


Fig. 10.16 (a) : A soap film with closed loop of thread

You will find that the loop of cotton thread takes a circular shape as shown in Fig. 10.16 (b). Initially there was soap film on both sides of the thread. The surface on both sides pulled it and net forces of surface tension were zero. When inner side was punctured by the needle, the outside surface pulled the thread to bring it into the circular shape, so that it may acquire minimum area.

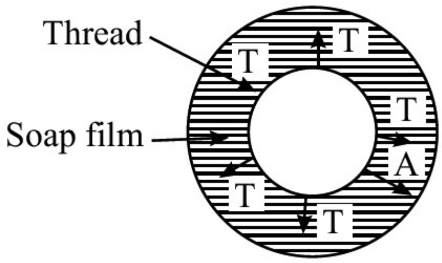
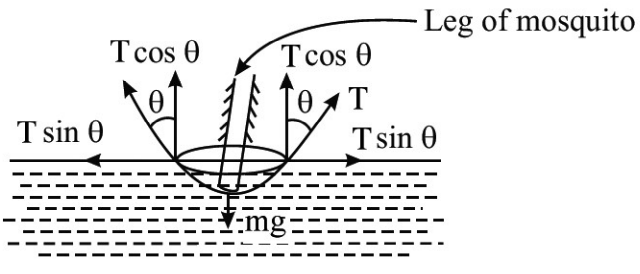


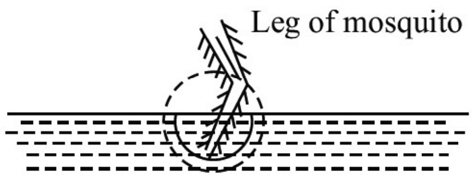
Fig. 10.16 (b) : The shape of the thread without inner soap film

10.4.2 Applications of Surface Tension

(A) Mosquitoes sitting on water : In rainy season, we witness spread of diseases like dengue, malaria and chikungunya by mosquito breeding on fresh stagnant water. Have you seen mosquitoes sitting on water surface? They do not sink in water due to surface tension. At the points where the legs of the mosquito touch the liquid surface, the surface becomes concave due to the weight of the mosquito. The surface tension acting tangentially on the free surface, therefore, acts at a certain angle to the horizontal. Its vertical component acts upwards. The total force acting vertically upwards all along the line of contact of certain length balances the weight of the mosquito acting vertically downward, as shown in Fig. 10.17.



(a) Dip in the level to form concave surface

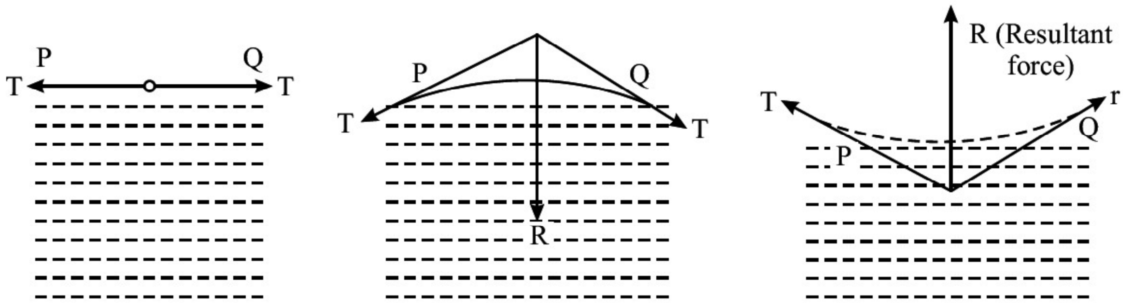


(b) Magnified image

Fig. 10.17 : The weight of a mosquito is balanced by the force of surface tension

(B) Excess pressure on concave side of a spherical surface : Consider a small surface element with a line PQ of unit length on a liquid surface as shown in Fig. 10.18. If the surface is plane, i.e. θ = 90°, the surface tension on the two sides tangential to the surface balances and the resultant tangential force is zero [Fig. 10.18 (a)]. If, however, the surface is convex, [Fig. 10.18 (b)] or concave [Fig. 10.18 (c)], the forces due to surface tension acting across the sides of the line PQ will have resultant force R towards the center of curvature of the surface.

Thus, whenever the surface is curved, the surface tension gives rise to a pressure directed towards the center of curvature of the surface. This pressure is balanced by an equal and opposite pressure acting on the surface. Therefore, there is always an excess pressure on the concave side of the curved liquid surface.



(a) plane surface (b) convex surface (c) concave surface

Fig. 10.18

(i) Spherical drop : A liquid drop has only one surface i.e. the outer surface (The liquid area in contact with air is called the surface of the liquid). Let r be the radius of a small spherical liquid drop and P be excess pressure inside the drop (which is concave on the inner side, but convex on the outside). Then

$$P = (P_i - P_0)$$

where P_i and P_0 are the inside and outside pressures of the drop respectively [Fig 10.19 (a)]. If the radius of the drop increases by Δr due to this constant excess pressure P, then increase in surface area of the spherical drop is given by

$$\Delta A = 4\pi (r + \Delta r)^2 - 4\pi r^2 = 8 \pi r \Delta r$$

where we have neglected the term containing second power of Δr since its value is very small.

The work done on the drop for this increase in area is given by

$$W = \text{Extra surface energy} = T\Delta A = T (8 \pi r \Delta r) \qquad (10.8)$$

If the drop is in equilibrium, this extra surface energy is equal to the work done due to expansion under the pressure difference or excess pressure P :

$$\text{Work done} = P\Delta V = P (4\pi r^2 \Delta r) \qquad (10.9)$$

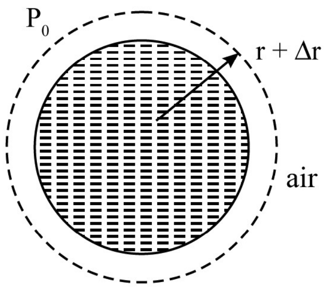


Fig. 10.19 (a) : A spherical drop

On combining Eqn.10.8 and Eqn. 10.9, we get

$$P\, 4\pi r^2 \Delta r = T\, 8 \pi r \Delta r$$

or
$$P = \frac{2T}{r} \qquad (10.10)$$

(ii) Air Bubble in water : An air bubble also has a single surface, which is the inner surface Fig. 10.19 (b). Hence, the excess of pressure P inside an air bubble of radius r in a liquid of surface tension T is given by

$$P = \frac{2T}{r} \qquad (10.11)$$

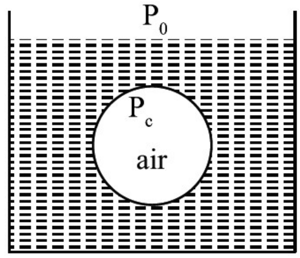


Fig. 10.19 (b) : Air Bubble

(iii) Soap bubble floating in air : The soap bubble has two surfaces of equal surface area (i. e. the outer and inner) as shown in Fig. 10.19 (c). Hence, excess pressure inside a soap bubble floating in air is given by

$$P = \frac{4T}{r} \qquad (10.12)$$

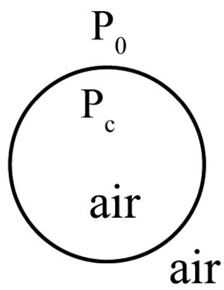


Fig. 10.19 (c) : Soap bubble in air

where T is surface tension of soap solution.

This is twice that inside a spherical drop of same radius or an air bubble in water. Now you can understand why a little extra pressure is needed to form a soap bubble.

Example 10.2

Calculate the difference of pressure between inside and outside of a (i) spherical soap bubble in air, (ii) air bubble in water, and (iii) spherical drop of water, each of radius 1 mm. Given surface tension of water is 7.2×10^{-2} Nm^{-1} and surface tension of soap solution is 2.5×10^{-2} Nm^{-1}.

Solution :

(i) Excess pressure inside a soap bubble of radius r is $P = \frac{4T}{r}$

$$P = \frac{4 \times 2.5 \times 10^{-2}\ Nm^{-1}}{1 \times 10^{-3}\ m} = 100\,Nm^{-2}$$

(ii) Excess pressure inside an air bubble in water is $P = \frac{2T'}{r}$

$$= \frac{2 \times 7.2 \times 10^{-2}\ Nm^{-1}}{1 \times 10^{-3}\ m} = 144\,Nm^{-2}$$

(iii) Excess pressure inside a spherical drop of water is $P = \frac{2T'}{r}$

$$= \frac{2 \times 7.2 \times 10^{-2}\ Nm^{-1}}{1 \times 10^{-3}\ m} = 144\,Nm^{-2}$$

(C) Detergents and surface tension : You may have seen different advertisements highlighting that detergents can remove oil stains from clothes. Water is used as cleaning agent. Soap and detergents lower the surface tension of water. This is desirable for washing and cleaning since high surface tension of pure water does not allow it to penetrate easily between the fibres of materials, where dirt particles or oil molecules are held up.

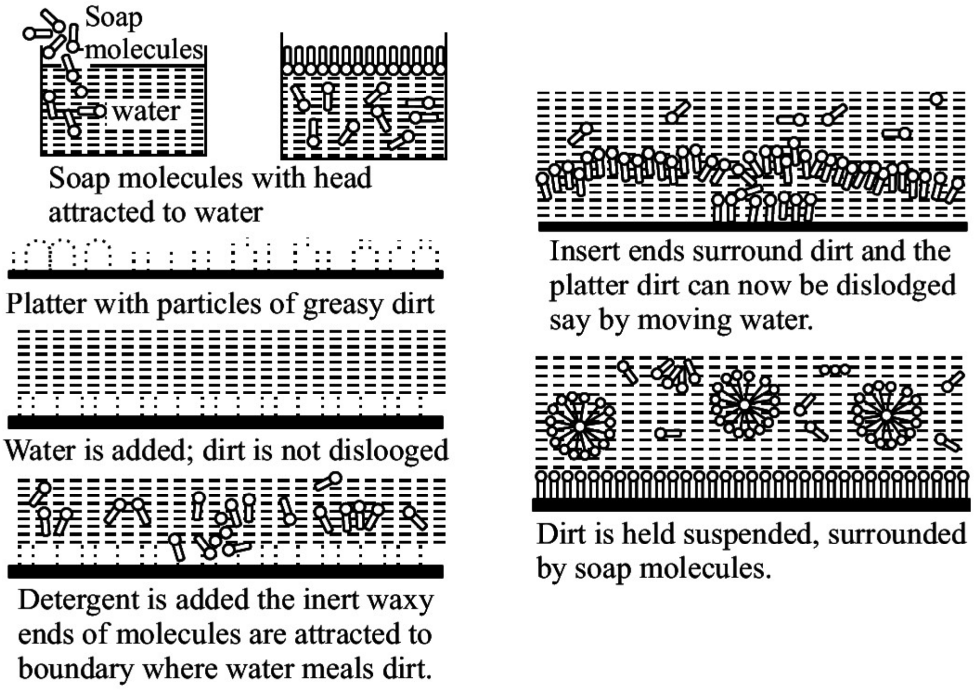


Fig. 10.20 : Detergent action

You now know that surface tension of soap solution is smaller than that of the pure water. But the surface tension of detergent solutions is smaller than that of the soap solution. That is why detergents are more effective than soap. A detergent dissolved in water weakens the hold of dirt particles on the cloth fibres which therefore, get easily detached on squeezing the cloth.

The addition of detergent, whose molecules attract water as well as oil, drastically reduces the surface tension (T) of water – oil. It may even become favourable to form such interfaces, i. e. globes of dirt surrounded by the detergent and then by the water. This kind of process using surface active detergents is important for not only cleaning the clothes but also in recovering oil, mineral ores etc.

(D) Wax-Duck floating on water : You have learnt that the surface tension of liquids decreases due to dissolved impurities. If you stick a tablet of camphor to the bottom of a wax-duck and float it on still water surface, you will observe that it begins to move randomly after a minute or two. This is because camphor dissolves in water and the surface tension of water just below the duck becomes smaller than the surrounding liquid. This creates a net difference of force of surface tension which makes the duck to move.

Now, it is time for you to check how much you have learnt. Therefore, answer the following questions.

Intext Questions 10.2

1. What is the difference between force of cohesion and force of adhesion?
2. Why do small liquid drops assume a spherical shape.
3. Do solids also show the property of surface tension? Why?
4. Why does mercury collect into globules when poured on plane surface?
5. Which of the following has more excess pressure? (i) An air bubble in water of radius 2 cm. Surface tension of water is 727×10^{-3} Nm^{-1} or (ii) A soap bubble in air of radius 4 cm. Surface tension of soap solution is 25×10^{-3} Nm^{-1}.

10.5 ANGLE OF CONTACT

You can observe that the free surface of a liquid kept in a container is curved. For example, when water is filled in a glass jar, it becomes concave but if we fill water in a paraffin wax container, the surface of water becomes convex. Similarly, when mercury is filled in a glass jar, its surface become convex. Thus, we see that shape of the liquid surface in a container depends on the nature of the liquid, material of container and the medium above free surface of the liquid. To characterize it, we introduce the concept of angle of contact.

It is the angle that the tangential plane to the liquid surface makes with the tangential plane to the wall of the container, to the point of contact, as measured from within the liquid, is known as angle of contact.

Fig. 10.21 shows the angles of contact for water in a glass jar and paraffin jar. The angle of contact is acute for concave spherical meniscus, Eg. water with glass and obtuse (or greater than 90°) for convex spherical meniscus, Eg. water in paraffin or mercury in glass tube.

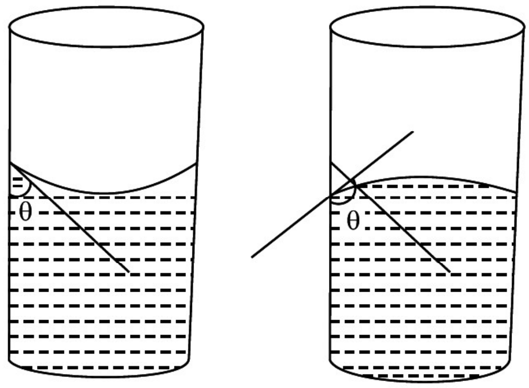


Fig. 10.21 : Nature of free surface when water is filled in (a) glass jar, (b) paraffin wax jar

Various forces act on a molecule in the surface of a liquid contained in a vessel near the boundary of the meniscus. As the liquid is present only in the lower quadrant, the resultant cohesive force F_c acts on the molecule at P symmetrically, as shown in the Fig. 10.22 (a). Similarly due to symmetry, the resultant adhesive force F_a acts outwards at right angles to the walls of the container vessel. The force F_c can be resolved into two mutually perpendicular components $F_c \cos \theta$ acting vertically downwards and $F_c \sin \theta$ acting at right angled to the boundary, The value of the angle of contact depends upon the relative values of F_c and F_a.

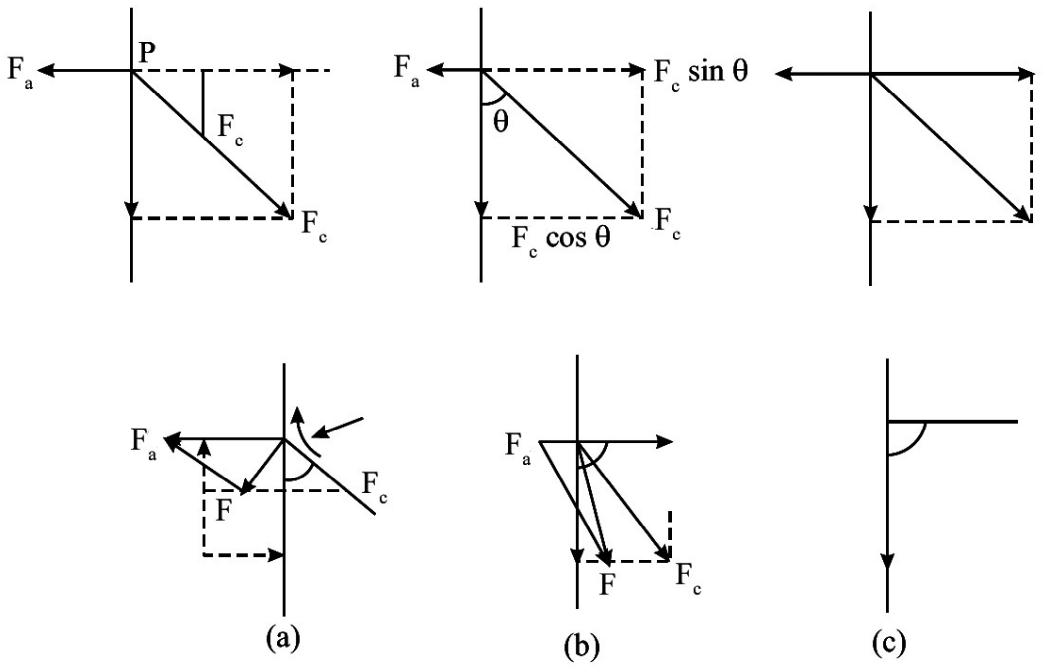


Fig. 10.22 : Different shapes of liquid meniscuses

Case 1 : If $F_a > F_c \sin \theta$, the net horizontal force is outward and the resultant F, of ($F_a - F_c \sin \theta$) and $F_c \cos \theta$ lies outside the wall. Since liquids cannot sustain constant shear, the liquid surface and hence all the molecules in it near the boundary adjust themselves at right angles to F_c so that no component of F acts tangential to the liquid surface. Obviously, such a surface at the boundary is concave spherical (Since radius of a circle is perpendicular to the circumference at every point). This is true in the case of water filled in a glass tube.

Case 2 : If $F_a < F_c \sin\theta$, the resultant F, of $(F_c \sin\theta - F_a)$ acting horizontally and $F_c \cos\theta$ acting vertically down wards is in the lower quadrant acting into the liquid. The liquid surface at the boundary, therefore, adjusts itself at right angles to this and hence becomes convex spherical. This is true for the case of mercury filled in the glass tube.

Case 3 : When $F_a = F_c \sin\theta$, the resultant force F acts vertically downwards and hence the liquid surface near the boundary becomes horizontal or plane.

10.6 CAPILLARY ACTION

You might have used blotting paper to absorb extra ink from your notebook. The ink rises in the narrow air gaps in the blotting paper. Similarly, if the lower end of a cloth gets wet, water slowly rises upward. Also, water given to the fields rises in the innumerable capillaries in the stems of plants and trees and reaches the branches and leaves. Do you know that farmers plough their fields only after rains so that the capillaries formed in the upper layers of the soil are broken. Thus, water trapped in the soil is taken up by the plants. On the other hand, we find that when a capillary tube is dipped into mercury, the level of mercury inside it is below the outside level. Such an important phenomenon of the elevation or depression of a liquid in an open tube of small cross-section (i.e., capillary tube) is basically due to surface tension and is known as capillary action.

The phenomenon of rise or depression of liquids in capillary tubes is known as capillary action or capillarity.

10.6.1 Rise of a Liquid in a Capillary Tube

Let us take a capillary tube dipped in a liquid, say water. The meniscus inside the tube will be concave, as shown in Fig. 10.23 (a). This is essentially because the forces of adhesion between glass and water are greater than cohesive forces.

Fig. 10.23 : Capillary action

Let us consider four points A, B, C and D near the liquid–air interface as shown in Fig. 10.23 (a). We know that the pressure just below the meniscus is less than the pressure just above it by $\frac{2T}{R}$, i.e.

$$P_B = P_A - \frac{2T}{R} \tag{10.13}$$

where T is surface tension at liquid – air interface and R is the radius of concave surface. But pressure at A is equal to the pressure at D and is equal to the atmospheric pressure P (say). And pressure at D is equal to pressure at C. Therefore, pressure at B is less than pressure at D. But we know that the pressure at all points at the same level in a liquid must be same. That's why water begins to flow from the outside region into the tube to make up the deficiency of pressure at point B.

Thus, liquid begins to rise in the capillary tube to a certain height h as shown in Fig. 10.23 (b) till the pressure of liquid column of height h becomes equal to $\frac{2T}{R}$. Thereafter, water stops rising. In this condition

$$h\rho g = \frac{2T}{R} \tag{10.14}$$

where ρ is the density of the liquid and g is the acceleration due to gravity.

If r be radius of capillary tube and θ be the angle of contact, then from Fig. 10.24, we can write

$$R = \frac{r}{\cos\theta}$$

Substituting this value of R in Eqn. 10.14, we have

$$h\rho g = \frac{2T}{r/\cos\theta}$$

$$h = \frac{2T\cos\theta}{r\rho g} \tag{10.15}$$

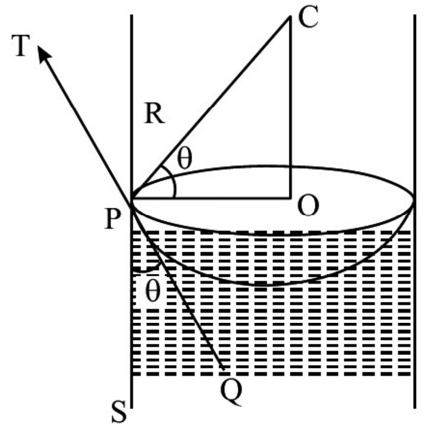


Fig. 10.24 : Angle of contact

It is clear from the above expression that if the radius of tube is less (i.e. in a very fine bore capillary), liquid rise will be high.

Intext Questions 10.3

1. Does the value of angle of contact depend on the surface tension of the liquid?
2. The angle of contact for a solid and liquid is less than the 90°. Will the liquid wet the solid? If a capillary is made of that solid, will the liquid rise or fall in it?
3. Why is it difficult to enter mercury in a capillary tube, by simply dipping it into a vessel containing mercury while designing a thermometer.

4. Calculate the radius of a capillary to have a rise of 3 cm when dipped in a vessel containing water of surface tension 7.2×10^{-2} Nm^{-1}. The density of water is 1000 kg m^{-3}, angle of contact is zero, and g = 10 ms^{-2}.

5. How does kerosene oil rise in the wick of a lantern?

10.7 VISCOSITY

If you stir a liquid taken in a beaker with a glass rod in the middle, you will note that the motion of the liquid near the walls and in the middle is not same. It can be seen in the Fig. 10.25. Next watch the flow of two liquids (eg. glycerine and water) through identical pipes. You will find that water flows rapidly out of the vessel whereas glycerine flows slowly. Drop a steel ball through each liquid. The ball falls more slowly in glycerine than in water. These observations indicate a characteristic property of the liquid that determines their motion. This property is known as viscosity. Let us now learn how it arises.

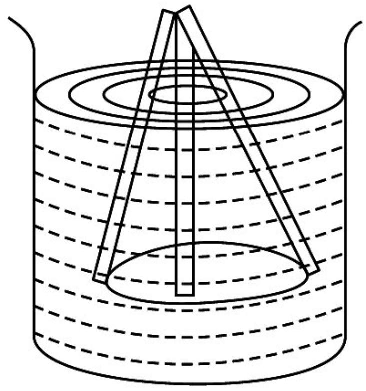


Fig. 10.25 : Water being stirred with a glass rod

We know that when one body slides over the other, a frictional force acts between them. Similarly, whenever a fluid flows, two adjacent layers of the fluid exert a tangential force on each other. This force acts as a drag and opposes the relative motion between them. The property of a fluid by virtue of which it opposes the relative motion in its adjacent layers is known as viscosity.

Derivation of expression for Viscous Force (F)

Consider a liquid flow through a tube as shown in the Fig. 10.26. The layer of the liquid in touch with the wall of the tube can be assumed to be stationary due to friction between the solid wall and the liquid. Other layers are in motion and have different velocities. Let v be the velocity of the layer at a distance x from the surface and v + dv be the velocity at a distance x + dx.

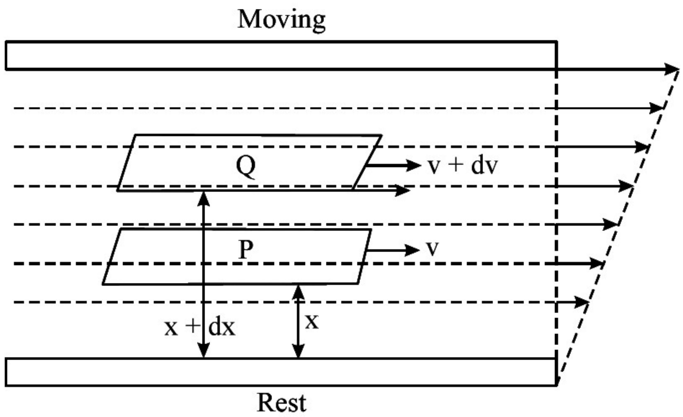


Fig. 10.26 : Flow of a liquid in a tube: Different layers move with different velocities

Thus, the velocity changes by dv in going through a distance dx perpendicular to it. The quantity dv/dx is called the velocity gradient. The viscous force F between two layers of the fluid is proportional to :

i. area (A) of the layer in contact,

$$F \alpha A$$

ii. velocity gradient (dv/dx) in a direction perpendicular to the flow of liquid,

$$F \alpha \frac{dv}{dx}$$

On combining these, we can write

$$F \alpha A \frac{dv}{dx}$$

$$F = -\eta A \frac{dv}{dx} \qquad (10.16)$$

where η is a constant of proportionality and is called coefficient of viscosity. The negative sign indicates that force is frictional in nature and opposes motion.

The SI unit of coefficient of viscosity is Nsm^{-2}. In CGS system, the unit of viscosity is *poise*. 1 *poise* = 0.1 Nsm^{-2}. Dimensions of coefficient of viscosity are $[ML^{-1} T^{-1}]$.

10.8 TYPES OF LIQUID FLOW

Have you ever seen a river in floods? Is it similar to the flow of water in a city water supply system? If not, how are the two different? To discover answer to such questions, let as study the flow of liquids.

10.8.1 Streamline Motion

The path followed by fluid particles is called line of flow. If every particle passing through a given point of the path follows the same line of flow as that of preceding particles, the flow is said to be streamlined. A streamline can be represented as the curve or path whose tangent at any point gives the direction of the velocity of the fluid at that point. In steady flow, the streamlines coincide with the line of flow and these are shown in the Fig. 10.27.

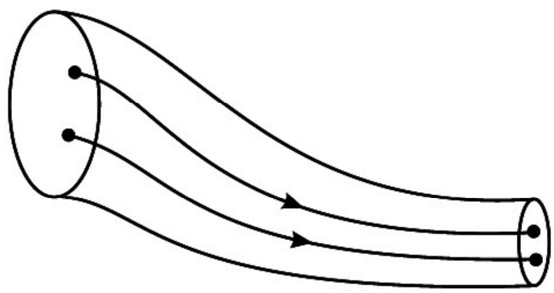


Fig. 10.27 : Streamline flow

Note that streamlines do not intersect each other because two tangents can then be drawn at the point of intersection giving two directions of velocities, which is not possible.

When the velocity of flow is less than the critical velocity (v_c) of a given liquid flowing through a tube, the motion is streamlined. In such a case, we can imagine the entire thickness of the stream of the liquid to be made up of a large number of plane layers (laminae) one sliding past the other, i.e. one flowing over the other. Such a flow is called laminar flow.

If the velocity of flow exceeds the critical velocity (v_c), the mixing of streamlines takes place and the flow path becomes zig-zag. Such a motion is said to be turbulent.

10.8.2 Equation of Continuity

If an incompressible, non-viscous fluid flows through a tube of non-uniform cross section, the product of the area of cross section and the fluid speed at any point in the tube is constant for a streamline flow.

Let A_1 and A_2 denote the areas of cross section of the tube where the fluid is entering and leaving, as shown in Fig. 10.28. If v_1 and v_2 are the speeds of the fluid at the ends A and B, respectively and ρ is the density of the fluid, then the liquid entering the tube at A covers a distance v_1 in one second. So, volume of the liquid entering per second is A_1v_1.

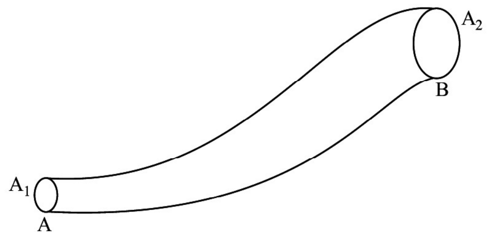


Fig. 10.28 : Liquid flowing through a tube

Therefore, mass of the liquid entering per second at point A = $A_1V_1\rho$

Similarly, mass of the liquid leaving per second at point B = $A_2V_2\rho$

Since there is no accumulation of fluid inside the tube, the mass of the liquid crossing any section of the tube must be same. Therefore, we have

$$A_1V_1\rho = A_2V_2\rho$$

$$A_1V_1 = A_2V_2 \qquad (10.17)$$

This expression is called equation of continuity.

10.8.3 Critical Velocity and Reynolds's Number

We now know that when the velocity of flow is less than a certain value, called critical velocity, the flow remains streamlined. But when the velocity of flow exceeds the critical velocity, the flow becomes turbulent. The value of the critical angle depends on the nature of the liquid and dimensions of the tube through which the liquid flows. Experiments show that the value of critical velocity (v_c) of any liquid is:

- directly proportional to the coefficient of viscosity (η) of the liquid,

$$v_c \ \alpha \ \eta$$

- inversely proportional to the density of the liquid (ρ),

$$v_c \ \alpha \frac{1}{\rho} \qquad \text{and}$$

- inversely proportional to the diameter of the tube (d) through which the liquid flows,

$$v_c \ \alpha \frac{1}{d}$$

Hence, we can write

$$v_c = \frac{R\eta}{\rho d} \qquad (10.18)$$

where R is constant of proportionality and is called Reynolds's Number. It has no dimensions. Experiments show that if R is below 1000, the flow is laminar. The flow becomes unsteady when R is between 1000 and 2000 and the flow becomes turbulent for R greater than 2000.

Example 10.3

The average speed of blood in the artery (d = 2.0 cm) during the resting part of heart's cycle is about 30 cm s^{-1}. Is the flow laminar or turbulent? Density of blood 1.05 gm-cm^{-3} and η = 4.0 × 10^{-2} poise.

Solution :

From Eqn. 10.18, we recall that Reynold's number $R = \frac{v_c \rho d}{\eta}$. On substituting the given values, we have

$$R = \frac{(30\,\text{cm}\,\text{s}^{-1}) \times (1.05\,\text{gm-cm}^{-3}) \times (2.0\text{ cm})}{4.0 \times 10^{-2}\text{ poise}} = 1575$$

Since 1575 < 2000, the flow is unsteady.

10.9 STOKE'S LAW

George Stokes gave an empirical law for the magnitude of the tangential backward viscous force F acting on a freely falling smooth spherical body of radius r in a highly viscous liquid of coefficient of viscosity η moving with velocity v. This is known as Stokes' law.

According to Stokes' law

$$F \propto \eta r v$$

$$F = K \eta r v \qquad (10.19)$$

Where K is constant of proportionality. It has been found experimentally that K = 6π. Hence Stokes' law can be written as :

$$F = 6 \pi \eta r v \qquad (10.20)$$

Stokes' Law can also be derived using the method of dimensions as follows

According to Stokes, the viscous force depends on :

- coefficient of viscosity (η) of the medium
- radius of the spherical body (r)
- velocity of the body (v)

Then

$$F \propto \eta^a r^b v^c$$

$$F = K\eta^a r^b v^c$$

where K is constant of proportionality. Taking dimensions on both the sides, we get

$$[MLT^{-2}] = [ML^{-1}\ T^{-1}]^a\ [L]^b\ [LT^{-1}]^c$$

$$[MLT^{-2}] = [M^a\ L^{-a+b+c}\ T^{-a-c}]$$

Comparing the exponents on both the sides of the above equation and solving we get $a = b = c = 1$. Hence,

$$F = K\ \eta\ r\ v$$

10.9.1 Terminal Velocity

Let us consider a spherical body of radius r and density ρ falling through a liquid of density σ. The forces acting on the body will be

i. viscous force F acting vertically upward

ii. weight of the body W acting downward

iii. buoyant force B acting upward

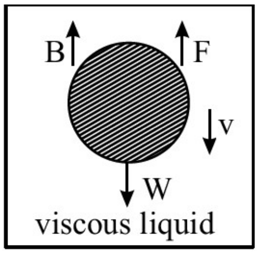


Fig. 10.29 : Force acting on a sphere falling in viscous fluid

Under the action of these forces, at some instant the net force on the body becomes zero, (since the viscous force increases with the increase of velocity). Then, the body falls with a constant velocity known as terminal velocity. We know that magnitude of these forces is :

$$F = 6\pi\ \eta\ r\ v_0$$

where v_0 is the terminal velocity.

$$W = \left(\frac{4}{3}\pi\ r^3\ \rho\right)g$$

And

$$B = \left(\frac{4}{3}\pi\ r^3\ \sigma\right)g$$

The net force is zero (i.e. total upward forces is equal to total downward forces), when object attains terminal velocity. Hence,

$$6\pi\eta r v_0 + \frac{4}{3}\pi r^3\ \sigma g = \frac{4}{3}\pi r^3\ \rho g$$

On rearranging the above equation, we can write it as

$$6\pi\eta r v_0 = \frac{4}{3}\pi r^3\ \rho g - \frac{4}{3}\pi r^3\ \sigma g$$

$$v_0 = \frac{2r^2\,(\rho - \sigma)\,g}{9\eta} \qquad (10.21)$$

10.9.2 Applications of Stokes' Law

(A) Parachute : When a soldier jumps from a flying aeroplane, he falls with acceleration due to gravity g but due to viscous drag in air, the acceleration goes on decreasing till he acquires terminal velocity. The soldier then descends with constant velocity and opens his parachute close to the ground at a precalculated moment, so that he may land safely near his destination.

(B) Velocity of rain drops : When raindrops fall under gravity, their motion is opposed by the viscous drag in air. When viscous force becomes equal to the force of gravity, the drop attains a terminal velocity. That is why rain drops reaching the earth do not have very high kinetic energy.

Example 10.4

Determine the radius of a drop of rain falling through air with terminal velocity 0.12 ms^{-1}. Given $\eta = 1.8 \times 10^{-5}$ kg m^{-1} s^{-1}, $\rho = 1.21$ kg m^{-3}, $\sigma = 1.0 \times 10^{3}$ kg m^{-3} and $g = 9.8$ ms^{-2}.

Solution :

We know that terminal velocity is given by

$$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

On rearranging terms, we can write,

$$r = \sqrt{\frac{9(1.8 \times 10^{-5}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1})(0.12\,\text{ms}^{-1})}{2(1.0 \times 10^{3}\,\text{kg}\,\text{m}^{-3} - 1.21\,\text{kg}\,\text{m}^{-3})(9.8\,\text{m}\,\text{s}^{-2})}} = 10^{-5}\,\text{m}$$

Intext Questions 10.4

1. Differentiate between streamline flow and turbulent flow?
2. Can two streamlines cross each other in a flowing liquid?
3. Name the physical quantities on which critical velocity of a viscous liquid depends.
4. Calculate the terminal velocity of a rain drop of radius 0.01 m if the coefficient of viscosity of air is 1.8×10^{-5} Ns m^{-2} and its density is 1.2 kg m^{-3}. Density of water is 1000 kg m^{-3}. Take g = 10 m s^{-2}.
5. When a liquid contained in a tumbler is stirred and placed for some time, it comes to rest, why?

Daniel Bernoulli (1700-1782)

Daniel Bernoulli, a Swiss Physicist and mathematician was born in a family of mathematicians on February 8, 1700. He made important contributions in hydrodynamics. His famous work, **Hydrodyanamica** was published in 1738. He also explained the behavior of gases with changing pressure and temperature, which led to the development of kinetic theory of gases.

He is known as the founder of mathematical physics. Bernoulli's principle is used to produce vacuum in chemical laboratories by connecting a vessel to a tube through which water is running rapidly.

10.10 BERNOULLI'S PRINCIPLE

Have you ever thought how air circulates in a dog's burrow, smoke comes quickly out of a chimney or why car's convertible top bulges upward at high speed? You must have definitely experienced the bulging upwards of your umbrella on a stormy – rainy day. All these can be understood on the basis of Bernoulli's principle.

Bernoulli's Principle states that where the velocity of a fluid is high, the pressure is low and where the velocity of the fluid is low, pressure is high.

10.10.1 Energy of a Flowing Fluid

Flowing fluids possess three types of energy. We are familiar with the kinetic and potential energies. The third type of energy possessed by the fluid is pressure energy. It is due to the pressure of the fluid. The pressure energy can be taken as the product of pressure difference and its volume. If an element of liquid of mass m, and density d is moving under a pressure difference P, then

$$\text{Pressure energy} = p \times \left(\frac{m}{d}\right) \text{joule} \qquad (10.22)$$

$$\text{Pressure energy per unit mass } = \frac{p}{d} \text{ J kg}^{-1} \qquad (10.23)$$

10.10.2 Bernoulli's Equation

Bernoulli developed an equation that expresses this principle quantitatively. The following three important assumptions were made to develop this equation.

1) The fluid is incompressible, i.e. its density does not change when it passes from a wide bore tube to a narrow bore tube.
2) The fluid is non-viscous or the effect of viscosity is not to be taken into account.
3) The motion of the fluid is streamlined.

Consider an incompressible, non-viscous and streamlined flow of a fluid through a tube of varying cross section shown in the Fig. 10.30. Suppose at point A the pressure is P_1, area of cross section A_1, velocity of flow v_1, height above the ground h_1 and at B, the pressure is P_2, area of cross-section A_2 velocity of flow v_2, and height above the ground h_2.

Since points A and B can be any two points along a tube of flow, we write Bernoulli's equation

$$P+\frac{1}{2}dv^2+hdg=\text{constant} \qquad (10.24)$$

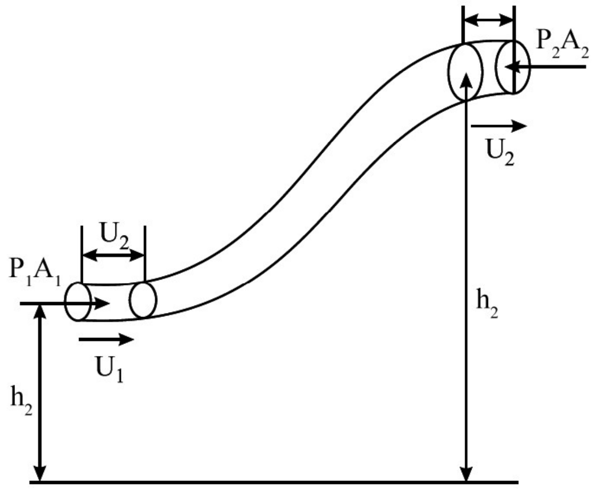


Fig. 10.30 : Streamlined flow of a fluid through a tube of varying cross section

That is, the sum of pressure energy, kinetic energy and potential energy of a fluid remains constant in streamline motion.

Activity 10.4

1. Take a sheet of paper in your hand.
2. Press down lightly on horizontal part of the paper as shown in Fig. 10.31 so that the paper curves down.
3. Blow on the paper along the horizontal line.

Watch the paper. It lifts up because speed increases and pressure on the upper side of the paper decreases.

Fig. 10.31

10.10.3 Applications of Bernoulli's Theorem

Bernoulli's theorem finds many applications in our lives. Some commonly observed phenomena can also be explained on the basis of Bernoulli's theorem.

(A) Flow meter or Venturi meter : It is a device used to measure the rate of flow of liquids through pipes. The device is inserted in the flow pipe, as shown in the Fig. 10.32.

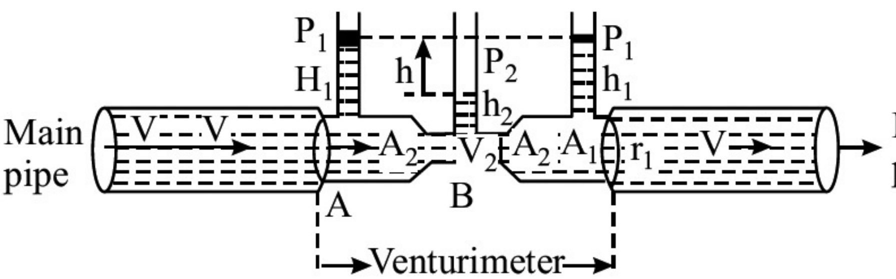


Fig. 10.32 : Venturi meter

It consists of a manometer, whose two limbs are connected to a tube having two different cross-sectional areas say A_1 and A_2 at A and B, respectively. Suppose the main pipe is horizontal at a height h above the ground, then applying Bernoulli's theorem for the steady flow of liquid through the Venturi meter at A and B, we can write

Total energy at A = Total energy at B

$$\frac{1}{2}mv_1^2 + mgh + \frac{mP_1}{d} = \frac{1}{2}mv_2^2 + mgh + \frac{mP_2}{d}$$

On rearranging terms, we can write,

$$P_1 - P_2 = \frac{d}{2}(v_2^2 - v_1^2) = \frac{d}{2}v_1^2\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right] \qquad (10.25)$$

It shows that points of higher velocities are the points of lower pressure (because of the sum of pressure energy and K.E. remain constant). This is called Venturi's Principle.

For steady flow through the Venturi meter, volume of liquid entering per second at A = volume of liquid leaving per second at B. Therefore

$$A_1v_1 = A_2v_2 \qquad (10.26)$$

The liquid is assumed incompressible i.e., velocity is more at narrow ends and vice versa. Using this result in Eqn. 10.25, we conclude that pressure is lesser at the narrow ends.

$$P_1 - P_2 = \frac{d}{2}v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{d\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}} \qquad (10.27)$$

If h denotes level difference between the two limbs of the Venturi meter, then

$$P_1 - P_2 = hdg$$

And

$$v_1 = \sqrt{\frac{2hdg}{d\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}} = \sqrt{\frac{2hg}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

From this we note that $v_1 \alpha \sqrt{h}$, since all other parameters are constant for a given Venturi meter. Thus

$$v_1 = K\sqrt{h}$$

where K is constant.

The volume of liquid flowing per second is given by

$$V = A_1 v_1 = A_1 \times Kh$$

or

$$V = K' h \qquad (10.28)$$

Where $K' = kA_1$ is another constant.

Bernoulli's principle has many applications in the design of many useful appliances like Atomizer, Spray gun, Bunsen burner, Carburettor, Aerofoil, etc.

(B) Speed of Efflux of a liquid: Torricelli's Law : Torricelli's law has in many applications in our daily life. This law describes the relationship between the speed of efflux of a liquid and its height in the container. The word efflux means outward flow from a fluid.

Consider a tank containing a liquid of density ρ has a hole in its side at a height h_1 (say point 1) and the liquid level at a height h_2 (say point 2) from its bottom as shown in the Fig. 10.33. The side hole is open to the atmosphere and its diameter is much smaller than the diameter of the tank. The air above the liquid is at a pressure P. Suppose A_1 is cross-sectional area of the side hole and A_2 is the cross – sectional area of the tank, from the equation of continuity, we have

$$A_1 v_1 = A_2 v_2$$

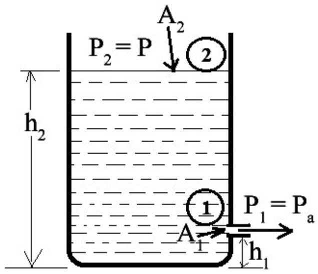


Fig. 10.33 : A tank contained liquid and a side hole at a height h_1 and liquid level at a height h_2 from its bottom

Where v_1 is the speed of the efflux from the side hole of the tank and v_2 is the speed of the decrease in the liquid level of the tank. As A_2 is much greater than A_1, $v_1 >> v_2$. This means that the decrease in the speed of the liquid level in the tank is very small and can be neglected i.e. the liquid level in the tank can be assumed to be at rest ($v_2 = 0$).

Now, applying the Bernoulli's equation [Eqn. 10.24] at points 1 and 2 we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \qquad (10.29)$$

As the side hole is free to air, the pressure at point 1 is equal to the atmospheric pressure i.e. $P_1 = P_a$ and due to liquid in the tank, the pressure at point 2 i.e. container pressure, $P_2 = P$ (= ρgh). Substituting the above values in Eqn. 10.29, we have

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P + \rho g h_2$$

$$P - P_a = \frac{1}{2}\rho v_1^2 + \rho g (h_2 - h_1)$$

Here say $(h_2 - h_1) = h$, we have

$$P - P_a = \frac{1}{2}\rho v_1^2 + \rho g h$$

$$v_1^2 = 2gh + \frac{2(P - P_a)}{\rho}$$

$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}} \tag{10.30}$$

In general, $P >> P_a$ and 2gh may be ignored, then,

$$v_1 = \sqrt{\frac{2P}{\rho}} \tag{10.31}$$

Hence, the speed of efflux is determined by the container pressure only and we know the container pressure $P = \rho g h$. Then from the Eqn. 10.31, we have

$$v_1 = \sqrt{\frac{2\rho g h}{\rho}} = \sqrt{2gh}$$

$$v_1 = \sqrt{2gh} \tag{10.32}$$

In other words, for an open tank, the speed of efflux through a hole at a depth h from the liquid level is equal to that acquired by an object falling freely through a vertical distance h. The Eqn. 10.32 represents Torricelli's law.

(C) Atomizer : An atomizer is shown in Fig. 10.34. When the rubber bulb A is squeezed, air blows through the tube B and comes out of the narrow orifice with larger velocity creating a region of low pressure in its neighbourhood. The liquid (scent or paint) from the vessel is, therefore, sucked into the tube to come out to the nozzles N. As the liquid reaches the nozzle, the air stream from the tube B blows it into a fine spray.

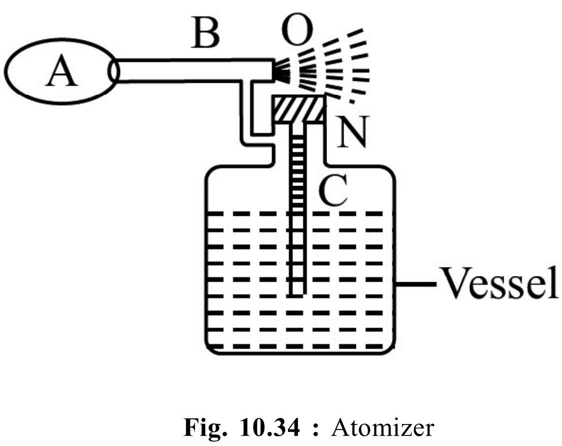


Fig. 10.34 : Atomizer

(D) Spray gun : When the piston is moved in, it blows the air out of the narrow hole O with large velocity creating a region of low pressure in its neighbourhood. The liquid (e.g. insecticide) is sucked through the narrow tube attached to the vessel end having its opening just below O. The liquid on reaching the end gets sprayed by out blown air from the piston Fig. 10.35.

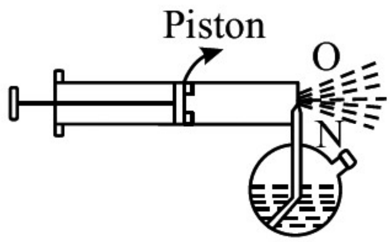


Fig. 10.35 : Spray gun

(E) Bunsen Burner : When the gas emerges out of the nozzle N, its velocity being high the pressure becomes low in its vicinity. The air, therefore, rushed in through the side hole A and gets mixed with the gas. The mixture then burns at the mouth when ignited, to give a hot blue flame Fig.10.36.

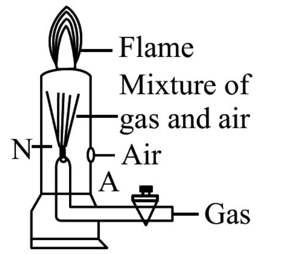


Fig. 10.36 : Bunsen Burner

(F) Carburettor : The carburettor shown in Fig. 10.37. is a device used in motor cars for supplying a proper mixture of air and petrol vapours to the cylinder of the engine. The energy is supplied by the explosion of this mixture inside the cylinders of the engine. Petrol is contained in the float chamber. There is a decrease in the pressure on the side A due to motion of the piston. This causes the air from outside to be sucked in with large velocity. This causes a low pressure near the nozzle B (due to constriction, velocity of air sucked is more near B) and, therefore, petrol comes out of the nozzle B which gets mixed with the incoming air. The mixture of vaporized petrol and air forming the fuel then enters the cylinder through the tube A.

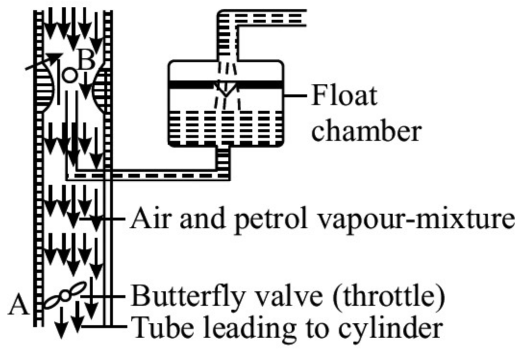


Fig. 10.37 : Carburettor

Sometimes when the nozzle B gets choked due to deposition of carbon or some impurities, it checks the flow of petrol and the engine not getting fuel, stops working. The nozzle has therefore, to be opened and cleaned.

(G) Aerofoil : When a solid moves in air, streamlines are formed. The shape of the body of the aeroplane is designed specially as shown in the Fig. 10.38. When the aeroplane runs on its runway, high velocity streamlines of air are formed. Due to crowding of more streamlines on the upper side, it becomes a region

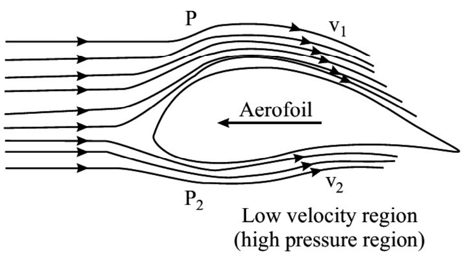


Fig. 10.38 : Crowding of streamlines on the upper side

of more velocity and hence of comparatively low-pressure region than below it. This pressure difference gives the lift to the aeroplane and this is known as the dynamic lift. Dynamic lift is the force that acts on a body by virtue of its motion through a fluid.

Based on this very principle, i.e. the regions of high velocities due to crowding of streamlines are the regions of low pressure, following are interesting demonstrations.

10.10.4 Some of the demonstrations of Bernoulli's Theorem

(A) Attracted disc paradox : When air is blown through a narrow tube handle into the space between two cardboard sheets Fig. 10.39 placed one above the other and the upper disc is lifted with the handle, the lower disc is attracted to stick to the upper disc and is lifted with it. This is called attracted disc paradox.

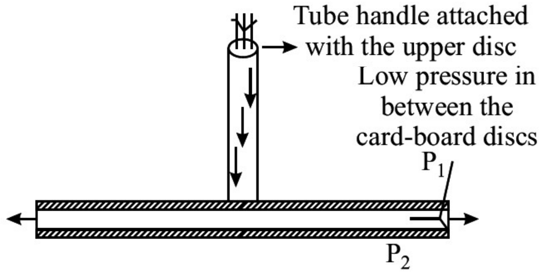


Fig. 10.39 : Attracted disc paradox

(B) Dancing of a ping pong ball on a jet of water : If a light hollow spherical ball (ping-pong ball or table tennis ball) is gently put on a vertical stream of water coming out of a vertically upward directed jet end of a tube, it keeps on dancing this way and that way without falling to the ground Fig. 10.40. When the ball shifts to the lefts, then most of the jet streams pass by its right side thereby creating a region of high velocity and hence low pressure on its right side in comparison to that on the left side and the ball is again pushed back to the center of the jet stream.

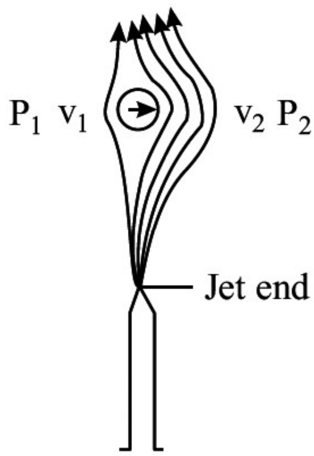


Fig. 10.40 : Dancing Ping-Pong ball

(C) Water vacuum pump or aspirator or filter pump : Fig. 10.41 shows a filter pump used for producing moderately low pressures. Water from the tap is allowed to come out of the narrow jet end of the tube A. Due to small aperture of the nozzle, the velocity becomes high and hence a low-pressure region is created around the nozzle N. Air is, therefore, sucked from the vessel to be evacuated through the tube B and gets mixed with the steam of water and goes out through the outlet. After a few minutes., the pressure of air in the vessel is decreased to about 1 cm of mercury by such a pump.

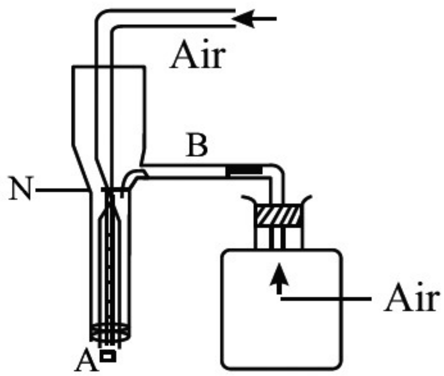


Fig. 10.41 : Filter Pump

(D) Swing of a cricket ball : When a cricketer throws a spinning ball, it moves along a curved path in the air. This is called swing of the ball. It is clear from Fig. 10.42. That when a ball is moved forward, the air occupies the space left by the ball with a velocity v (say). When the ball spins, the layer of air around it also moves with the ball, say with the velocity u. So, the resultant velocity of air above the ball becomes (v – u) and below the ball becomes (v + u). Hence, the pressure difference above and below the ball moves the ball in a curved path. The difference in air velocities results in a pressure disparity between above and below the ball, creating a net upward force on it. This phenomenon is known as the **Magnus effect**, i.e. the dynamic lift associated with a spinning object is called Magnus effect.

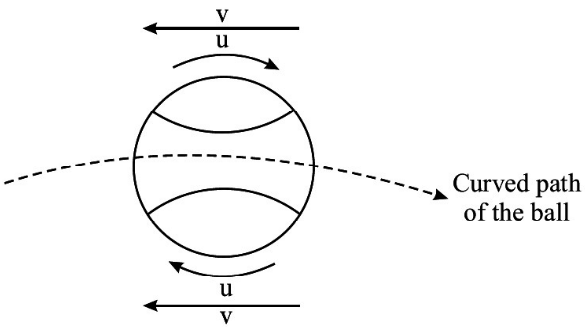


Fig. 10.42 : Swing of a cricket ball

Example 10.5

Water flows out of a small hole in the wall of a large tank near its bottom (Fig. 10.43). What is the speed of efflux of water when the height of water level in the tank is 2.5m?

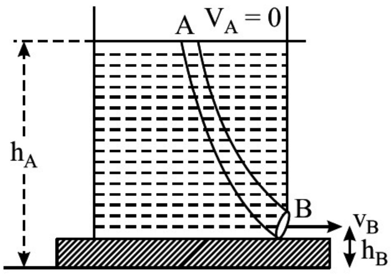


Fig. 10.43

Solution :

Let B be the hole near the bottom. Imagine a tube of flow A to B for the water to flow from the surface point A to the hole B. We can apply the Bernoulli's theorem to the points A and B for the streamline flow of small mass m.

Applying the Bernoulli's theorem to the points A and B, we have

Total energy at B = Total energy at A

$$P_B + \frac{1}{2}dv_B^2 + h_Bdg = P_A + \frac{1}{2}dv_A^2 + h_Adg$$

At A, $v_A = 0$, $P_A = P$ = atmospheric pressure, h_A = height from the ground

At B, $v_B = ?$, $P_B = P$, h_B = height of the hole above the ground

Let $h_A - h_B = H$ = height of the water level in the vessel = 2.5 m and

d = density of the water

Substituting the values we get,

$$\frac{1}{2}m v_B^2 = mg(h_A - h_B)$$

$$v_B = \sqrt{2g(h_A - h_B)}$$

$$v_B = \sqrt{2 \times 9.8 \times 2.5} = 7ms^{-1}$$

Intext Questions 10.5

1. The windstorm often blows off the tin roof of the houses. How does Bernoulli's equation explain the phenomenon?
2. When you press the mouth of a water pipe used for watering the plants, water goes to a longer distance, why?
3. What are the conditions necessary for the application of Bernoulli's theorem to solve the problems of flowing liquid?
4. Water flows along a horizontal pipe having non-uniform cross section. The pressure is 20 mm of mercury where the velocity is 0.20 m/s. Find the pressure at a point where the velocity is 1.50 m/s?
5. Why do bowlers in a cricket match shine only one side of the ball?

WHAT YOU HAVE LEARNT

- Hydrostatic pressure P at a depth h below the free surface of a liquid of density ρ is given by P = ρgh.
- The upward force acting on an object submerged in a fluid is known as buoyant force.
- According to Pascal's law, when pressure is applied to any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.
- The liquid molecules in the liquid surface have potential energy called surface energy.
- The surface tension of a liquid may be defined as force per unit length acting on an imaginary line drawn in the surface. It is measured in Nm^{-1}.
- Surface tension may also define as the surface energy per unit area.
- Surface tension of any liquid is the property by virtue of which a liquid surface acts like a stretched membrane.
- Angle of contact is defined as the angle between the tangent to the liquid surface and the wall of the container at the point of contact as measured from within the liquid.
- The liquid surface in a capillary tube is either concave or convex. This curvature is due to surface tension. The rise in capillary is given by

$$h = \frac{2T\cos\theta}{rdg}$$

- The excess pressure P on the concave side of the liquid surface is given by,

  $P = \frac{2T}{r}$, where T is surface tension of the liquid

  $P = \frac{2T}{r}$, for air bubble in the liquid and

  $P = \frac{4T}{r}$, where T is surface tension of soap solution, for soap bubble in air

- Detergents are considered better cleaner of clothes because they reduce the surface tension of water-oil.
- The property of a fluid by virtue of which it opposes the relative motion between its adjacent layers is known as viscosity.
- The flow of liquid becomes turbulent when the velocity is greater than a certain value called critical velocity (v_c) which depends upon the nature of the liquid and the diameter of the tube i.e. viscosity (η), Pressure (P) and diameter of the tube (d).
- Coefficient of viscosity of any liquid may be defined as the magnitude of tangential backward viscus force acting between two successive layers of unit area in contact with each other moving in a region of unit velocity gradient.
- Stokes' law states that tangential backward viscous force acting on a spherical mass of radius r falling with velocity v in a liquid of coefficient of viscosity η is given by $F = 6 \pi \eta r v$
- Bernoulli's theorem states that the total energy of an element of mass (m) of an incompressible liquid moving steadily, remains constant throughout the motion. Mathematically, Bernoulli's equation as applied to any two points A and B of tube of flow

$$P_A + \frac{1}{2} dv_A^2 + h_A dg = P_B + \frac{1}{2} dv_B^2 + h_B dg$$

  Where d is the density of the liquid.

TERMINAL EXERCISE

1. Derive an expression for hydrostatic pressure due to a liquid column.
2. State Pascal's law. Explain the working of hydraulic press.
3. Define surface tension. Find its dimensional formula.
4. Describe an experiment to show that liquid surfaces behave like a stretched membrane.
5. The hydrostatic pressure due to a liquid filled in a vessel at a depth 0.9 m is 3.0 Nm^{-2}. What will be the hydrostatic pressure at a hole in the side wall of the same vessel at a depth of 0.8 m.

6. In a hydraulic lift, how much weight is needed to lift a heavy stone of mass 1000 kg? Given the ratio of the areas of cross section of the two pistons is 5. Is the work output greater than the work input? Explain.

7. A liquid filled in a capillary tube has convex meniscus. If F_a is force of adhesion, F_c is force of cohesion and θ is the angle of contact, which of the following relations should hold good?

   (a) $F_a > F_c \sin\theta$; (b) $F_a < F_c \sin\theta$; (c) $F_a \cos\theta = F_c$; (d) $F_a \sin\theta > F_c$

8. 1000 drops of water of same radius coalesce to form a larger drop. What happens to the temperature of the water drop? Why?

9. What is capillary action? What are the factors on which the rise or fall of a liquid in a capillary tube depends?

10. Calculate the approximate rise of a liquid of density 10^3 kg m^{-3} in a capillary tube of length 0.05 m and radius 0.2×10^{-3} m. Given surface tension of the liquid for the material of that capillary is 7.27×10^{-2} Nm^{-1}.

11. Why is it difficult to blow water bubbles in air while it is easier to blow soap bubble in air?

12. Why the detergents have replaced soaps to clean oily clothes.

13. Two identical spherical balloons have been inflated with air to different sizes and connected with the help of a thin pipe. What do you expect out of the following observations?

    (i) The air from smaller balloon will rush into the bigger balloon till whole of its air flows into the later.

    (ii) The air from the bigger balloon will rush into the smaller balloon till the sizes of the two become equal.

    What will be your answer if the balloons are replaced by two soap bubbles of different sizes.

14. Which process involves more pressure to blow an air bubble of radius 3 cm inside a soap solution or a soap bubble in air? Why?

15. Differentiate between laminar flow and turbulent flow and hence define critical velocity.

16. Define viscosity and coefficient of viscosity. Derive the units and dimensional formula of coefficient of viscosity. Which is more viscous: water or glycerine? Why?

17. What is Reynold's number? What is its significance? Define critical velocity on the basis of Reynold's number.

18. State Bernoulli's principle. Explain its application in the design of the body of an aeroplane.

19. Explain Why:

    (i) A spinning tennis ball curves during the flight?

    (ii) A ping pong ball keeps on dancing on a jet of water without falling on to either side?

(iii) The velocity of flow increases when the aperture of water pipe is decreased by squeezing its opening.

(iv) A small spherical ball falling in a viscous fluid attains a constant velocity after some time.

(v) If mercury is poured on a flat glass plate, it breaks up into small spherical droplets.

20. Calculate the terminal velocity of an air bubble with in diameter which rises in a liquid of viscosity 0.15 kg m^{-1} s^{-1} and density 0.9 gm cm^{-3}. What will be the terminal velocity of the same bubble while rising in water? For water $\eta = 10^{-2}$ kg m^{-1} s^{-1}.

21. A pipe line 0.2 m in diameter, flowing full of water has a constriction of diameter 0.1 m. If the velocity in the 0.2 m pipe-line is 2 m s^{-1}, calculate (i) the velocity in the constriction, and (ii) the discharge rate in cubic meters per second.

22. (i) At what velocity does a steel ball with a radius of 1 mm fall in a tank of glycerine when its acceleration is half that of a freely falling body

(ii) What is the terminal velocity of the ball? The density of steel and of glycerine are 8.5 gm cm^{-3} and 1.32 gm cm^{-3} respectively, viscosity of glycerine is 8.3 poise.

23. Water at 20°C flows with a speed of 50 cm s^{-1} through a pipe of diameter of 3 mm.

(i) What is Reynold's number? (ii) What is the nature of flow?

Given, viscosity of water at 20°C as 1.005×10^{-2} poise and Density of water at 20°C as 1 gm cm^{-3}.

24. Modern aeroplane design calls for a lift of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the velocity of flow past the lower wing surface is 100 ms^{-1}, what is the required velocity over the upper surface to give a desired lift of 1000 Nm^{-2}? The density of air is 1.3 kg m^{-3}.

25. Water flows horizontally through a pipe of varying cross-section. If the pressure of water equals 5 cm of mercury at a point where the velocity of flow is 28 cm s^{-1}, then what is the pressure at another point, where the velocity of flow is 70 cm s^{-1}? [Tube density of water 1 gm cm^{-3}].

ANSWERS TO INTEXT QUESTIONS

10.1

1. Because then the weight of the person applies on a larger area hence pressure on snow decreases.

2. $P = P_a + \rho gh$

   $P = 1.5 \times 10^7$ Pa

3. Pressure applied by the weight of the boy $= \dfrac{25}{0.05} = 500\,\text{Nm}^{-2}$.

Pressure due to the weight of the elephant $= \dfrac{5000}{10} = 500\,\text{Nm}^{-2}$.

$\therefore$ The boy can balance the elephant.

4. Because of the larger area of the rod, pressure on the skin is small.

5. $\dfrac{50}{0.1} = \dfrac{w}{10}$, w = 5000 kg wt

10.2

1. The force of attraction between molecules of the same substance is called the force of cohesion, while the force of attraction between molecules of different substances is called the force of adhesion.
2. Surface tension leads to the minimum surface area and for a given volume, sphere has minimum surface area.
3. No, they have tightly bound molecules.
4. Due to surface tension forces.
5. For air bubble in water.

$$P = \frac{2T}{r} = \frac{2 \times 727 \times 10^{-3}}{2 \times 10^{-2}} = 72.7\,\text{Nm}^{-2}$$

For soap bubble in air,

$$P' = \frac{4T'}{r'} = \frac{4 \times 25 \times 10^{-3}}{4 \times 10^{-2}} = 2.5\,\text{Nm}^{-2}$$

10.3

1. No.
2. Yes, the liquid will rise.
3. Mercury has a convex meniscus and the angle of contact is obtuse. The fall in the level of mercury in capillary makes it difficult to enter.
4. $r = \dfrac{2T}{h\rho g} = \dfrac{2 \times 7.2 \times 10^{-2}}{3 \times 1000 \times 10} = 4.8 \times 10^{-6}\,\text{m}$.
5. Due to capillary action.

10.4

1. If every particle passing through a given point of path follows the same line of flow as that of preceding particle the flow is stream lined, if it is zig-zag, the flow is turbulent.
2. No, otherwise the same flow will have two directions.
3. Critical velocity depends upon the viscous nature of the liquid, the diameter of the tube and density of the liquid.
4. 0.012 ms^{-1}.
5. Due to viscous force.

10.5

1. High velocity of air creates low pressure on the upper part.
2. Decreasing in the area creates large pressure.
3. The fluid should be incompressible and non-viscous or (very less). The motion should be streamlined.
4. $P_1 - P_2 = \frac{1}{2} d (v_2^2 - v_1^2)$
5. So that the stream lines with the two surfaces are different. More swing in the ball will be obtained.

ANSWERS TO TERMINAL EXERCISE

5. 2.67 Nm^{-2}.
6. 200 N, No.
20. 2.1 mm s^{-1}, 35 cm s^{-1}.
21. 8 ms^{-1}, 6.3×10^{-2} m^3 s^{-1}.
22. 7.8 mm s^{-1}, 0.19 ms^{-1}.
23. 1500, Unsteady.
24. 2 cm of mercury.

11 THERMAL PROPERTIES OF MATTER

INTRODUCTION

The energy from the sun is responsible for life on our beautiful planet. Before reaching the earth, it passes through vacuum as well as material medium between the earth and the sun. Do you know that each one of us also radiates energy at the rate of nearly 70 watt? Here we will study the radiation in detail. This study enables us to determine the temperatures of stars even though they are very far away from us. Another process of heat transfer is conduction, which requires the presence of a material medium. When one end of a metal rod is heated, its other end also becomes hot after some time. That is why we use handles of wood or similar other bad conductor of heat in various appliances. Heat energy falling on the walls of our homes also enters inside through conduction. But when you heat water in a pot, water molecules near the bottom get the heat first. They move from the bottom of the pot to the water surface and carry heat energy. This mode of heat transfer is called convection. These processes are responsible for various natural phenomena, like monsoon which are crucial for existence of life on the globe. You will learn more about these processes of heat transfer in this unit.

OBJECTIVES

After studying this lesson, you should be able to

- *distinguish between conduction, convection and radiation;*
- *define the coefficient of thermal conductivity;*
- *describe greenhouse effect and its consequences for life on earth;*
- *apply laws governing black body radiation.*

11.1 PROCESSES OF HEAT TRANSFER

The second law postulates that the natural tendency of heat is to flow spontaneously from a body at higher temperature to a body at lower temperature. The transfer of heat continues until the temperatures of the two bodies become equal. From kinetic theory, we know that temperature of a gas is related to its average kinetic energy. It means that molecules of a gas at different temperatures have different average kinetic energies. There are three processes by which transfer of heat takes place. These are: conduction, convection and radiation. In conduction and convection, heat transfer takes place through molecular motion. Let us understand how this happens. Heat transfer through conduction is more common in solids. We know that atoms in solids are tightly bound. When heated, they cannot leave their sites; they are constrained to vibrate about their respective equilibrium positions. Let

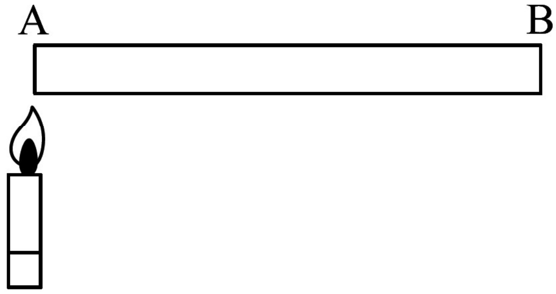


Fig. 11.1 : Head conduction in a metal rod

us understand as to what happens to their motion when we heat a metal rod at one end (Fig.11.1). The atoms near the end A become hot and their kinetic energy increases. They vibrate about their mean positions with increased kinetic energy and being in contact with their nearest neighboring atoms, pass on some of their kinetic energy (K.E.) to them. These atoms further transfer some K.E. to their neighbors and so on. This process continues and kinetic energy is transferred to atoms at the other end B of the rod. As average kinetic energy is proportional to temperature, the end B gets hot. Thus, heat is transferred from atom to atom by conduction. In this process, the atoms do not bodily move but simply vibrate about their mean equilibrium positions and pass energy from one to another. In convection, molecules of fluids receive thermal energy and move up bodily. To see this, take some water in a flask and put some grains of potassium permanganate ($KMnO_4$) at its bottom. Put a Bunsen flame under the flask. As the fluid near the bottom gets heated, it expands. The density of water decreases and the buoyant force causes it to move upward (Fig.11.2). The space occupied by hot water is taken by the cooler and denser water, which moves downwards. Thus, a convection current of hotter water going up and cooler water coming down is set up. The water gradually heats up. These convection currents can be seen as $KMnO_4$ colors them red. In radiation, heat energy moves in the form of waves. These waves can pass through vacuum and do not require the presence of any material medium for their propagation. Heat from the sun comes to us mostly by radiation.

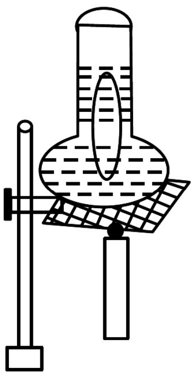


Fig. 11.2 : Convection currents are formed in water when heated

11.1.1 Conduction

Consider a rectangular slab of area of cross-section A and thickness d. Its two faces are maintained at temperatures T_h and T_c ($< T_h$), as shown in Fig. 11.3. Let us consider all the factors on which the quantity of heat Q transferred from one face to another depends. We can intuitively feel that larger the area A, the greater will be the heat transferred ($Q \alpha A$). Also, greater the thickness, lesser will be the heat transfer ($Q \alpha 1/d$). Heat transfer will be more if the temperature difference between the faces, ($T_h - T_c$), is large. Finally longer the time t allowed for heat transfer, greater will be the value of Q. Mathematically, we can write

$$Q \alpha \frac{A(T_h - T_c)t}{d}$$

$$Q = \frac{KA(T_h - T_c)t}{d} \qquad (11.1)$$

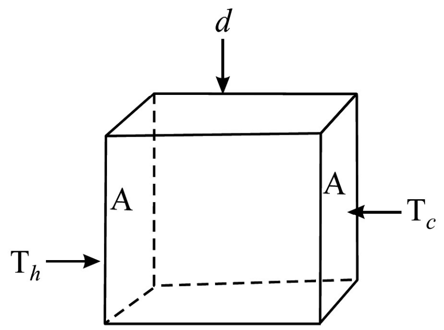


Fig. 11.3 : Heat conduction through a slab of thickness d and surfaces area A, when the faces are kept at temperatures T_h and T_c.

where K is a constant which depends on the nature of the material of the slab. It is called the coefficient of thermal conductivity, or simply, thermal conductivity of the material. Thermal conductivity of a material is defined as the amount of heat transferred in one second across a piece of the material having area of cross-section 1 m^2 and edge 1m when its opposite faces are maintained at a temperature difference of 1 K. The SI unit of thermal conductivity is $Wm^{-1}K^{-1}$. The value of K for some materials is given in Table 11.1

Table - 11.1 : Thermal Conductivity of some materials

Material	**Thermal conductivity ($Wm^{-1}K^{-1}$)**
Copper	400
Aluminium	240
Concrete	1.2
Glass	0.8
Water	0.6
Air	0.025
Thermocol	0.01

Example 11.1

A cubical thermocol box, full of ice has side 30 cm and thickness of 5 cm. If outside temperature is 45°C, estimate the amount of ice melted in 6 h. (K for thermocol is 0.01 $Js^{-1}m^{-1}{}^\circ C^{-1}$ and latent heat of fusion of ice is 335 Jg^{-1}.

Solution :

The quantity of heat transferred into the box through its one face can be obtained using

Eqn. (11.1) $$Q = \frac{KA(T_h - T_c)t}{d}$$

$K = 0.01\ Js^{-1}m^{-1}{}^\circ C^{-1}$

$T_h - T_c = 45^\circ C - 0^\circ C = 45^\circ C$

$t = 6 \text{ hours} = 6 \times 60 \times 60 \text{ sec}$

$A = 30 \text{ cm} \times 30 \text{ cm} = 900 \times 10^{-4}\ m^2$

$d = 5 \text{ cm} = 5 \times 10^{-2}\ m$

$L = 335\ J\ g^{-1}$

$$Q = \frac{0.01 \times 900 \times 10^{-4} \times 45 \times 6 \times 60 \times 60}{5 \times 10^{-2}}$$

$Q = 10496\ J$

Since the box has six faces, total heat passing into the box

$Q = 10496 \times 6\ J$

The mass of ice melted m, can be obtained by dividing Q by L

$$m = \frac{Q}{L} = \frac{10496 \times 6}{335} = 1878\,g$$

$$m = 1.878 \text{ kg}$$

We can see from Table 11.1 that metals such as copper and aluminium have high thermal conductivity. This implies that heat flows with more ease through copper. This is the reason why cooking vessels and heating pots are made of copper. On the other hand, air and thermocol have very low thermal conductivities. Substances having low value of K are sometimes called thermal insulators. We wear woolen clothes during winter because air trapped in wool fibers prevents heat loss from our body. Wool is a good thermal insulator because air is trapped between its fibers. The trapped heat gives us a feeling of warmth. Even if a few cotton clothes are put on one above another, the air trapped in-between layers stops cold. In the summer days, to protect a slab of ice from melting, we put it in an ice box made of thermocol. Sometimes we wrap the ice slab in jute bag, which also has low thermal conductivity.

11.1.2 Convection

It is common experience that while walking by the side of a lake or a sea shore on a hot day, we feel a cool breeze. Do you know the reason? Due to continuous evaporation of water from the surface of lake or sea, the temperature of water falls. Warm air from the shore rises and moves upwards (Fig.11.4). This creates low pressure area on the shore and causes cooler air from water surface to move to the shore. The net effect of these convection currents is the transfer of heat from the shore, which is hotter, to water, which is cooler. The rate of heat transfer depends on many factors. There is no simple equation for convection as for conduction. However, the rate of heat transfer by convection depends on the temperature difference between the surfaces and also on their areas.

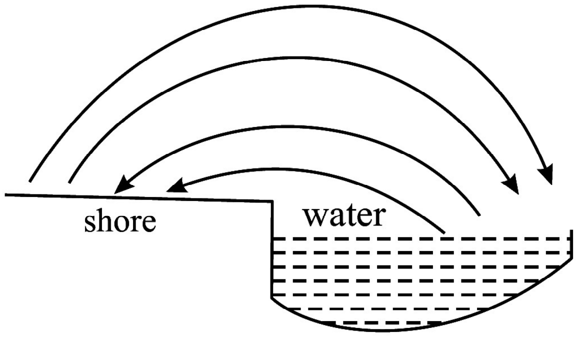


Fig. 11.4 : Convection currents. Hot air from the shore rises and moves towards cooler water. The convection current from water to the shores is experienced as cool breeze.

Intext Questions 11.1

1. Distinguish between conduction and convection.
2. Verify that the units of K are $Js^{-1}\ m^{-1}\ °C^{-1}$.
3. Explain why do humans wrap themselves in woolens in winter season?
4. A cubical slab of surface area 1 m^2, thickness 1 m, and made of a material of thermal conductivity K. The opposite faces of the slab are maintained at 1°C temperature

difference. Compute the energy transferred across the surface in one second, and hence give a numerical definition of K.

5. During the summer, the land mass gets very hot. But the air over the ocean does not get as hot. This results in the onset of sea breezes. Explain.

11.1.3 Radiation

Radiation refers to continuous emission of energy from the surface of a body. This energy is called radiant energy and is in the form of electromagnetic waves. These waves travel with the velocity of light (3×10^8 m s^{-1}) and can travel through vacuum as well as through air. They can easily be reflected from polished surfaces and focused using a lens.

All bodies emit radiation with wavelengths that are characteristic of their temperature. The sun, at 6000 K emits energy mainly in the visible spectrum. The earth at an ideal radiation temperature of 295 K radiates energy mainly in the far infra-red (thermal) region of electromagnetic spectrum. The human body also radiates energy in the infra-red region.

Let us now perform a simple experiment. Take a piece of blackened platinum wire in a dark room. Pass an electrical current through it. You will note that the wire has become hot. Gradually increase the magnitude of the current. After sometime, the wire will begin to radiate. When you pass a slightly stronger current, the wire will begin to glow with dull red light. This shows that the wire is just emitting red radiation of sufficient intensity to affect the human eye. This takes place at nearly 525°C. With further increase in temperature, the color of the emitted radiation will change from dull red to cherry red (at nearly 900°C) to orange (at nearly 1100°C), to yellow (at nearly 1250°C) until at about 1600°C, it becomes white. What do you infer from this? It shows that the temperature of a luminous body can be estimated from its color. Secondly, with increase in temperature, waves of shorter wavelengths (since red light is of longer wavelength than orange. yellow etc.) are also emitted with sufficient intensity. Considering in reverse order, you may argue that when the temperature of the wire is below 525°C, it emits waves longer than red but these waves can be detected only by their heating effect.

11.2 RADIATION LAWS

At any temperature, the radiant energy emitted by a body is a mixture of waves of different wavelengths. The most intense of these waves will have a particular wavelength (say λ_m). At 400°C, the λ_m will be about 5×10^{-4} cm or 5 μm (1 micron (μ) = 10^{-6}m) for a copper block. The intensity decreases for wavelengths either greater or less than this value (Fig. 11.5). Evidently area between each curve and the horizontal axis represents the total rate of radiation at that temperature. You may study the curves shown in Fig. 11.5 and verify the following two facts. 1) The rate of radiation at

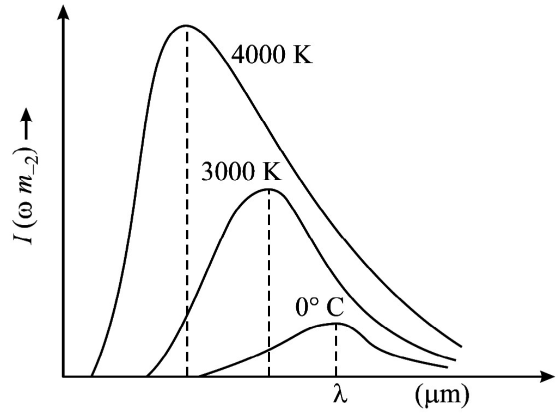


Fig. 11.5 : Variation in intensity with wavelength for a black body at different temperatures

a particular temperature (represented by the area between each curve and the horizontal axis) increases rapidly with temperature. 2) Each curve has a definite energy maximum and a corresponding wavelength λ_m (i.e. wavelength of the most intense wave). The λ_m shifts towards shorter wavelengths with increasing temperature. This second fact is expressed quantitatively by what is known as Wien's displacement law. It states that λ_m shifts towards shorter wavelengths as the temperature of a body is increased. This law is, strictly valid only for black bodies. Mathematically, we say that the product $\lambda_m T$ is constant for a body emitting radiation at temperature T.

$$\lambda_m T = \text{constant} \qquad (11.2)$$

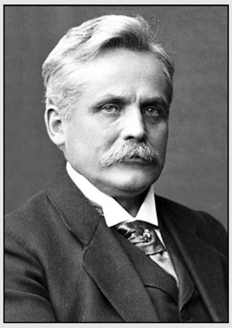
The constant in Eqn. (11.2) has a value b = 2.884×10^{-3} m K. This law furnishes us with a simple method of determining the temperature of all radiating bodies including those in space. The radiation spectrum of the moon has a peak at λ_m = 14 micron.

Using Eqn. (11.2), we get $$T = \frac{b}{\lambda_m} = \frac{2884 \times 10^{-6}\,\text{m}}{14 \times 10^{-6}\,\text{m}}$$

$$T = \frac{2884}{14} = 206\,\text{K}$$

i.e. the temperature of the lunar surface is 206K

Wilhelm Wien (1864 – 1928)



The 1911 Nobel Laureate in physics, Wilhelm Wien, was son of a land owner in East Prussia. After schooling at Prussia, he went to Germany for his college. At the University of Berlin, he studied under great physicist Helmholtz and got his doctorate on diffraction of light from metal surfaces in 1886. He had a very brilliant professional career. In 1896, he succeeded Philip Lenard as Professor of Physics at Aix-la-Chappelle. In 1899, he become Professor of Physics at University of Giessen and in 1900, he succeeded W.C. Roentgen at Wurzburg. In 1902, he was invited to succeed Ludwig Boltzmann at University of Leipzig and in 1906 to succeed Drude at University of Berlin. But he refused these invitations. In 1920, he was appointed Professor of Physics at Munich and he remained there till his last.

11.2.1 Kirchhoff's Law

As pointed out earlier, when radiation falls on matter, it may be partly reflected, partly absorbed and partly transmitted. If for a particular wavelength λ and a given surface, r_λ, a_λ and t_λ respectively denote the fraction of total incident energy reflected, absorbed and transmitted, we can write $$r_\lambda + a_\lambda + t_\lambda = 1 \qquad (11.3)$$

A body is said to be perfectly black, if $r_\lambda = t_\lambda = 0$ and $a_\lambda = 1$. It means that radiations incident on black bodies will be completely absorbed. As such, perfectly black body does not exist in nature. Lamp black is the nearest approximation to a black body. It absorbs about 96% of visible light and platinum black absorbs about 98%. It is found to transmit light of long wavelength.

A perfectly white body, in contrast, defined as a body with $a_\lambda = 0$, $t_\lambda = 0$ and $r_\lambda = 1$. A piece of white chalk approximates to a perfectly white body. This implies that good emitters are also good absorbers. But each body must either absorb or reflect the radiant energy reaching it. So we can say that a good absorber must be a poor reflector (or good emitter).

Designing a Black Body

Uniformly heated enclosure with a small cavity behaves as a black body for emission. Such an enclosure behaves as a perfectly black body towards incident radiation also. Any radiation passing into the hole will undergo multiple reflections internally within the enclosure and will be unable to escape outside. This may be further improved by blackening the inside. Hence the enclosure is a perfect absorber and behaves as a perfectly black body. Fig. 11.6 shows a black body due to Ferry. There is a cavity in the form of a hollow sphere and its inside is coated with black material. It has a small conical opening O. Note the conical projection P opposite the hole O. This is to avoid direct reflection from the surface opposite the hole which would otherwise render the body not perfectly black.

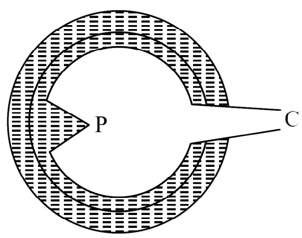


Fig. 11.6 : Ferry's black body

Activity 11.1

You have studied that black surface absorbs heat radiations more quickly than a shiny white surface. You can perform the following simple experiment to observe this effect. Take two metal plates A and B. Coat one surface of A as black and polish one surface of B. Take an electric heater. Support these on vertical stands such that the coated black surface and coated white surface face the heater. Ensure that coated plates are equidistant from the heater. Fix one cork each with wax on the uncoated sides of the plates. Switch on the electric heater. Since both metal plates are identical and placed at the same distance from the heater, they receive the same amount of radiation from it. You will observe that the cork on the blackened plate falls first. This is because the black surface absorbs more heat than the white surface. This proves that black surfaces are good absorbers of heat radiations.

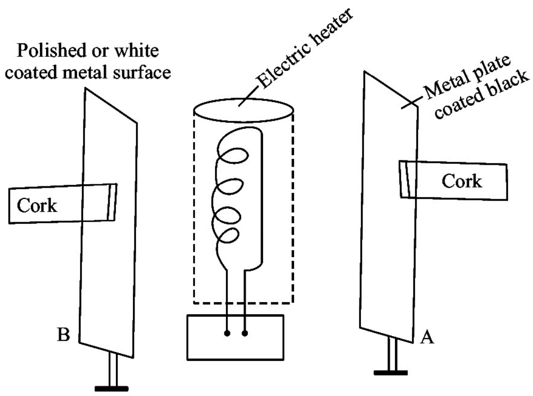


Fig. 11.7 : Showing the difference in heat absorption of a black and a shining surface

11.2.2 Stefan-Boltzmann Law

On the basis of experimental measurements, Stefan and Boltzmann concluded that the radiant energy emitted per second from a surface of area A is proportional to fourth power of temperature. $E = Ae\sigma T^4$ (11.4)

where σ is Stefan-Boltzmann constant and has the value $5.672 \times 10^{-8} Jm^{-2}s^{-1} K^{-4}$.

The temperature is expressed is kelvin, e is emissivity. It depends on the nature of the surface and temperature. The value of e lies between 0 and 1, being small for polished metals and 1 for perfectly black materials.

From Eqn. (11.4) you may think that if the surfaces of all bodies are continually radiating energy, why don't they eventually radiate away all their internal energy and cool down to absolute zero. They would have done so if energy were not supplied to them in some way. In fact, all objects radiate and absorb energy simultaneously. If a body is at the same temperature as its surroundings, the rate of emission is same as the rate of absorption; there is no net gain or loss of energy and no change in temperature. However, if a body is at a lower temperature than its surroundings, the rate of absorption will be greater than the rate of emission. Its temperature will rise till it is equal to the room temperature. Similarly, if a body is at higher temperature, the rate of emission will be greater than the rate of absorption. There will be a net energy loss. Hence, when a body at a temperature T_1 is placed in surroundings at temperature T_2, the amount of net energy loss per second is given by

$$E_{net} = Ae\sigma\left(T_1^4 - T_2^4\right) \quad \text{for } T_1 > T_2 \qquad (11.5)$$

Example 11.2

Determine the surface area of the filament of a 100 W incandescent lamp at 3000 K. Given $\sigma = 5.7 \times 10^{-8}$ $Wm^{-2} K^{-4}$, and emissivity e of the filament = 0.3.

Solution :

According to Stefan-Boltzmann law $E = Ae\sigma T^4$

where E is rate at which energy is emitted, A is surface area, and T is temperature of the surface.

Hence we can rewrite it as

$$A = \frac{E}{e\sigma T^4} = \frac{100}{0.3 \times 5.7 \times 10^{-8} \times 3000^4}$$

$$E = 7.25 \times 10^{-5} \text{ m}^2$$

Intext Questions 11.2

1. At what wavelength does a cavity radiator at 300 K emit most radiation?
2. Why do we wear light color clothing during summer?
3. State the important fact which we can obtain from the experimental study of the spectrum of black body radiation.

4. A person with skin temperature 28° C is present in a room at temperature 22° C. Assuming the emissivity of skin to be unity and surface area of the person as 1.9 m^2, compute the radiant power of this person.

11.3 SOLAR ENERGY

Sun is the core source of all energy available on the earth. The sun is radiating enormous amount of energy in the form of light and heat and even the small fraction of that radiation received by earth is more than enough to meet the needs of living beings on its surface. The effective use of solar energy, therefore, may someday provide solution to our energy needs. Some of the basic issues related with solar radiations are discussed below.

1. Solar Constant :

To calculate the total solar energy reaching the earth, we first determine the amount of energy received per unit area in one second. The energy is called solar constant. Solar constant for earth is found to be 1.36×10^3 W m^{-2}. Solar constant multiplied by the surface area of earth gives us the total energy received by earth per second.

Mathematically, $Q = 2\pi R_e^2 S$; where R_e is radius of earth and S is solar constant.

Note that only half of the earth's surface has been taken into account as only this much of the surface is illuminated at one time. Therefore,

$$Q = 2 \times 3.14 \times (6.4 \times 10^6)^2 \times (1.36 \times 10^3)$$

$$Q = 6.28 \times 55.71 \times 10^{15}$$

$$Q = 3.49 \times 10^{17} \text{ W}$$

$$Q = 3.49 \times 10^{11} \text{ MW}$$

2. Greenhouse Effect :

The solar radiations in appropriate amount are necessary for life to flourish on earth. The atmosphere of earth plays an important role to provide a comfortable temperature for the living organisms. One of the processes by which this is done is greenhouse effect. In a greenhouse, plants, flowers, grass etc. are enclosed in a glass structure. The glass allows short wavelength radiation of light to enter. This radiation is absorbed by plants. It is subsequently re-radiated in the form of longer wavelength heat radiations – the infrared. The longer wavelength radiations are not allowed to escape from the greenhouse as glass is effectively opaque to heat. These heat radiations are thus trapped in the greenhouse keeping it warm. An analogous effect takes place in our atmosphere. The atmosphere, which contains a trace of carbon dioxide, is transparent to visible light. Thus, the sun's light passes through the atmosphere and reaches the earth's surface. The earth absorbs this light and subsequently emits it as infrared radiation. But carbon dioxide in air is opaque to infra-red radiations.CO_2 reflects these radiations

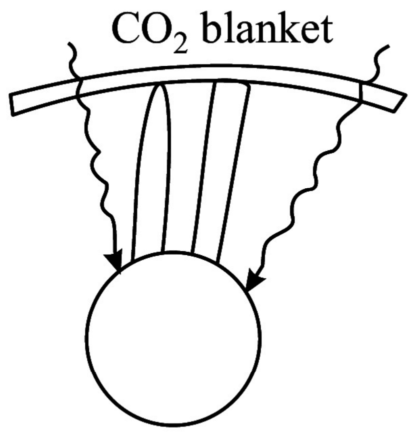


Fig. 11.8 : Greenhouse Effect

back rather than allowing them to escape into the atmosphere. As a result, the temperature of earth increases. This effect is referred to as the ***greenhouse effect***.

Due to emission of huge quantities of CO_2 in our atmosphere by the developed as well as developing countries, the greenhouse effect is adding to global warming and likely to pose serious problems to the existence of life on the earth. A recent report by the UN has urged all countries to cut down on their emissions of CO_2, because glaciers have begun to shrink at a rapid rate. In the foreseeable future, these can cause disasters beyond imagination beginning with flooding of major rivers and rise in the sea level. Once the glaciers melt, there will be scarcity of water and erosion in the quality of soil. There is a lurking fear that these together will create problems of food security. Moreover, changing weather patterns can cause droughts & famines in some regions and floods in others. In Indian context, it has been estimated that lack of positive action can lead to serious problems in Gangetic plains by 2030. Additionally, the sea will reclaim vast areas along our coast, inundating millions of people and bringing unimaginable misery and devastation.

11.4 NEWTON'S LAW OF COOLING

Newton's law of cooling states that the rate of cooling of a hot body is directly proportional to the mean excess temperature of the hot body over that of its surroundings provided the difference of temperature is small. The law can be deduced from Stefan-Boltzmann law. Let a body at temperature T be surrounded by another body at T_o. The rate at which heat is lost per unit area per second by the hot body is

$$E = e\sigma(T^4 - T_0^4)A \qquad (11.6)$$

As $(T^4 - T_0^4) = (T^2 - T_0^2)\,(T^2 + T_0^2) = (T - T_o)(T + T_o)(T^2 + T_0^2)$

Equation (11.6) becomes $E = e\sigma(T - T_0)(T + T_0)(T^2 + T_0^2)A$

$$E = e\sigma(T - T_0)(T^3 + TT_0^2 + T_0T^2 + T_0^3)A \qquad (11.7)$$

If $(T - T_0)$ is very small so the terms T^3, TT_0^2, and T_0T^2 is approximated to T_0^3

So that $E = e\sigma(T - T_0)4AT_0^3$ (11.8)

Where $k = 4e\sigma AT_0^3$ is constant

$$E = K\,(T - T_0)$$

$$E \propto (T - T_0) \qquad (11.9)$$

This is Newton's law of cooling.

"Greater the difference in temperature between the system and its surrounding, the more rapidly the heat is transferred, i.e., the more rapidly temperature of body changes."

Intext Questions 11.3

1. Calculate the power received from sun by a region 40 m wide and 50 m long located on the surface of the earth?
2. What threats are being posed for life on the earth due to rapid consumption of fossil fuels by human beings?
3. What will be shape of cooling curve of a liquid?

WHAT YOU HAVE LEARNT

- Heat flows from a body at higher temperature to a body at lower temperature.
- There are three processes by which heat is transferred: conduction, convection and radiation.
- In conduction, heat is transferred from one atom/ molecule to another atom/molecule which vibrates about their fixed positions.
- In convection, heat is transferred by bodily motion of molecules.
- In radiation, heat is transferred through electromagnetic waves.
- The quantity of heat transferred by conduction is given by $Q = \frac{K(T_h - T_c)At}{d}$
- Wien's Law : The spectrum of energy radiated by a body at temperature T(K) has a maxima at wavelength λ_m such that $\lambda_m T$ = constant (= 2880 μK)
- Stefan-Boltzmann Law. The rate of energy radiated by a source at T(K) is given by $E = e\sigma AT^4$
- The absorptive power a is defined as

$$a = \frac{\text{Total amount of energy absorbed between } \lambda \text{ and } \lambda + d\lambda}{\text{Total amount of incident energy between } \lambda \text{ and } \lambda + d\lambda}$$

- The emissive power of a surface e_λ is the amount of radiant energy emitted per square meter area per second per unit wavelength range at a given temperature.
- The solar constant for the earth is 1.36×10^3 Jm^{-2} s^{-1}
- Newton's Law of cooling states that the rate of cooling of a body is linearly proportional to the excess of temperature of the body above its surroundings.

TERMINAL EXERCISE

1. A thermos flask (Fig. 11.9) is made of a double walled glass bottle enclosed in metal container. The bottle contains some liquid whose temperature we want to maintain. Look at the diagram carefully and explain how the construction of the flask helps in minimizing heat transfer due to conduction, convection and radiation.

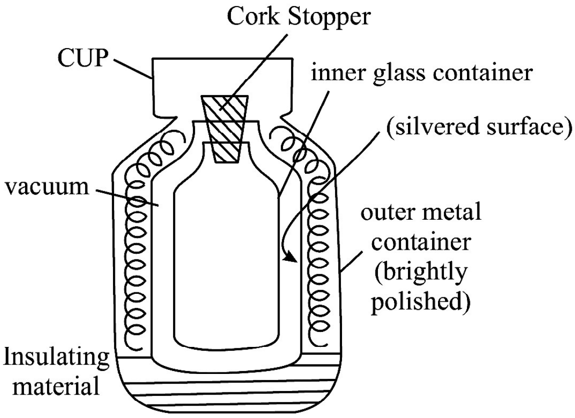


Fig. 11.9

2. The wavelength corresponding, to emission of maximum energy of a star is 4000A°. Compute the temperature of the star. (1 A° = 10^{-8} cm).
3. A blackened solid copper sphere of radius 2cm is placed in an evacuated enclosure whose walls are kept at 1000°C. At what rate must energy be supplied to the sphere to keep its temperature constant at 127°C.
4. Comment on the statement "A good absorber must be a good emitter".
5. A copper pot whose bottom surface is 0.5 cm thick and 50 cm in diameter rests on a burner which maintains the bottom surface of the pot at 110°C. A steady heat flows through the bottom into the pot where water boils at atmospheric pressure. The actual temperature of the inside surface of the bottom of the pot is 105°C. How many kilograms of water boil off in one hour?
6. Define the coefficient of thermal conductivity. List the factors on which it depends.
7. Distinguish between conduction and convection methods of heat transfer.
8. If two or more rods of equal area of cross-section are connected in series, show that their equivalent thermal resistance is equal to the sum of thermal resistance of each rod. [Hint: Thermal resistance is reciprocal of thermal conductivity]
9. Ratio of coefficient of thermal conductivities of the different materials is 4 : 3, to have the same thermal resistance of the two rods of these materials of equal thickness. What should be the ratio of their lengths?
10. Why do we feel warmer on a winter night when clouds cover the sky than when the sky is clear?

11. Why does a piece of copper or iron appear hotter to touch than a similar piece of wood even when both are at the same temperature?
12. Why is it more difficult to sip hot tea from a metal cup than from a china-clay cup?
13. Why are the woolen clothes warmer than cotton clothes?
14. Why do two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double the thickness?
15. Can the water be boiled by convection inside an earth satellite?
16. A 500 W bulb is glowing. We keep our one hand 5 cm above it and other 5 cm below it. Why more heat is experienced at the upper hand?
17. Two vessels of different materials are identical in size and in dimensions. They are filled with equal quantity of ice at O°C. If ice in both vessels melts completely in 25 minutes and in 20 minutes respectively compare the (thermal conductivities) of metals of both vessels.
18. Calculate the thermal resistivity of a copper rod 20 cm length and 4 cm in diameter. Thermal conductivity of copper = 9.2×10^{-2}, temperature difference across the ends of the rod be 50°C. Calculate the rate of heat flow.

ANSWERS TO INTEXT QUESTIONS

11.1

1. Conduction is the principal mode of transfer of heat in solids in which the particles transfer energy to the adjoining molecules.

   In convection the particles of the fluid bodily move from high temperature region to low temperature region and vice-versa.

2. $$K = \frac{Qd}{tA(Q_2 - Q_1)}$$

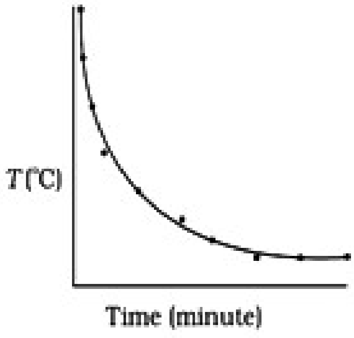
   $$= \frac{J}{s}\frac{m}{m^2\ {}^{o}C} = Jm^{-1}\,s^{-1}\ {}^{o}C^{-1}$$

3. The trapped air in wool fibers prevents body heat from escaping out and thus keeps the wearer warm.
4. The coefficient of thermal conductivity is numerically equal to the amount of heat energy transferred in one second across the faces of a cubical slab of surface area 1 m^2 and thickness 1 m, when they are kept at a temperature difference of 1°C.
5. During the day, land becomes hotter than water and air over the ocean is cooler than the air near the land. The hot dry air over the land rises up and creates a low pressure region. This cause see breeze because the moist air from the ocean moves to the land. Since specific thermal capacity of water is higher than that of sand, the latter

gets cooled faster and is responsible for the reverse process during the night causing land breezes.

12.2

1. $\lambda_m = \dfrac{\text{Wien's constant}}{T} = \dfrac{2880 \times 10^{-6}}{300} = 9.6\ \mu m$



2. We prefer light color clothing in summer, as they are good reflectors of heat and light. (they absorb less heat) Thus, light colored clothes keep us cool, and hence, are preferred during summer season.

3. $\lambda_m T$ = constant and $E = \sigma e A T^4$

4. 66.4 W.

12.3

1. Solar constant x area = 2.7×10^5W
2. Constant addition of CO_2 in air will increase greenhouse effect causing global warming due to which glaciers are likely to melt and flood the land mass of the earth.
3. Exponential decay

ANSWERS TO TERMINAL EXERCISE

2. 7210 K
3. 71.6×10^{-11} W
5. 4.7×10^5 kg
9. 3 : 4
17. 4 : 5
18. 10.9 m $°C^{-1}W^{-1}$, 0.298 W

12

THERMODYNAMICS

INTRODUCTION

You are familiar with the sensation of hotness and coldness. When you rub your hands together, you get the feeling of warmth. You will agree that the cause of heating in this case is mechanical work. This suggests that there is a relationship between mechanical work and thermal effect. A study of phenomena involving thermal energy transfer between bodies at different temperatures forms the subject matter of thermodynamics, which is a phenomenological science based on experience. A quantitative description of thermal phenomena requires a definition of temperature, thermal energy and internal energy. And the laws of thermodynamics provide relationship between the direction of flow of heat, work done on/by a system and the internal energy of a system. In this lesson you will learn three laws of thermodynamics: the zeroth law, the first law and the second law of thermodynamics. These laws are based on experience and need no proof. As such, the zeroth, first and second law introduces the concept of temperature, internal energy and entropy, respectively. While the first law is essentially the law of conservation of energy for a thermodynamic system, the second law deals with conversion of heat into work and vice versa. You will also learn that Carnot's engine has maximum efficiency for conversion of heat into work.

OBJECTIVES

After studying this lesson, you should be able to

- *draw indicator diagrams for different thermodynamic processes and show that the area under the indicator diagram represents the work done in the process;*
- *explain thermodynamic equilibrium and state the Zeroth law of thermodynamics;*
- *explain the concept of internal energy of a system and state first law of thermodynamics;*
- *apply first law of thermodynamics to simple systems and state its limitations;*
- *define triple point;*
- *state the second law of thermodynamics in different forms;*
- *describe Carnot cycle and calculate its efficiency.*

12.1 CONCEPT OF HEAT AND TEMPERATURE

12.1.1 Heat

Energy has pervaded all facets of human activity ever since man lived in caves. In its manifestation as heat, energy is intimate to our existence. The energy that cooks our food, lights our houses, runs trains and aeroplanes originates in heat released in burning of wood, coal, gas or oil. You may like to ask: What is heat? To discover answer to this question, let us consider as to what happens when we inflate the tyre of a bicycle using a pump. If you touch the nozzle, you will observe that pump gets hot. Similarly, when you rub your hands together, you get the feeling of warmth. You will agree that in these processes heating is not caused by putting a flame or something hot underneath the pump or the hand. Instead, heat is arising as a result of mechanical work that is done in compressing the gas in the pump and forcing the hand to move against friction. These examples, in fact, indicate a relation between mechanical work and thermal effect. We know from experience that a glass of ice cold water left to itself on a hot summer day eventually warms up. But a cup of hot coffee placed on the table cools down. It means that energy has been exchanged between the system – water or coffee – and its surrounding medium. This energy transfer continues till thermal equilibrium is reached. That is until both – the system and the surroundings – are as the same temperature. It also shows that the direction of energy transfer is always from the body at high temperature to a body at lower temperature. You may now ask: In what form is energy being transferred? In the above examples, energy is said to be transferred in the form of heat. So we can say that heat is the form of energy transferred between two (or more) systems or a system and its surroundings because of temperature difference. You may now ask. What is the nature of this form of energy? The answer to this question was provided by Joule through his work on the equivalence of heat and mechanical work: Mechanical motion of molecules making up the system is associated with heat. The unit of heat is calorie. "*One calorie is defined as the quantity of heat energy required to raise the temperature of 1 gram of water from* 14.5°C *to* 15.5°C. *It is denoted as cal"*.

Kilocalorie (kcal) is the larger unit of heat energy

1 kcal = 10^3 cal. and 1 cal = 4.18 J

12.1.2 Concept of Temperature

While studying the nature of heat, you learnt that energy exchange between a glass of cold water and its surroundings continues until thermal equilibrium was reached. All bodies in thermal equilibrium have a common property, called temperature, whose value is same for all of them. Thus, we can say that temperature of a body is the property which determines whether or not it is in thermal equilibrium with other bodies.

12.1.3 Thermodynamic Terms

(i) Thermodynamic system : A thermodynamic system refers to a definite quantity of matter which is considered unique and separated from everything else, which can influence it. Every system is enclosed by an arbitrary surface, which is called its boundary. The boundary may enclose a solid, a liquid or a gas. It may be real or imaginary, either at rest or in motion and may change its size and shape. The region of space outside the boundary of a system constitutes its surroundings.

- **(a) Open System :** It is a system which can exchange mass and energy with the surroundings. A water heater is an open system.
- **(b) Closed system :** It is a system which can exchange energy but not mass with the surroundings. A gas enclosed in a cylinder fitted with a piston is a closed system.

(c) **Isolated system :** It is a system which can exchange neither mass nor energy with the surrounding. A filled thermos flank is an ideal example of an isolated system.

(ii) **Thermodynamic Variables or Coordinates :** To describe a thermodynamic system, we use its physical properties such as temperature (T), pressure (P), volume (V), and Entropy (S). These are called thermodynamic variables.

(iii) **Indicator diagram :** For a thermodynamic system a plot between pressure (P) on Y-axis and volume (V) on X-axis is called P-V diagram or indicator diagram. This graph indicates how pressure of a system varies with its volume during a thermodynamic process. The indicator diagram can be used to obtain an expression for the work done. It is equal to the area under the P-V diagram (Fig. 12.1). Suppose that pressure is P at the start of a very small expansion ΔV. Then, work done by the system.

$$\Delta W = P\Delta V = \text{Area of the shaded portion ABCD under P-V curve}$$

Now total work done by the system when it expands from V_1 to V_2 = Area of $P_1\,P_2\,V_2\,V_1\,P_1$ Note that the area depends upon the shape of the indicator diagram. The indicator diagram is widely used in calculating the work done in the process of expansion or compression. It is found more useful in processes where relationship between P and V is not known. The work done on the system increases its energy and work done by the system reduces it. For this reason, work done on the system is taken as negative. The area enclosed by an isotherm (plot of p versus V at constant temperature) depends on its shape. We may conclude that work done by or on a system depends on the path. That is, work does not depend on the initial and final states.

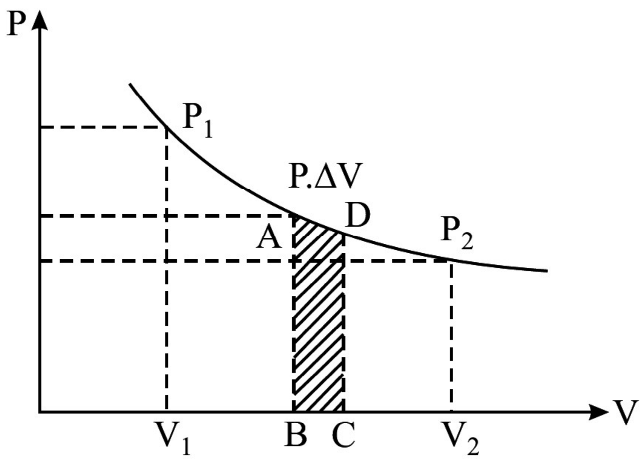


Fig. 12.1 : Indicater Diagram

12.2 THERMODYNAMIC EQUILIBRIUM

Imagine that a container is filled with a liquid (water, tea, milk, coffee) at 60° C. If it is left to itself, it is common experience that after some time, the liquid attains the room temperature. We then say that water in the container has attained thermal equilibrium with the surroundings. If within the system, there are variations in pressure or elastic stress, then parts of the system may undergo some changes. However, these changes cease ultimately, and no unbalanced force will act on the system. Then we say that it is in mechanical equilibrium. (Do you know that our earth bulged out at the equator in the process of attaining mechanical equilibrium in its formation from a molten state.) If a system has components which react chemically, after some time, all possible chemical reactions will cease to occur. Then the system is said to be in chemical equilibrium. A system which exhibits thermal, mechanical and chemical equilibriums is said to be in thermodynamic equilibrium. The macroscopic properties of a system in this state do not change with time.

12.2.1 Thermodynamic Process

If any of the thermodynamic variables of a system change while going from one equilibrium state to another, the system is said to execute a thermodynamic process. For example, the expansion of a gas in a cylinder at constant pressure due to heating is a thermodynamic process. A graphical representation of a thermodynamic process is called a path. Now we will discuss different types of thermodynamic processes.

(i) **Reversible process :** If a process is executed so that all intermediate stages between the initial and final states are equilibrium states and the process can be executed back along the same equilibrium states from its final state to its initial state, it is called reversible process. A reversible process is executed very slowly and in a controlled manner.

Examples:

❒ Take a piece of ice in a beaker and heat it. You will see that it changes to water. If you remove the same quantity of heat of water by keeping it inside a refrigerator, it again changes to ice (initial state).

❒ Consider a spring supported at one end. Put some masses at its free end one by one. You will note that the spring elongates (increases in length). Now remove the masses one by one. You will see that spring retraces its initial positions. Hence it is a reversible process. As such, a reversible process can only be idealized and never achieved in practice.

(ii) **Irreversible process :** A process which cannot be retraced along the same equilibrium state from final to the initial state is called irreversible process. All natural processes are irreversible. For example, heat produced during friction, sugar dissolved in water, or rusting of iron in the air.

(iii) **Isothermal process :** A thermodynamic process that occurs at constant temperature is an isothermal process. The expansion and compression of a perfect gas in a cylinder made of perfectly conducting walls are isothermal processes. The change in pressure or volume is carried out very slowly so that any heat developed is transferred into the surroundings and the temperature of the system remains constant. The thermal equilibrium is always maintained. In such a process, ΔQ, ΔU and ΔW are finite.

(iv) **Adiabatic process :** A thermodynamic process in which no exchange of thermal energy occurs is an adiabatic process. For example, the expansion and compression of a perfect gas in a cylinder made of perfect insulating walls. The system is isolated from the surroundings. Neither any amount of heat neither leaves the system nor enters it from the surroundings. In this process, therefore $\Delta Q = 0$ and $\Delta U = -\Delta W$.

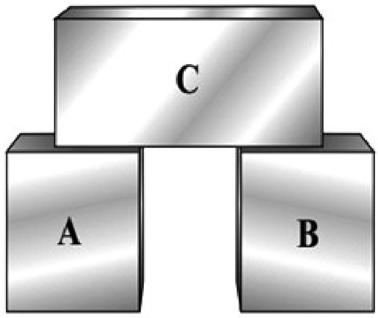
The change in the internal energy of the system is equal to the work done on the system. When the gas is compressed, work is done on the system. So, ΔU becomes positive and the internal energy of the system increases. When the gas expands, work is done by the system. It is taken as positive and ΔU becomes negative. The internal energy of the system decreases.

(v) **Isobaric process :** A thermodynamic process that occurs at constant pressure is an isobaric process. Heating of water under atmospheric pressure is an isobaric process.

(vi) **Isochoric process :** A thermodynamic process that occurs at constant volume is an isochoric process. For example, heating of a gas in a vessel of constant volume is an isochoric process. In this process, volume of the gas remains constant so that no work is done, i.e. $\Delta W = 0$. We therefore get $\Delta Q = \Delta U$.

(vii) **Cyclic process :** For a thermodynamic process the initial and final states are coincides is a cyclic process. In a Cyclic Process the system returns back to its initial state. It means that there is no change in the internal energy of the system. $\Delta U = 0$. $\therefore\ \Delta Q = \Delta W$.

12.2.2 Zeroth Law of Thermodynamics



Let us consider three metal blocks A, B and C. Suppose block A is in thermal equilibrium with block C. Further suppose that block B is also in thermal equilibrium with block C. It means the temperature of the block C is equal to the temperature of block B as well as of block A. It follows that the temperatures of blocks A and B are equal. We summarize this result in the statement known as Zeroth Law of Thermodynamics: *"If two bodies or systems A and B are separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other."*

12.2.3 Phase Change and Phase Diagram

You have learnt that at STP, matter exists in three states : solid, liquid and gas. The different states of matter are called its phases. For example, ice (solid), water (liquid) and steam (gas) are three phases of water. We can discuss these three phases using a three dimensional diagram drawn in pressure (P), temperature (T) and volume (V). It is difficult to draw three dimensional diagrams. Thus, we discuss the three phases of matter by drawing a pressure-temperature diagram. This is called phase diagram

Refer to Fig. 12.3, which shows phase diagram of water. You can see three curves CD; AB and EF. Curve CD shows the variation of melting point of ice with pressure. It is known as a fusion curve. Curve AB shows variation of boiling point of water with pressure. It is known as vaporization curve. Curve EF shows change of ice directly to steam. It is known as a sublimation curve. This curve is also known as Hoarfrost Line. If you extend the curve AB, CD and EF (as shown in the figure with dotted lines), they meet at point P. This point is called ***Triple point***. At triple point, all three phases co-exist. When we heat a solid, its temperature increases till it reaches a temperature at which it starts melting. This temperature is called melting point of the solid. During this change of state, we supply heat continuously but the temperature does not rise. The heat required to completely change unit mass of a solid into its corresponding liquid state at its melting point is called latent heat of fusion of the solid. On heating a liquid, its temperature also rises till its boiling point is reached. At the boiling point, the heat we supply is used up in converting the liquid into its gaseous state. The amount of heat required to convert unit mass of liquid in its gaseous state at constant temperature is called latent heat of vaporization of the liquid.

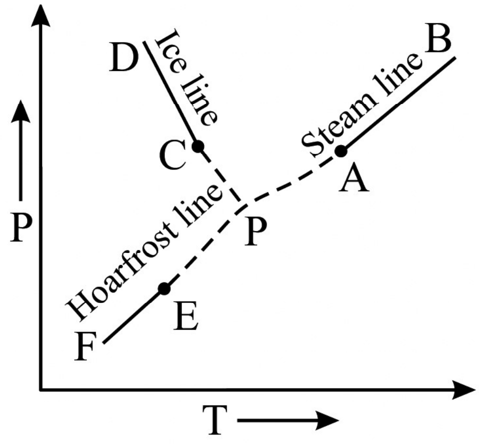


Fig. 12.2 : Phase diagram of water

12.2.3.1 Triple Point of Water

Triple point of a pure substance is a very stable state signified by precisely constant temperature and pressure values. For this reason, in kelvin's scale of thermometry, triple point of water is taken as the upper fixed point. On increasing pressure, the melting point of a solid decreases and boiling point of the liquid increases. It is possible that by adjusting temperature and pressure, we can obtain all the three states of matter to co-exist simultaneously. These values of temperature and pressure signify the triple point. The values of Triple point are T = 273.16 K and P = 0.006 atm.

Intext Questions 12.1

1. Fill in the blanks
   (i) Zeroth law of thermodynamics provides the basis for the concept of ....
   (ii) If a system A is in thermal equilibrium with a system B and B is in thermal equilibrium with another system C, then system A will also be in thermal equilibrium with system ....
   (iii) The unit of heat is ....

2. Fig. 12.3 is an indicator diagram of a thermodynamic process. Calculate the work done by the system in the process:
   (a) Along the path ABC from A to C
   (b) If the system is returned from C to A along the same path, how much work is done by the system.

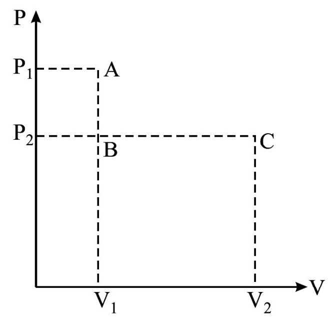


Fig. 12.3

3. Fill in the blanks.
   (i) A reversible process is that which can be .... in the opposite direction from its final state to its initial state.
   (ii) An .... process is that which cannot be retraced along the same equilibrium states from final state to the initial state.

4. State the basic difference between isothermal and adiabatic processes. ....
5. State one characteristic of the triple point. ....

12.3 INTERNAL ENERGY OF A SYSTEM

Have you ever thought about the energy which is released when water freezes into ice? Don't you think that there is some kind of energy stored in water? This energy is released when water changes into ice. This stored energy is called the internal energy. On the basis of kinetic theory of matter, we can discuss the concept of internal energy as sum of the energies of individual components/constituents. This includes kinetic energy due to their random motion and their potential energy due to interactions amongst them.

(a) Internal kinetic energy : According to kinetic theory, matter is made up of a large number of molecules. These molecules are in a state of constant rapid motion and hence possess kinetic energy. The total kinetic energy of the molecules constitutes the internal kinetic energy of the body.

(b) Internal potential energy : The energy arising due to the inter-molecular forces is called the internal potential energy.

The internal energy of a metallic rod is made up of the kinetic energies of conduction electrons, potential energies of atoms of the metal and the vibrational energies about their equilibrium positions. The energy of the system may be increased by causing its molecules to move faster (gain in kinetic energy by adding thermal energy). It can also be increased by causing the molecules to move against inter-molecular forces, i.e., by doing work on it. Internal energy is denoted by the letter U.

Internal energy of a system = Kinetic energy of molecules + Potential energy of molecules

Let us consider an isolated thermodynamic system subjected to an external force. Suppose W amount of work is done on the system in going from initial state i to final state f adiabatically. Let U_i and U_f be internal energies of the system in its initial and final states respectively. Since work is done on the system, internal energy of final state will be higher than that of the initial state.

According to the law of conservation of energy, $U_i - U_f = -W$

Negative sign signifies that work is done on the system. We may point out here that unlike work, internal energy depends on the initial and final states, irrespective of the path followed. We express this fact by saying that U is a function of state and depends only on state variables P, V, and T. Note that if some work is done by the system; its internal energy will decrease.

12.4 FIRST LAW OF THERMODYNAMICS

You now know that the zeroth law of thermodynamics tells us about thermal equilibrium among different systems characterized by same temperature. However, this law does not tell us anything about the non-equilibrium state. Let us consider two examples: (i) Two systems at different temperatures are put in thermal contact and (ii) Mechanical rubbing between two systems. In both cases, change in their temperatures occurs but it cannot be explained by the Zeroth law. To explain such processes, the first law of thermodynamics was postulated. The first law of thermodynamics is, in fact, the law of conservation of energy for a thermodynamic system. It states that change in internal energy of a system during a thermodynamic process is equal to the sum of the heat given to it and the work done on it. Suppose that ΔQ amount of heat is given to the system and $-\Delta W$ work is done on the system. Then increase in internal energy of the system, ΔU, according to the first law of thermodynamics is given by

$$\Delta U = \Delta Q - \Delta W \qquad (12.3\ [a])$$

This is the mathematical form of the first law of thermodynamics. Here ΔQ, ΔU and ΔW all are in SI units. The first law of thermodynamics can also be written as

$$\Delta Q = \Delta U + \Delta W \qquad (12.3\ [b])$$

The signs of ΔQ, ΔU and ΔW are known from the following sign conventions:

1. Work done (ΔW) by a system is taken as positive whereas the work done on a system is taken as negative. The work is positive when a system expands. When a system is compressed, the volume decreases, the work done is negative. The work done does not depend on the initial and final thermodynamic states; it depends on the path followed to bring a change.
2. Heat gained by (added to) a system is taken as positive, whereas heat lost by a system is taken as negative.
3. The increase in internal energy is taken as positive and a decrease in internal energy is taken as negative. If a system is taken from state 1 to state 2, it is found that both ΔQ and ΔW depend on the path of transformation. However, the difference ($\Delta Q - \Delta W$) which represents ΔU, remains the same for all paths of transformations.

We therefore say that the change in internal energy ΔU of a system does not depend on the path of the thermodynamic transformations.

12.4.1 Limitations of the First Law of Thermodynamics

The first law of thermodynamics asserts the equivalence of heat and other forms of energy. This equivalence makes the world around us work. The electrical energy that lights our houses, operates machines and runs trains originates in heat released in burning of fossil or nuclear fuel. In a sense, it is universal. It explains the fall in temperature with height; the adiabatic lapse rate in upper atmosphere. Its applications to flow process and chemical reactions are also very interesting. However, it has limitations

- You know that heat always flows from a hot body to a cold body. But first law of thermodynamics does not prohibit flow of heat from a cold body to a hot body. It means that this law fails to indicate the direction of heat flow.
- You know that when a bullet strikes a target, the kinetic energy of the bullet is converted into heat. This law does not indicate as to why heat developed in the

target cannot be changed into the kinetic energy of bullet to make it fly. It means that this law fails to provide the conditions under which heat can be changed into work.

Moreover, it has obvious limitations in indicating the extent to which heat can be converted into work.

Intext Questions 12.2

1. Fill in the blanks
   (i) The total of kinetic energy and potential energy of molecules of a system is called its ....
   (ii) Work done = –W indicates that work is done......................... the system.
2. The first law of thermodynamics states that ....

12.5 SECOND LAW OF THERMODYNAMICS

You now know that the first law of thermodynamics has inherent limitations in respect of the direction of flow of heat and the extent of convertibility of heat into work. So a question may arise in your mind: Can heat be wholly converted into work? Under what conditions this conversion occurs? The answers of such questions are contained in the postulate of Second law of thermodynamics. The second law of thermodynamics is stated in several ways. However, here you will study Kelvin-Planck and Clausius statements of second law of thermodynamics.

The Kelvin-Planck's statement is based on the experience about the performance of heat engines. In a heat engine, the working substance extracts heat from the source (hot body), converts a part of it into work and rejects the rest of heat to the sink (environment). There is no engine which converts the whole heat into work, without rejecting some heat to the sink. These observations led Kelvin and Planck to state the second law of thermodynamics as "It is impossible for any system to absorb heat from a reservoir at a fixed temperature and convert whole of it into work".

Clausius statement of second law of thermodynamics is based on the performance of a refrigerator. A refrigerator is a heat engine working in the opposite direction. It transfers heat from a colder body to a hotter body when external work is done on it. Here concept of external work done on the system is important. To do this external work, supply of energy from some external source is a must. These observations led Clausius to state the second law of thermodynamics in the following form. "It is impossible for any process to have as its sole result to transfer heat from a colder body to a hotter body without any external work". Thus, the second law of thermodynamics plays a unique role for practical devices like heat engine and refrigerator.

12.5.1 Carnot Cycle

You must have noticed that when water is boiled in a vessel having a lid, the steam generated inside throws off the lid. This shows that high pressure steam can be made to do useful work. A device which can convert heat into work is called a heat engine. Modern engines which we use in our daily life are based on the principle of heat engine. These may be categorized in three types: steam engine, internal combustion engine and gas turbine. However, their working can be understood in terms of Carnot's reversible engine.

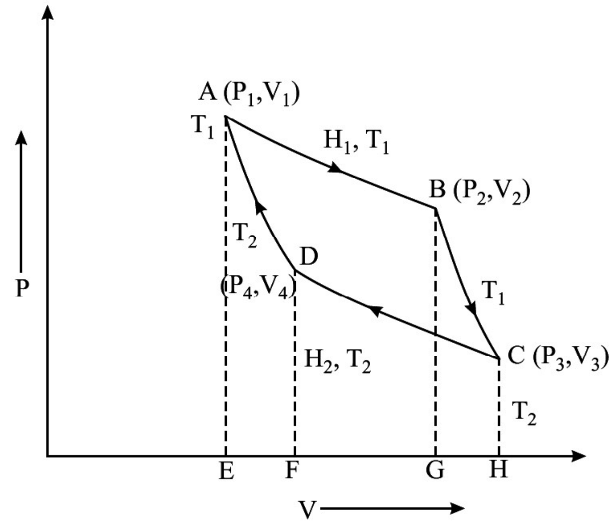


Fig. 12.4 : Indicator diagram of Carnot cycle

In Carnot cycle, the working substance is subjected to four operations: (a) isothermal expansion, (b) adiabatic expansion, (c) isothermal compression and (d) adiabatic compression. Such a cycle is represented on the P-V diagram in (Fig. 12.4). To describe four operations of Carnot's cycle, let us fill one gram. mol. of the working substance in the cylinder (Fig. 12.5). Original condition of the substance is represented by point A on the indicator diagram. At this point, the substance is at temperature T_1, pressure P_1 and volume V_1.

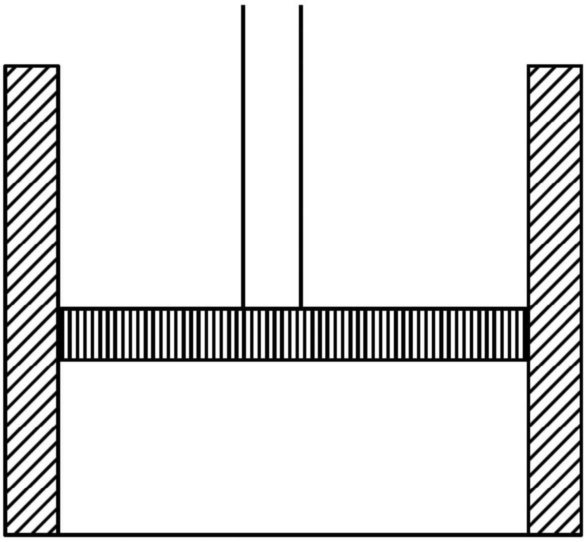


Fig. 12.5 : The cylinder with working substance

(a) Isothermal expansion : The cylinder is put in thermal contact with the source and allowed to expand. The volume of the working substance increases to V_2. Thus working substance does work in raising the piston. In this way, the temperature of the working substance would tend to fall. But it is in thermal contact with the source.

So it will absorb a quantity of heat H_1 from the source at temperature T_1. This is represented by the point B. At B, the values of pressure and volume are P_2 and V_2 respectively. On the indicator diagram (Fig. 12.4), you see that in going from A to B, temperature of the system remains constant and working substance expands. We call it isothermal expansion process. H_1 is the amount of heat absorbed in the isothermal expansion process. Then, in accordance with the first law of thermodynamics, H_1 will be equal to the external work done by the gas during isothermal expansion from A to B at temperature T_1. Suppose W_1 is the external work done by the gas during isothermal expansion AB. Then it will be equal to the area ABGEA.

$$\text{Hence } W_1 = \text{Area ABGEA}$$

(b) Adiabatic expansion : Next the cylinder is removed from the source and placed on a perfectly non-conducting stand. It further decreases the load on the piston to P_3. The expansion is completely adiabatic because no heat can enter or leave the working substance. Therefore, the working substance performs external work in raising the piston at the expense of its internal energy. Hence its temperature falls. The gas is thus allowed to expand adiabatically until its temperature falls to T_2, the temperature of the sink. It has been represented by the adiabatic curve BC on the indicator diagram. We call it adiabatic expansion. If the pressure and volume of the substance are P_3 and V_3, respectively at C, and W_2 is the work done by the substance from B to C, then

$$W_2 = \text{Area BCHGB.}$$

(c) Isothermal compression : Remove the cylinder from the non-conducting stand and place it on the sink at temperature T_2. In order to compress the gas slowly, increase the load (pressure) on the piston until its pressure and volume become P_4 and V_4, respectively. It is represented by the point D on the indicator diagram (Fig. 12.4). The heat developed (H_2) due to compression will pass to the sink. Thus, there is no change in the temperature of the system. Therefore, it is called an isothermal compression process. It is shown by the curve CD (Fig. 12.4). The quantity of heat rejected (H_2) to the sink during this process is equal to the work done (say W_3) on the working substance.

$$\text{Hence } W_3 = \text{Area CHFDC}$$

(d) Adiabatic compression : Once again place the system on the non-conducting stand. Increase the load on the piston slowly. The substance will undergo an adiabatic compression. This compression continues until the temperature rises to T_1 and the substance comes back to its original pressure P_1 and volume V_1. This is an adiabatic compression process and represented by the curve DA on the indicator diagram (Fig. 12.4). Suppose W_4 is the work done during this adiabatic compression from D to A.

$$\text{Then } W_4 = \text{Area DFEAD}$$

During the above cycle of operations, the working substance takes H_1 amount of heat from the source and rejects H_2 amount of heat to the sink. Hence the net amount of heat absorbed by the working substance is

$$\Delta H = H_1 - H_2$$

Also the net work done (say W) by the engine in one complete cycle

$$W = \text{Area ABCHEA} - \text{Area CHEADC}$$
$$= \text{Area ABCD}$$

Thus, the work done in one cycle is represented on a P-V diagram by the area of the cycle. You have studied that the initial and final states of the substance are the same. It means that its internal energy remains unchanged. Hence according to the first law of thermodynamics

$$W = H_1 - H_2$$

Therefore, heat has been converted into work by the system, and any amount of work can be obtained by merely repeating the cycle.

12.5.2 Efficiency of Carnot Engine

Efficiency is defined as the ratio of heat converted into work in a cycle to heat taken from the source by the working substance. It is denoted as

$$\eta = \frac{\text{Heat converted into work}}{\text{Heat taken from source}}$$

$$\eta = \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1}$$

For Carnot's engine $\dfrac{H_1}{H_2} = \dfrac{T_1}{T_2}$

Hence $\eta = 1 - \dfrac{T_2}{T_1}$

Note that efficiency of Carnot engine does not depend on the nature of the working substance. Further, if no heat is rejected to the sink, η will be equal to one. But for H_2 to be zero, T_2 must be zero. It means that efficiency η can be 100% only when $T_2 = 0$. The entire heat taken from the hot source is converted into work. This violates the second law of thermodynamics. Therefore, a steam engine can operate only between finite temperature limits and its efficiency will be less than one. It can also be argued that the Carnot cycle, being a reversible cycle, is most efficient; no engine can be more efficient than a Carnot engine operating between the same two temperatures.

12.5.3 Limitation of Carnot's Engine

You have studied about Carnot's cycle in terms of isothermal and adiabatic processes. Here it is important to note that the isothermal process will take place only when piston moves very slowly. It means that there should be sufficient time for the heat to transfer from the working substance to the source. On the other hand, during the adiabatic process, the piston moves extremely fast to avoid heat transfer. In practice, it is not possible to fulfill these vital conditions. Due to these reasons, all practical engines have efficiency less than that of Carnot's engine.

Intext Questions 12.3

1. State whether the following statements are true or false.

   (i) In a Carnot engine, when heat is taken by a perfect gas from a hot source, the temperature of the source decreases. …………..

   (ii) In Carnot engine, if temperature of the sink is decreased the efficiency of engine also decreases. ………………

2. (i) A Carnot engine has the same efficiency between 1000 K and 500 K and between TK and 1000 K. Calculate T. …………………

   (ii) A Carnot engine working between an unknown temperature T and ice point gives an efficiency of 0.68. Deduce the value of T. ……………………

WHAT YOU HAVE LEARNT

- Heat is a form of energy which produces in us the sensation of warmth.
- The energy which flows from a body at higher temperature to a body at lower temperature because of temperature difference is called heat energy.
- The most commonly known unit of heat energy is calorie. 1 cal = 4.18 J and 1 k cal = 10^3 cal.
- A graph which indicates how the pressure (P) of a system varies with its volume during a thermodynamic process is known as indicator diagram.
- Work done during expansion or compression of a gas is $P \Delta V = P (V_f - V_i)$.
- Zeroth law of thermodynamics states that if two systems are separately in thermal equilibrium with a third system, then the two systems also be in thermal equilibrium with each other.
- The sum of kinetic energy and potential energy of the molecules of a body gives the internal energy. The relation between internal energy and work is $U_i - U_f = -W$.
- The first law of thermodynamics states that the amount of heat given to a system is equal to the sum of change in internal energy of the system and the external work done.
- First law of thermodynamics tells nothing about the direction of the process.
- The process which can be retraced in the opposite direction from its final state to initial state is called a reversible process.
- The process which cannot be retraced along the same equilibrium state from final to the initial state is called an irreversible process.
- A process that occurs at constant temperature is an isothermal process.
- Any thermodynamic process that occurs at constant heat is an adiabatic process.
- The different states of matter are called its phase and the pressure and temperature diagram showing three phases of matter is called a phase diagram.
- Triple point is a point (on the phase diagram) at which solid, liquid and vapor states of matter can co-exist. It is characterized by a particular temperature and pressure.
- According to Kelvin-Planck's statement of second law, it is not possible to obtain a continuous supply of work from a single source of heat.
- According to Clausius statement of second law, heat cannot flow from a colder body to a hotter body without doing external work on the working substance.

- The three essential requirements of any heat engine are: (i) source from which heat can be drawn (ii) a sink into which heat can be rejected. (iii) Working substance which performs mechanical work after being supplied with heat.
- Carnot's engine is an ideal engine in which the working substance is subjected to four operations (i) Isothermal expansion (ii) adiabatic expansion (iii) isothermal compression and (iv) adiabatic compression. Such a cycle is called a Carnot cycle.
- Efficiency of a Carnot engine is given $\eta = 1 - \frac{H_2}{H_1}$, H_1 = Amount of heat absorbed and H_2 = Amount of heat rejected.
- $\eta = 1 - \frac{T_2}{T_1}$, T_1 = Temperature of the source, and T_2 = Temperature of the sink.
- Efficiency does not depend upon the nature of the working substance.

TERMINAL EXERCISE

1. Distinguish between the terms internal energy and heat energy
2. What do you mean by an indicator diagram? Derive an expression for the work done during expansion of an ideal gas.
3. Define temperature using the Zeroth law of thermodynamics.
4. State the first law of thermodynamics and its limitations.
5. What is the difference between isothermal, adiabatic, isobaric and isochoric processes?
6. State the Second law of thermodynamics.
7. Discuss reversible and irreversible processes with examples.
8. Explain Carnot's cycle. Use the indicator diagram to calculate its efficiency.
9. Calculate the change in the internal energy of a system when (a) the system absorbs 2000 J of heat and produces 500 J of work (b) the system absorbs 1100 J of heat and 400 J of work is done on it.
10. A Carnot's engine whose temperature of the source is 400K takes 200 calories of heat at this temperature and rejects 150 calories of heat to the sink. (i) What is the temperature of the sink? (ii) Calculate the efficiency of the engine.

ANSWERS TO INTEXT QUESTIONS

12.1

1. (i) Temperature (ii) C (iii) Joule or Calorie
2. (a) $P_2 (V_2 - V_1)$ (b) $-P_2 (V_2 - V_1)$
3. (i) retrace (ii) irreversible
4. An isothermal process occurs at a constant temperature whereas an adiabatic process occurs at constant heat.
5. At triple point all three states of matter i.e. solid, liquid and vapor can co-exist.

12.2

1. (i) Internal energy (ii) on
2. It states that the amount of heat given to a system is equal to the sum of the change in internal energy of the system and the external energy.

12.3

1. (i) False (ii) True
2. (i) 2000 K (ii) 8583.1 K

ANSWERS TO TERMINAL EXERCISE

9. (a) 1500 J (b) 1500 J
10. 300 K, 25%

13

KINETIC THEORY OF GASES

INTRODUCTION

Matter exists in three states – solid, liquid and gas. These are composed of atoms/ molecules which are held together by intermolecular forces. At room temperature, these atoms/ molecules have finite thermal energy. If thermal energy increases, molecules begin to move more freely. This state of matter is said to be the gaseous state. In this state, intermolecular forces are very weak and very small compared to their kinetic energy. Gases can be studied by considering the small scale action of individual molecules or by considering the large scale action of the gas as a whole. We can directly measure or sense, the large scale action of the gas. But to study the action of the molecules, we must use a theoretical model. The model, called the kinetic theory of gases.

Under different conditions of temperature, pressure and volume, gases exhibit different properties. In this lesson you will learn the kinetic theory of gases which is based on certain simplifying assumptions. You will also learn the kinetic interpretation of temperature and its relationship with the kinetic energy of the molecules. Why the gases have two types of heat capacities will also be explained in this lesson.

OBJECTIVES

After studying this lesson, you should be able to

- *know the primary objective of the kinetic theory of gases and to relate the temperature, volume and pressure of a gas to its speed, position and momentum;*
- *understand the basic ideas of the kinetic theory;*
- *state the assumptions of kinetic theory of gases;*
- *derive the expression for pressure;*
- *explain how r.m.s velocity and average velocity are related to temperature;*
- *derive gas laws on the basis of kinetic theory of gases;*
- *give kinetic interpretation of temperature and compute the mean kinetic energy of a gas;*
- *explain the law of equi-partition of energy;*
- *explain why a gas has two heat capacities.*

13.1 KINETIC THEORY OF GASES

You now know that matter is composed of very large number of atoms and molecules. Each of these molecules shows the characteristic properties of the substance of which it is a part. Kinetic theory of gases attempts to relate the macroscopic or bulk properties such as pressure, volume and temperature of an ideal gas with its microscopic properties such as speed and mass of its individual molecules. The kinetic theory is based on certain assumptions.

A gas at room temperature and atmospheric pressure (low pressure) behaves like an ideal gas.

13.1.1 Assumptions of Kinetic Theory of Gases

The simplest kinetic model is based on the assumptions that:

(1) The gas is composed of a large number of identical molecules moving in random directions, separated by distances that are large compared with their size

(2) The molecules undergo perfectly elastic collisions (no energy loss) with each other and with the walls of the container

(3) The transfer of kinetic energy between molecules is heat.

(4) The intermolecular forces between them are negligible.

(5) Between collisions, molecules move in straight lines with uniform velocities.

(6) Distribution of molecules is uniform throughout the container.

These simplifying assumptions bring the characteristics of gases within the range of mathematical treatment.

13.1.2 Pressure exerted by gas

To derive an expression for the pressure exerted by a gas on the walls of the container, we consider the motion of only one molecule because all molecules are identical. Moreover, since a molecule moving in space will have velocity components along x, y and z–directions, in view of assumption (6) it is enough for us to consider the motion only along one dimension, say x-axis. (Fig.13.1). Note that if there were $N = 6.023 \times 10^{23}$ molecules m^{-3}, instead of considering 3N paths, the assumptions have reduced the problem to only one molecule in one dimension. Let us consider a molecule having velocity C in the face LMNO. Its x, y and z components are u, v and w, respectively. If the mass of the molecule is m and it is moving with a speed u along x–axis, its momentum will be mu towards the wall and normal to it. On striking the wall, this molecule will rebound in the opposite direction with the same speed u, since the collision has been assumed to be perfectly elastic. The momentum of the molecule after it rebounds is (–mu). Hence, the change in momentum of a molecule is

$$mu - (-mu) = 2\ mu$$

If the molecule travels from face LMNO to the face ABCD with speed u along x–axis and rebounds back without striking any other molecule on the way, it covers a distance 2l in time $2l/u$. That is, the time interval between two successive collisions of the molecules with the wall is $2l/u$.

According to Newton second law of motion, the rate of change of momentum is equal to applied force.

$$\text{Rate of change of momentum at ABCD} = \frac{\text{Change in momentum}}{\text{Time}}$$

$$= \frac{2mu}{2l/u} = \frac{mu^2}{l}$$

This is the rate of change of momentum of one molecule. Since there are N molecules, the total rate of change of momentum or the total force on the wall ABCD of all molecules moving along x-axis with speeds u_1, u_2, u_3, ……. u_N is given by

$$\text{Force on ABCD} = \frac{m}{l}(u_1^2 + u_2^2 + u_3^2 + ... + u_N^2)$$

We know that pressure is force per unit area. Therefore, the pressure P exerted on the wall ABCD of area l^2 by the molecules moving along x-axis is given by

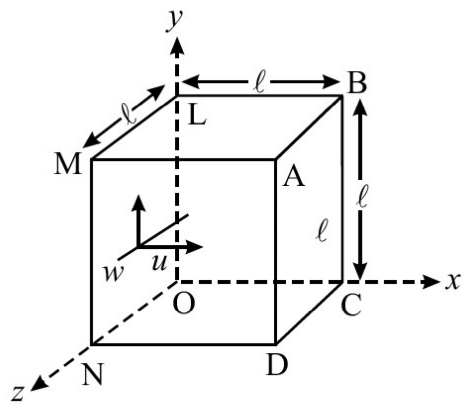


Fig. 13.1 : Motion of a molecule in a container

$$P = \frac{\frac{m}{l}\left(u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2\right)}{l^2}$$

$$P = \frac{m\left(u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2\right)}{l^3} \qquad (13.1)$$

If $\overline{u^2}$ represents the mean value of the squares of all the speed components along x-axis is

$$\overline{u^2} = \frac{(u_1^2 + u_2^2 + u_3^2 + ... + u_N^2)}{N}$$

$$N\overline{u^2} = (u_1^2 + u_2^2 + u_3^2 + ... + u_N^2)$$

Substitute above equation in (13.1)

$$P = \frac{Nm\overline{u^2}}{l^3} \qquad (13.2)$$

Since u, v and w are components of c along three axes, so that $c^2 = u^2 + v^2 + w^2$

This is also true for mean square values $\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$

Since the molecular distribution has been assumed to be isotropic, there is no preferential motion along any one edge of the cube. This means that the mean value of u^2, v^2, and w^2 are equal

$$\overline{u^2} = \overline{v^2} = \overline{w^2}$$

So that $\overline{u^2} = \dfrac{\overline{c^2}}{3}$

Substitute above equation in (13.2)

$$P = \frac{Nm}{l^3}\frac{\overline{c^2}}{3}$$

In the above equation l^3 represents volume of gas, and then the above equation takes the form

$$P = \frac{Nm}{V}\frac{\overline{c^2}}{3} \qquad (13.3)$$

Density of the gas $\rho = \dfrac{mN}{V}$ then

$$P = \frac{1}{3}\rho\overline{c^2}$$

$$\overline{c^2} = \frac{3P}{\rho} \qquad (13.4)$$

The ratio N/V is called number of molecules per unit volume (n), so that equation. (13.3) can also be expressed as

$$P = \frac{1}{3}mn\overline{c^2} \qquad (13.5)$$

Equation (13.3) can also be expressed as $PV = \dfrac{1}{3}Nm\overline{c^2} = \dfrac{1}{3}M\overline{c^2}$ (13.6)

In the above equation left hand side has macroscopic properties i.e. pressure and volume, and the right hand side has only microscopic properties i.e. mass and mean square speed of the molecules.

The following points about the above derivation should be noted:

(i) From Eqn. (13.4) it is clear that the shape of the container does not play any role in kinetic theory.

(ii) We ignored the intermolecular collisions but these would not have affected the result, because, the average momentum of the molecules on striking the walls is unchanged by their collision; same is the case when they collide with each other.

(iii) The mean square speed is not the same as the square of the mean speed.

Mean Free Path: The average distance between two successive collisions of the molecules is called mean free path. The mean free path of a molecule is given by

$$\sigma = \frac{1}{\sqrt{2}n\pi d^2}$$

where n is the number density and d the diameter of the molecules.

Example 13.1

Five molecules have speeds 1, 2, 3, 4 and 5 units. Calculate mean square speed and square of the mean speed.

Soution :

$$\text{Mean square speed} = \frac{\left(1^2+2^2+3^2+4^2+5^2\right)}{5} = \frac{\left(1+4+9+16+25\right)}{5} = \frac{55}{5} = 11\text{units}$$

$$\text{Mean or average speed} = \frac{\left(1+2+3+4+5\right)}{5} = \frac{15}{5} = 3\,\text{units}$$

Square of mean speed = 3^2 = 9 units

Thus, we see that mean square speed is not the same as square of mean speed.

Example 13.2

Calculate the pressure exerted by 10^{22} molecules of oxygen, each of mass 5×10^{-26} kg, in a hollow cube of side 10 cm where the average translational speed of molecule is 500 ms^{-1}.

Solution :

Change in momentum 2m u = $2 \times (5 \times 10^{-26}\text{ kg}) \times (500\text{ ms}^{-1})$

$$= 5 \times 10^{-23}\text{ kg ms}^{-1}$$

Time taken to make successive impacts on the same face is equal to the time spent in travelling a distance of 2×10 cm or 2×10^{-1} m.

$$\text{Hence Time} = \frac{2 \times 10^{-2}}{500} = 4\times10^{-4}\text{s}$$

$$\therefore \quad \text{Rate of change of momentum} = \frac{5 \times 10^{-23}}{4\times10^{-4}} = 1.25 \times 10^{-19}\,\text{N}$$

The force on the side due to one third molecules

$$F = \frac{1}{3} \times 1.25 \times 10^{-19} \times 10^{22} = 416.7\,\text{N}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{417}{100 \times 10^{-4}}$$

$$= 4.2 \times 10^{4}\ \text{N-m}^{-2}$$

Intext Questions 13.1

1. (i) A gas fills a container of any size but a liquid does not. Why?

   (ii) Solids have more ordered structure than gases. Why?

2. What is an ideal gas?

3. How is pressure related to density of molecules?

13.2 KINETIC INTERPRETATION OF TEMPERATURE

From Eqn. (13.6) we recall that

$$PV = \frac{1}{3} Nm\overline{c^2}$$

Also, for n moles of a gas, the equation of state is PV = n RT, where gas constant R is equal to 8.3 J mol^{-1} K^{-1}. On combining this result with the expression for pressure, we get

$$nRT = \frac{1}{3} m N \overline{c^2}$$

Multiplying both sides by $\frac{3}{2n}$ we have

$$\frac{3}{2} RT = \frac{1}{2} \frac{Nm\overline{c^2}}{n} = \frac{1}{2} m N_A \overline{c^2}$$

where $\frac{N}{n} = N_A$ is Avogadro's number. It denotes the number of atoms or molecules in one mole of a substance. Its value is 6.023×10^{23} per gram mole. In terms of N_A, we can write

$$\frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{1}{2} m \overline{c^2}$$

But $\frac{1}{2}m\overline{c^2}$ is the mean kinetic energy of a molecule. Therefore, we can write

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}kT \qquad (13.7)$$

$k = \frac{R}{N_A}$ is Boltzmann constant = $1.38\times10^{-23}JK^{-1}$.

In terms of k, the mean kinetic energy of a molecule of the gas is given as

$$\bar{\varepsilon} = \frac{1}{2}m\overline{c^2} = \frac{3}{2}kT \qquad (13.8)$$

Hence, kinetic energy of a gram mole of a gas is $\frac{3}{2}kT$.

This relationship tells us that the kinetic energy of a molecule depends only on the absolute temperature T of the gas and it is quite independent of its mass. This fact is known as the ***kinetic interpretation of temperature***.

Clearly, at T = 0, the gas has no kinetic energy. In other words, all molecular motion ceases to exist at absolute zero and the molecules behave as if they are frozen in space. According to modern concepts, the energy of the system of electrons is not zero even at the absolute zero. The energy at absolute zero is known as ***zero point energy***.

From Eqn.(13.7), we can write the expression for the square root of $\overline{c^2}$, called **root mean square speed**

$$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \qquad (13.8(a))$$

This expression shows that at any temperature T, the c_{rms} is inversely proportional to the square root of molar mass. It means that lighter molecule, on an average, move faster than heavier molecules. For example, the molar mass of oxygen is 16 times the molar mass of hydrogen. So according to kinetic theory, the hydrogen molecules should move 4 times faster than oxygen molecules. ***It is for this reason that lighter gases are in the above part of our atmosphere***. This observed fact provided an early important evidence for the validity of kinetic theory.

13.3 DEDUCTION OF GAS LAWS FROM KINETIC THEORY

(i) Boyle's Law

We know that the pressure P exerted by a gas is given by Eqn. (13.6)

$$PV = \frac{1}{3}M\overline{c^2}$$

When the temperature of a given mass of the gas is constant, the mean square speed is constant. Thus, both M and $\overline{c^2}$, on the right hand side of above Eqn. are constant. Thus, we can write

$$PV = \text{Constant (or) } P \propto \frac{1}{V} \qquad (13.9)$$

This is Boyle's law, which states that "at constant temperature, the pressure of a given mass of a gas is inversely proportional to the volume of the gas".

(ii) Charle's Law

From Eqn. (13.6) we know that

$$PV = \frac{1}{3}M\overline{c^2}$$

$$V = \frac{1}{3}\frac{M}{P}\overline{c^2}$$

For constant M and P, $V \propto \overline{c^2}$ and $\overline{c^2} \propto T$ so that $V \propto T$ (13.10)

This is Charles's law; "the volume of a given mass of a gas at constant pressure is directly proportional to temperature".

Robert Boyle (1627–1691)

British experimentalist Robert Boyle is famous for his law relating the pressure and volume of a gas (PV = constant). Using a vacuum pump designed by Robert Hook, he demonstrated that sound does not travel in vacuum. He proved that air was required for burning and studied the elastic properties of air. A founding fellow of Royal Society of London, Robert Boyle remained a bachelor throughout his life to pursue his scientific interests. Crater Boyle on the moon is named in his honor.

(iii) Gay Lussac's Law

According to kinetic theory of gases, for an ideal gas

$$P = \frac{1}{3}\frac{M}{V}\overline{c^2}$$

For constant M and V $\quad P \propto \overline{c^2}$

But $\overline{c^2} \propto T$ so that $P \propto T$ (13.11)

Which is Gay Lussac's law. It states that the pressure of a given mass of a gas is directly proportional to its absolute temperature T, if its volume remains constant.

(iv) Avogadro's Law

Let us consider two different gases 1 and 2. Then from Eqn. (13.3), we recall that

$$P_1 V_1 = \frac{1}{3} m_1 N_1 \overline{c_1^2}$$

and

$$P_2 V_2 = \frac{1}{3} m_2 N_2 \overline{c_2^2}$$

If their pressure and volume are the same, we can write $P_1V_1 = P_2V_2$

Hence
$$\frac{1}{3} m_1 N_1 \overline{c_1^2} = \frac{1}{3} m_2 N_2 \overline{c_2^2} \qquad (13.12)$$

Since the temperature is constant, their kinetic energies will be the same, i.e.

$$\frac{1}{2} m_1 \overline{c_1^2} = \frac{1}{2} m_2 \overline{c_2^2}$$

Now from equation (13.12) $N_1 = N_2$ (13.13)

That is, equal volume of ideal gases under the same conditions of temperature and pressure contains equal number of molecules. This statement is Avogadro's Law.

(v) Dalton's Law of Partial Pressure

Suppose we have a number of gases or vapors, which do not react chemically. Let their densities be ρ_1, ρ_2, ρ_3 ... and mean square speeds $\overline{c_1^2}, \overline{c_2^2}, \overline{c_3^2}$ ... respectively. We put these gases in the same enclosure. They all will have the same volume. Then the resultant pressure P will be given by

$$P = \frac{1}{3}\rho_1 \overline{c_1^2} + \frac{1}{3}\rho_2 \overline{c_2^2} + \frac{1}{3}\rho_3 \overline{c_3^2} + ...$$

Here $\frac{1}{3}\rho_1 \overline{c_1^2}, \frac{1}{3}\rho_2 \overline{c_2^2}, \frac{1}{3}\rho_3 \overline{c_3^2}$ ... signify individual (or partial) pressures of different gases or vapors. If we denote these by P_1, P_2, P_3, ... respectively we get

$$P = P_1 + P_2 + P_3 + ... \qquad (13.14)$$

In other words, the total pressure exerted by a gaseous mixture is the sum of the partial pressures that would be exerted, if individual gases occupied the space in turn. This is Dalton's law of partial pressures.

(vi) Graham's law of diffusion of gases

Graham investigated the diffusion of gases through porous substances and found that the rate of diffusion of a gas through a porous partition is inversely proportional to the square root of its density. This is known as Graham's law of diffusion. On the basis of kinetic theory of gases, the rate of diffusion through a fine hole will be proportional to the average or root mean square velocity c_{rms}.

From Eqn. (13.4) we recall that

$$\overline{c^2} = \frac{3P}{\rho}$$

or $$\sqrt{\overline{c^2}} = c_{rms} = \sqrt{\frac{3P}{\rho}}$$

That is, the root mean square velocities of the molecules of two gases of densities ρ_1 and ρ_2 respectively at a pressure P are given by

$$(c_{rms})_1 = \sqrt{\frac{3P}{\rho_1}} \quad \text{and} \quad (c_{rms})_2 = \sqrt{\frac{3P}{\rho_2}}$$

Thus $$\frac{\text{Rate of diffusion of one gas}}{\text{Rate of diffusion of other gas}} = \frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad (13.15)$$

Thus, rate of diffusion of gases is inversely proportional to the square root of their densities at the same pressure, which is Graham's law of diffusion.

Example 13.3

Calculate the root mean square speed of hydrogen molecules at 300 K. Take $m(H_2)$ as 3.347×10^{-27} kg and k $=1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

Solution :

We know that

$$c_{rms} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23}\,\text{J K}^{-1})\,(300\,\text{K})}{3.347 \times 10^{-27}\,\text{kg}}} = 1927\,\text{m s}^{-1}$$

Intext Questions 13.2

1. Five gas molecules chosen at random are found to have speeds 500 ms^{-1}, 600 ms^{-1}, 700 ms^{-1}, 800 ms^{-1}, and 900 ms^{-1}. Calculate their RMS speed.
2. If equal volumes of two non–reactive gases are mixed, what would be the resultant pressure of the mixture?
3. When we blow air in a balloon, its volume increases and the pressure inside is also more than when air was not blown in. Does this situation contradict Boyle's law?

Example 13.4

At what temperature will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure being constant (STP = Standard temperature and pressure).

Solution :

From Eqn. (13.8a), we recall that $c_{rms} \; \alpha \sqrt{T}$

Let the rms velocity at S.T.P. be c_0.

If T is the required temperature, the velocity $c = 2c_0$ as given in the problem

$$\frac{c}{c_0} = \frac{2c_0}{c_0} = \sqrt{\frac{T}{T_0}}$$

Squaring both sides, we get $4 = \frac{T}{T_0}$ (Since T_0 = 273K)

$$T = 4T_0 = 4 \times 273 = 1092 \text{ K}$$

$$T = 1029 - 273 = 819°\text{C}$$

Example 13.5

Calculate the average kinetic energy of a gas at 300 K. Given $k = 1.38 \times 10^{-23}$ JK^{-1}.

Solution :

We know that

$$\frac{1}{2}M\overline{c^2} = \frac{3}{2}kT$$

$$\bar{E} = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J}\,\text{K}^{-1})(300\,\text{K})$$

$$\bar{E} = 6.21 \times 10^{-21}\text{J}$$

13.4 THE LAW OF EQUIPARTITION OF ENERGY

We now know that kinetic energy of a molecule of a gas is given by

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

Since the motion of a molecule can be along x, y, and z directions equally probable, the average value of the components of velocity c (i.e., u, v and w) along the three directions should be equal. That is to say, for a molecule all the three directions are equivalent :

$$\bar{u} = \bar{v} = \bar{w}$$

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{\overline{c^2}}{3} \qquad (13.16)$$

$$c^2 = u^2 + v^2 + w^2$$

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Multiplying Eqn. (13.16) throughout by (1/2) m, where m is the mass of a molecule, we have

$$\frac{1}{2}m\overline{u^2} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}m\overline{w^2}$$

But $\frac{1}{2}m\overline{u^2} = E =$ total mean kinetic energy of a molecule along x–axis. Therefore, $E_x = E_y = E_z$. But the total mean kinetic energy of a molecule is (3/2) k T. Hence, we get an important result :

$$E_x = E_y = E_z = \frac{1}{2}kT$$

Since three velocity components u, v and w correspond to the three degree of freedom of the molecule, we can conclude that total kinetic energy of a dynamical system is equally divided among all its degrees of freedom and it is equal to 1/2 kT for each degree of freedom. This is the law of equipartition of energy and was deduced by Ludwig Boltzmann.

Let us apply this law for different types of gases.

So far we have been considering only translational motion. For a monoatomic molecule, we have only translational motion because they are not capable of rotation (although they can spin about any one of the three mutually perpendicular axes if it is like a finite sphere). Hence, for one molecule of a monoatomic gas, total energy

$$E = \frac{3}{2}kT \tag{13.17}$$

A diatomic molecule can be visualized as if two spheres are joined by a rigid rod. Such a molecule can rotate about any one of the three mutually perpendicular axes. However, the rotational inertia about an axis along the rigid rod is negligible compared to that about an axis perpendicular to the rod. It means that rotational energy consists of two terms such as

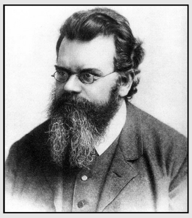
$$\frac{1}{2}I\omega_y^2,\ \frac{1}{2}I\omega_z^2$$

Now the special description of the centre of mass of a diatomic gas molecules will require three coordinates. Thus, for a diatomic gas molecule, both rotational and translational motion are present but it has 5 degrees of freedom. Hence

$$E = 3\left(\frac{1}{2}kT\right) + 2\left(\frac{1}{2}kT\right)$$

$$E = \frac{5}{2}kT \tag{13.18}$$

Ludwig Boltzmann (1844 – 1906)



Born and brought up in Vienna (Austria), Boltzmann completed his doctorate under the supervision of Josef Stefan in 1866. He also worked with Bunsen, Kirchhoff and Helmholtz. A very emotional person, he tried to commit suicide twice in his life and succeeded in his second attempt. The cause behind these attempts, people say, were his differences with Mach and Ostwald. He is famous for his contributions to kinetic theory of gases, statistical mechanics and thermodynamics. Crater Boltzmann on moon is named in his memory and honor.

13.5 HEAT CAPACITIES OF GASES

We know that the temperature of a gas can be raised under different conditions of volume and pressure. For example, the volume or the pressure may be kept constant or both may be allowed to vary in some arbitrary manner. In each of these cases, the amount of thermal energy required to increase unit rise of temperature in unit mass is different. Hence, we say that a gas has two different heat capacities.

If we supply an amount of heat ΔQ to a gas to raise its temperature through ΔT, the heat capacity is defined as

$$\text{Heat capacity} = \frac{\Delta Q}{\Delta T}$$

The heat capacity of a body per unit mass of the body is termed as specific heat capacity of the substance and is usually denoted by c. Thus

$$\text{Specific heat capacity, c} = \frac{\text{heat capacity}}{\text{m}}$$

From above two equations

$$C = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Thus, specific heat capacity of a material is the heat required to raise the temperature of its unit mass by 1°C (or 1 K). The SI unit of specific heat capacity is kilo calories per kilogram per kelvin (kcal kg^{-1} K^{-1}). It may also be expressed in joules per kg per K. For example the specific heat capacity of water is

$$1 \text{ kilo cal kg}^{-1}\text{ K}^{-1} = 4.2 \times 10^3 \text{ J kg}^{-1}\text{ K}^{-1}.$$

The above definition of specific heat capacity holds good for solids and liquids but not for gases, because it can vary with external conditions. In order to study the heat capacity of a gas, we keep the pressure or the volume of the gas constant. Consequently, we define two specific heat capacities:

(i) Specific heat at constant volume, denoted as c_V.

(ii) Specific heat at constant pressure, denoted as c_P.

(i) The specific heat capacity of a gas at constant volume (c_v) is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1K, when its volume is kept constant.

$$c_v = \left(\frac{\Delta Q}{\Delta T}\right)_v \tag{13.19}$$

(ii) The specific heat capacity of a gas at constant pressure (c_P) is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1K when its pressure is kept constant.

$$c_P = \left(\frac{\Delta Q}{\Delta T}\right)_P \tag{13.20}$$

When 1 mole of a gas is considered, we define molar heat capacity. We know that when pressure is kept constant, the volume of the gas increases. Hence in the second case note that the heat required to raise the temperature of unit mass through 1 degree at constant pressure is used up in two parts: (i) heat required to do external work to produce a change in volume of the gas, and (ii) heat required to raise the temperature of the gas through one degree (c_v). This means the specific heat capacity of a gas at constant pressure is greater than its specific heat capacity at constant volume by an amount which is thermal equivalent of the work done in expending the gas against external pressure. That is

$$c_p = W + c_v \tag{13.21}$$

13.6 RELATION BETWEEN C_P AND C_V

Let us consider one mole of an ideal gas enclosed in a cylinder fitted with a frictionless movable piston (Fig.13.2). Since the gas has been assumed to be ideal (perfect), there is no intermolecular force between its molecules. When such a gas expands, some work is done in overcoming internal pressure.

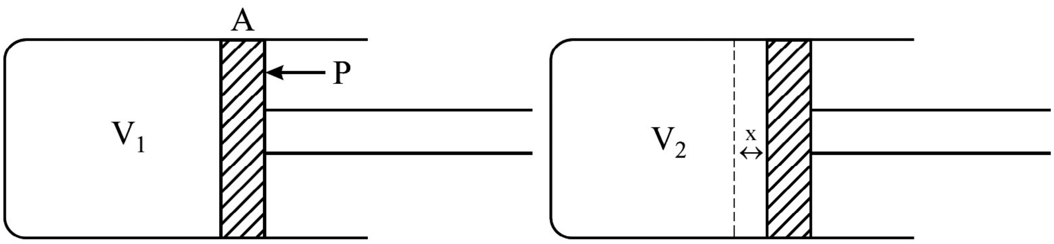


Fig.13.2 : Gas heated at constant pressure

Let P be the external pressure and A be the cross sectional area of the piston. The force acting on the piston = P × A. Now suppose that the gas is heated at constant pressure by 1 K and as a result, the piston moves outward through a distance x, as shown in Fig.13.2.

Let V_1 be the initial volume of the gas and V_2 be the volume after heating. Therefore, the work W done by the gas in pushing the piston through a distance x, against external pressure P is given by

$$W = P \times (A \times x)$$

$$W = P \times (\text{increase in volume})$$

$$W = P(V_2 - V_1)$$

We know from Eqn. (13.21) that $c_p - c_v$ = Work done (W) against the external pressure in raising the temperature of 1 mol of a gas through 1 K, i.e.

$$c_p - c_v = P(V_2 - V_1) \qquad (13.22)$$

Now applying perfect gas equation to these two stages of the gas i.e. before and after heating, we have

$$PV_1 = RT \qquad (13.23)$$

$$PV_2 = R(T+1) \qquad (13.24)$$

Subtracting Eqn. (13.23) from Eqn.(13.24), we get

$$P(V_2 - V_1) = R \qquad (13.25)$$

From Eqn. (13.22) and (13.25)

$$c_P - c_V = R \qquad (13.26)$$

Where R is in J mol^{-1} K^{-1}, Converting joules into calories, we can write

$$c_p - c_v = \frac{R}{J} \qquad (13.27)$$

Where J = 4.18 cal is the mechanical equivalent of heat.

Example 13.6

Calculate the value of c_p and c_v for a monoatomic, diatomic and triatomic gas molecules.

Solution :

We know that the average KE for 1 mole of a gas is given as

$$E = \frac{3}{2}RT$$

Now c_v is defined as the heat required to raise the temperature of 1 mole of a gas at constant volume by one degree i.e. if E_T denotes total energy of gas at TK and E_{T+1} signifies total energy of gas at (T + 1) K, then $c_v = E_{T+1} - E_T$

(i) Monoatomic gas total energy $E = \frac{3}{2}RT$

$$c_v = \frac{3}{2}R(T+1) - \frac{3}{2}RT$$

$$c_v = \frac{3}{2}R$$

Hence $\quad c_p = c_v + R$

$$c_p = \frac{3}{2}R + R = \frac{5}{2}R$$

(ii) Diatomic gas total energy $E = \frac{5}{2}RT$

$$c_v = \frac{5}{2}R(T+1) - \frac{5}{2}RT$$

$$c_v = \frac{5}{2}R$$

Hence $\quad c_p = c_v + R$

$$c_p = \frac{5}{2}R + R = \frac{7}{2}R$$

(iii) Triatomic gas total energy E = RT

$$c_v = 3R\,(T + 1) - 3RT$$

$$c_v = 3R$$

Hence $\quad c_p = c_v + R$

$$c_p = 3R + R = 4R$$

Intext Questions 13.3

1. What is the total energy of a nitrogen molecule?
2. Calculate the value of c_p and c_V for nitrogen (given, R = 8.3 J mol^{-1} K^{-1}).
3. Why do gases have two types of specific heat capacities?

WHAT YOU HAVE LEARNT

- Kinetic theory assumes the existence of atoms and molecules of a gas and applies the law of mechanics to large number of them using averaging technique.
- Kinetic theory relates macroscopic properties to microscopic properties of individual molecules.
- The pressure of a gas is the average impact of its molecules on the unit area of the walls of the container.

- Kinetic energy of a molecule depends on the absolute temperature T and is independent of its mass.
- At absolute zero of temperature, the kinetic energy of a gas is zero and molecular motion ceases to exist.
- Gas law can be derived on the basis of kinetic theory. This provided an early evidence in favor of kinetic theory.
- Depending on whether the volume or the pressure is kept constant, the amount of heat required to raise the temperature of unit mass of a gas by 1°C is different. Hence there are two specific heats of gas.

  i) Specific heat capacity at constant volume (c_V)

  ii) Specific heat capacity at constant pressure (c_P)

  These are related as $c_p - c_v = R$ and $c_p - c_v = \frac{R}{J}$

- The law of equipartition of the energy states that the total kinetic energy of a dynamical system is distributed equally among all its degrees of freedom and it is equal to (½) kT per degree of freedom.
- Total energy for a molecule of (i) a monatomic gas is (3/2) kT, (ii) a diatomic gas is (5/2) kT, and (iii) a triatomic gas is 3 kT.
- Gases have two types of specific heat capacities because they can absorb and release heat in two different ways : at constant volume (C_v) and at constant pressure (C_p).

TERMINAL EXERCISE

1. Can we use Boyle's law to compare two different ideal gases?
2. What will be the velocity and kinetic energy of the molecules of a substance at absolute zero temperature?
3. If the absolute temperature of a gas is raised four times, what will happen to its kinetic energy, root-mean square velocity and pressure?
4. What should be the ratio of the average velocities of hydrogen molecules (molecular mass = 2) and that of oxygen molecules (molecular mass = 32) in a mixture of two gases to have the same kinetic energy per molecule?
5. If three molecules have velocities 0.5, 1 and 2 kms^{-1} respectively, calculate the ratio between their root mean square and average speeds.
6. Explain what is meant by the root-mean square velocity of the molecules of a gas. Use the concepts of kinetic theory of gases to derive an expression for the root mean square velocity of the molecules in term of pressure and density of the gas.
7. (i) Calculate the average translational kinetic energy of a neon atom at 25°C. (ii) At what temperature does the average energy have half this value?

8. A container of volume of 50 cm^3 contains hydrogen at a pressure of 1.0 Pa and at a temperature of 27°C. Calculate (a) the number of molecules of the gas in the container, and (b) their root-mean square speed. ($R = 8.3$ J mol^{-1} K^{-1}, $N = 6 \times 10^{23}$ mol^{-1}. Mass of 1 mole of hydrogen molecule = 20×10^{-3} kg mol^{-1}).
9. A closed container contains hydrogen which exerts pressure of 20.0 mm Hg at a temperature of 50 K. (a) At what temperature will it exert pressure of 180 mm Hg? (b) If the root-mean square velocity of the hydrogen molecules at 10.0 K is 800 ms^{-1}, what will be their root-mean square velocity at this new temperature?
10. State the assumptions of kinetic theory of gases.
11. Find an expression for the pressure of a gas.
12. Deduce Boyle's law and Charles's law from kinetic the theory of gases.
13. What is the interpretation of temperature on the basis of kinetic theory of gases?
14. What is Avogadro's law? How can it be deduced from kinetic theory of gases.
15. Calculate the root-mean square velocity of the molecules of hydrogen at 0°C and at 100°C (Density of hydrogen at 0°C and 760 mm of mercury pressure = 0.09 kg m^{-3}).
16. Calculate the pressure in mm of mercury exerted by hydrogen gas if the number of molecules per m^3 is 6.8×10^{24} and the root-mean square speed of the molecules is 1.90×10^3 ms^{-1}. Avogadro's number 6.02×10^{23} and molecular weight of hydrogen = 2.02).
17. Define specific heat of a gas at constant pressure. Derive the relationship between c_p and c_V.
18. Define specific heat of gases at constant volume. Prove that for a triatomic gas $c_V = 3R$
19. Calculate c_P and c_V for argon. Given $R = 8.3$ J mol^{-1} K^{-1}.

ANSWERS TO INTEXT QUESTIONS

13.1

1. (i) Because in a gas the cohesive force between the molecules are extremely small as compared to the molecules in a liquid.

   (ii) Because the molecules in a solid are closely packed. The bonds between the molecules are stronger giving a ordered structure.
2. The gas which follows the kinetic theory of molecules is called as an ideal gas.
3. $P = \frac{1}{3}\rho\overline{c^2}$

13.2

1. Average speed $\overline{c} = \dfrac{500+600+700+800+900}{5} = \dfrac{3500}{5} = 700\,\text{m/s}$

   Average value of $\overline{c^2} = \dfrac{500^2+600^2+700^2+800^2+900^2}{5} = 510000\,\text{m}^2/\text{s}^2$

   $c_{rms} = \sqrt{\overline{c^2}} = \sqrt{510000} = 714\,\text{m/s}$

   c_{rms} and $\overline{c}$ are not same.

2. The resultant pressure of the mixture will be the sum of the pressure of gases 1 and 2 respectively i.e. $P = P_1 + P_2$.

3. Boyle's law is valid when the temperature of the gas remains constant, and the situation described involves changing the temperature by adding more air, which affects both the pressure and volume simultaneously. Therefore, Boyle's law is not applicable.

13.3

1. For each degree of freedom, energy $= \dfrac{1}{2}kT$

   $\therefore$ For 5 degrees of freedom for a molecule of nitrogen, total energy $= \dfrac{5}{2}kT$

2. c_V for a diatomic molecule $= \dfrac{5}{2}R$

   $c_v = \dfrac{5}{2} \times 8.3 = 20.75\,\text{J mol}^{-1}\text{K}^{-1}$

   $c_p = c_v + R = 20.75 + 8.3 = 29.05$ J mol^{-1} K^{-1}

3. Gases have two types of specific heat capacities because they can absorb and release heat in two different ways : at constant volume (C_v) and at constant pressure (C_p).

ANSWERS TO TERMINAL EXERCISE

2. Zero
3. Becomes 4 times, doubles, becomes 4 times.
4. 4 : 1
5. 2
7. 6.18×10^{-21} ms^{-1}, -124°C
8. (a) 12×10^{20} (b) 7.9×10^{11} ms^{-1}
9. (a) 2634°C (b) 2560 ms^{-1}
15. 1800 ms^{-1}, 2088 ms^{-1}
16. 3.97×10^3 Nm^{-2}
19. 12.45 J mol^{-1} K^{-1}, 20.75 J mol^{-1} K^{-1}.

14

SOUND WAVES

INTRODUCTION

Often we hear sound when people talk, listen to music, play a musical instrument, etc. But have you ever wondered what sound is and how it originated? In this chapter, let us find answers to all these questions. Before learning about sound waves lets us know the types of waves.

OBJECTIVES

After studying this lesson, you should be able to

- *differentiate waves and know the difference between transverse and longitudinal waves;*
- *define standing and Progressive waves;*
- *explain superposition and reflection of waves;*
- *explain modes in closed and open ended organ pipes;*
- *explain the phenomina of beats;*
- *explain Doppler effect and write equations for different cases.*

14.1 FUNDAMENTAL CONCEPT OF WAVES

Waves are a fundamental concept in physics and are essential for understanding many natural phenomena. They are disturbances that travel through a medium or space, carrying energy from one place to another without causing any net displacement of the medium itself. In simpler terms, waves transfer energy, not matter

Key characteristics of waves are :

1. **Amplitude :** The maximum displacement of a wave from its equilibrium position. In the case of a transverse wave, like a water wave, it is the height of the wave crest or depth of the wave trough. In a longitudinal wave, like a sound wave, it is the maximum compression or rarefaction of the medium.
2. **Wavelength :** The distance between two consecutive points that are in phase (e.g., two crests or two troughs) in a wave. It is often denoted by the Greek letter lambda (λ).
3. **Frequency :** The number of complete oscillations or cycles of a wave that occur in one second. It is measured in Hertz (Hz), where 1 Hz is equal to one cycle per second.

4. **Period :** The time taken to complete one full cycle of a wave. It is the reciprocal of frequency (T = 1/f).
5. **Speed :** The rate at which a wave travels through a medium. It is given by the product of wavelength and frequency ($v = \lambda f$).

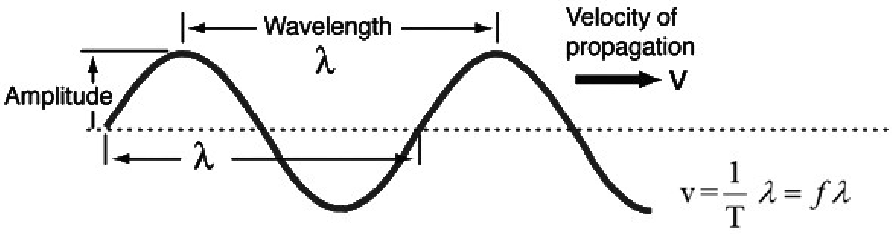


Fig. 14.1 : Wave

14.2 TYPES OF WAVES

14.2.1 Mechanical Waves

Mechanical waves require a physical medium to travel through. When the medium is disturbed, the wave travels through it, carrying energy. Mechanical waves can be further classified into two types :

a. **Transverse Waves :** In transverse waves, the particles of the medium oscillate perpendicular to the direction of wave propagation. Examples include water waves and electromagnetic waves.

b. **Longitudinal Waves :** In longitudinal waves, the particles of the medium oscillate parallel to the direction of wave propagation. An example is a sound wave traveling through air or any other material medium.

14.2.2 Electromagnetic Waves

Electromagnetic waves are a special type of transverse wave that can travel through a vacuum (empty space) as well as through various materials. They are created by the oscillation of electric and magnetic fields. Examples of electromagnetic waves include radio waves, microwaves, infrared waves, visible light, ultraviolet rays, X-rays, and gamma rays.

14.2.3 Surface Waves

Surface waves are a combination of transverse and longitudinal waves that occur at the interface between two different media, such as water and air. They travel along the boundary and have both horizontal and vertical motion. Examples include ocean waves and seismic waves (earthquakes).

14.2.4 Standing Waves

Standing waves are formed when two identical waves travelling in opposite directions superpose, creating specific points of no displacement called nodes and points of maximum displacement called antinodes. They do not propagate through space like travelling waves but appear to "stand still."

These are the primary types of waves encountered in physics. Each type of wave has its unique properties and behaviours, and they play a crucial role in various scientific and technological applications.

14.2.5 Difference between Transverse and Longitudinal wave

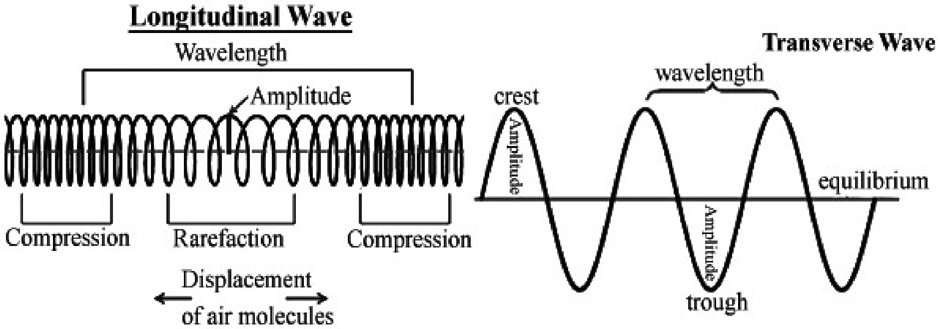


Fig. 14.2 : Transverse and Longitudinal Wave

The main difference between transverse and longitudinal waves lies in the direction of particle oscillation. Particle oscillation direction for Transverse Waves is perpendicular to the direction of wave propagation and for Longitudinal Waves is parallel to the direction of wave propagation. Transverse Waves is represented by crests and troughs, forming a wave-like pattern whereas Longitudinal Waves is represented by compressions and rarefactions, appearing as a series of close-together and spread-out regions.

14.3 PROGRESSIVE WAVE

In a progressive wave, the displacement relation describes how the individual particles of the medium oscillate as the wave travels through the medium. The displacement relation is a mathematical expression that relates the position of a particle to its time of oscillation.

Let's consider a one-dimensional progressive wave travelling along the x-axis. The general displacement relation for a progressive wave can be represented as

$$y\,(x,\,t) = A \sin\,(kx + \omega t + \phi) \qquad (14.1)$$

Where y (x, t) is the displacement of a particle in the medium at position x and time t. A is the amplitude of the wave, which represents the maximum displacement of the particles from their equilibrium position. Whereas k is the wave number, it is related to the wavelength as $k = \lambda/2\pi$. x is the position of the particle in the medium along the x-axis. ω is the angular frequency of the wave, which is related to the frequency (f) as $\omega = 2\,\pi f$. t is the time at which the displacement is measured. ϕ is the phase constant or phase angle, which accounts for the initial phase of the wave at $x = 0$ and $t = 0$.

The displacement relation describes the shape of the wave and how it propagates through the medium. As time progresses, each particle at a different position x will undergo a sinusoidal oscillation with the same frequency ω but with a phase shift determined by the value of kx and ϕ. In summary, the displacement relation in a progressive wave is a sinusoidal function that represents how the particles of the medium oscillate as the wave passes through them, and it depends on the amplitude, wave number, angular frequency, and phase constant of the wave.

14.3.1 Speed of travelling wave

The speed of a travelling wave depends on the medium through which it propagates. The general formula for the speed of a wave in a medium is given by:

$$v = \frac{1}{T}\lambda$$

Where v is the speed of the wave λ is the wavelength of the wave, T is the period of the wave.

Alternatively, the speed of a wave can also be expressed as $v = f \cdot \lambda$, where f is the frequency of the wave.

The speed of sound wave depends on the medium through which it travels. In general, the speed of sound in a particular medium is determined by the elasticity and density of that medium. In gases, the speed of sound depends on temperature, while in solids and liquids, it primarily depends on their elastic properties.

The formula for the speed of sound in a medium is given by:

$$v = \sqrt{\frac{E}{\rho}}$$

Where, v is the speed of sound in the medium, E is the elastic modulus (or Young's modulus) of the medium, which is a measure of the medium's elasticity, ρ is the density of the medium.

In gases, the speed of sound is approximately 343 meters per second (m/s) at room temperature (about 20° Celsius or 68° Fahrenheit). This value may vary slightly depending on the specific gas and its temperature.

In liquids, the speed of sound is generally higher than in gases and varies depending on the properties of the liquid. For example, in water at room temperature, the speed of sound is around 1482 m/s.

In solids, the speed of sound is much higher than in gases and liquids. It can range from a few thousand meters per second to tens of thousands of meters per second, depending on the type of material and its density.

Keep in mind that these values are approximate and can change based on the specific conditions of the medium, such as temperature and pressure.

14.3.2 The superposition of waves

The principle of superposition of waves states that when two or more waves pass through the same medium, their displacements at any point and time add algebraically to produce a new wave. This principle holds for linear waves, where the waves do not interact with each other or the medium they travel through.

Suppose two wave pulses travel in opposite directions on a slinky. What happens when they meet? Do they knock each other out? To answer these questions, let us perform an activity.

Activity 14.1

Produce two wave crests of different amptitudes on a stretched slinky, as shown in Fig. 14.3 and watch carefully. The crests are moving in the opposite directions. They meet and overlap at the point midway between them Fig. 14.3 (b) and then separate out. Thereafter, they continue to move in the same direction in which they were moving before crossing each other. Moreover, their shape also does not change Fig. 14.3 (c).

Now produce one crest and one trough on the slinky as shown in Fig. 14.3 (d). The two are moving in the opposite direction. They meet Fig. 14.3 (e), overlap and then separate out. Each one moves in the same direction in which it was moving before crossing and each one has the same shape as it was having before crossing. Repeat the experiment again and observe carefully what happens at the spot of overlapping of the two pulses Fig. 14.3(b) and (e). You will note that when crests overlap, the resultant is more and when crest overlaps the through, the resultant is on the side of crest but smaller size.We may summarize this result as : *At the points where the two pulses overlap, the resultant displacement is the vector sum of the displacements due to each of the two wave pulses.* ***This is called the principle of superposition.***

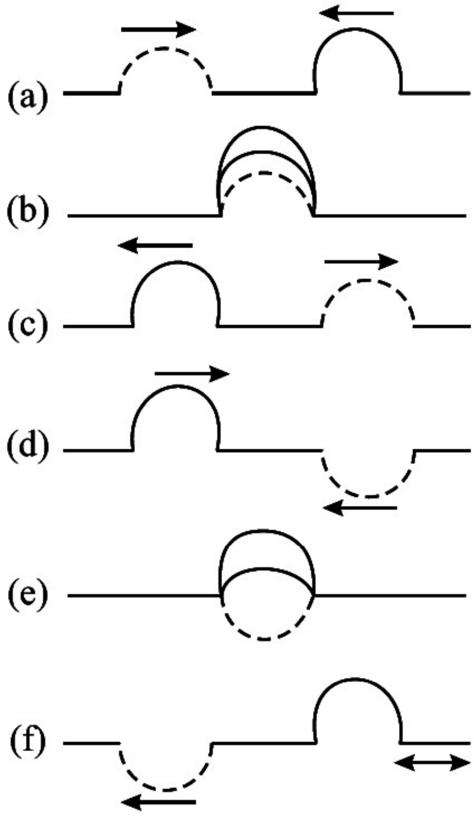


Fig. 14.3 : Illustrating principle of superpositionof waves

This activity demonstrates not only the principle of superposition but also shows that two or more waves can traverse the same space independent of each other. Each one travels as if the other were not present. This important property of the waves enable us to tune to a particular radio station even though the waves broadcast by a number of radio stations exist in space at the same time. We make use of this principle to explain the phenomena of *interference of waves, formation of beats and stationary or standing waves.*

Let's consider two simple mathematical examples to demonstrate the principle of superposition:

Example 14.1

Superposition of Two Waves with the Same Frequency and Amplitude

Suppose we have two waves described by the following equations

Wave 1 : $y_1(x,t) = A.\sin(kx - \omega t)$

Wave 2 : $y_2(x,t) = A.\sin(kx - \omega t + \phi)$

Where :

- A is the amplitude of both waves.
- k is the wave number (related to the wavelength as $k = \frac{2\pi}{\lambda}$)
- ω is the angular frequency (related to the frequency as $\omega = 2\pi f$).
- ϕ is the phase difference between the two waves.

The resulting wave obtained by superposition is :

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$y(x,t) = A.\sin(kx - \omega t) + A.\sin(kx - \omega t + \phi)$$

Using trigonometric identities (sum - to - product formula), we can simplify this expression:

$$y(x,t) = 2A.\cos\left(\frac{\phi}{2}\right).\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

This demonstrates constructive interference, where the two waves with the same frequency and amplitude reinforce each other, resulting in a new wave with an increased amplitude.

Example 14.2

Superposition of Two Waves with Different Frequencies and Amplitudes

Now, let's consider two waves with different frequencies and amplitudes:

Wave 1 : $y_1(x,t) = A_1.\sin(k_1 x - \omega_1 t)$

Wave 2 : $y_2(x,t) = A_2.\sin(k_2 x - \omega_2 t)$

Where

- A_1 and A_2 are the amplitude of both waves.
- k_1 and k_2 are the wave number of the two waves (related to their respective wavelength)
- ω_1 and ω_2 are the angular frequencies of the two waves (related to their respctive frequencies).

The resulting wave obtained by superposition is :

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$y(x,t) = A_1.\sin(k_1 x - \omega_1 t) + A_2.\sin(k_2 x - \omega_2 t)$$

There is no immediate simplification for this example, but it shows that the superposition of two waves with different frequencies and amplitudes create a more complex wave pattern, combining the characteristics of both individual waves.

These examples demonstrate how the principle of superposition allows us to analyse and understand the behaviour of waves when they interact, leading to various phenomena like interference patterns in sound and light waves.

14.4 REFLECTION OF WAVES

Reflection of waves is a phenomenon where a wave encounters a boundary or interface between two different media and bounces back into the original medium. When a wave reflects, it changes its direction but retains its original characteristics, such as frequency, wavelength, and amplitude.

The reflection of waves follows specific principles based on the type of wave and the properties of the medium. Here are some key points about wave reflection:

14.4.1 Reflection of Mechanical Waves

When mechanical waves, such as water waves or sound waves, encounter a reflecting surface, they bounce back.

The angle of incidence (the angle between the incident wave and the normal to the reflecting surface) is equal to the angle of reflection (the angle between the reflected wave and the normal).

This phenomenon obeys the law of reflection, which states that the incident angle (θ_i) and the reflected angle (θ_r) are related as $\theta_i = \theta_r$.

14.4.1.1 Reflection of waves - Different cases

When a wave propagates through a medium, it reflects when it encounters the boundary of the medium. The wave before hitting the boundary is known as the incident wave. The wave after encountering the boundary is known as the reflected wave. How the wave is reflected at the boundary of the medium depends on the boundary conditions; waves will react differently if the boundary of the medium is fixed in place or free to move Fig. 14.4. A fixed boundary condition exists when the medium at a boundary is fixed in place so that it cannot move. A free boundary condition exists when the medium at the boundary is free to move.

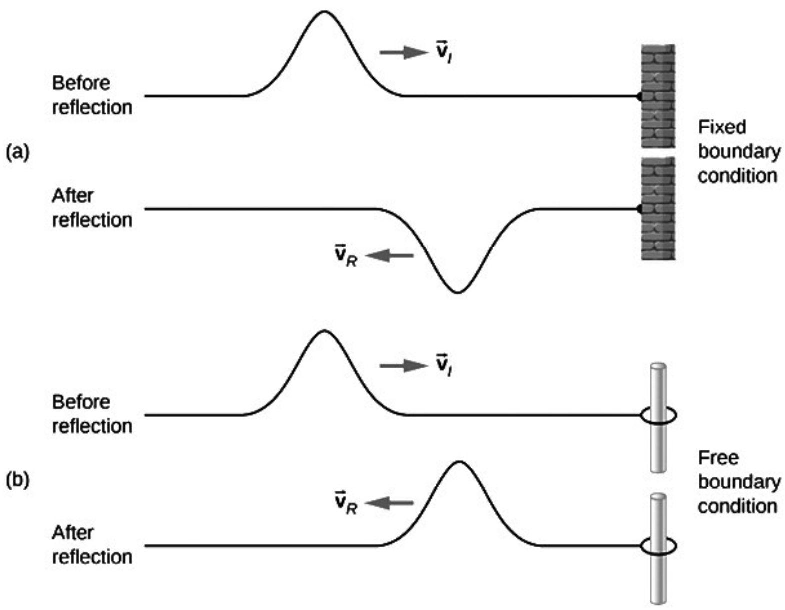


Fig. 14.4 : Fixed and Free boundary condition

Fig. 14.4 (a) One end of a string is fixed so that it cannot move. A wave propagating on the string, encountering this fixed boundary condition, is reflected 180° (π rad) out of phase with respect to the incident wave. Fig. 14.4 (b) One end of a string is tied to a solid ring of negligible mass on a friction less lab pole, where the ring is free to move. A wave propagating on the string, encountering this free boundary condition, is reflected in phase 0° (0 rad) with respect to the wave.

In some situations, the boundary of the medium is neither fixed nor free. Consider Fig. 14.5 (a), where a low-linear mass density string is attached to a string of a higher linear mass density. In this case, the reflected wave is out of phase with respect to the incident wave. There is also a transmitted wave that is in phase with respect to the incident wave. Both the incident and the reflected waves have amplitudes less than the amplitude of the incident wave. If the tension is the same in both strings, the wave speed is higher in the string with the lower linear mass density.

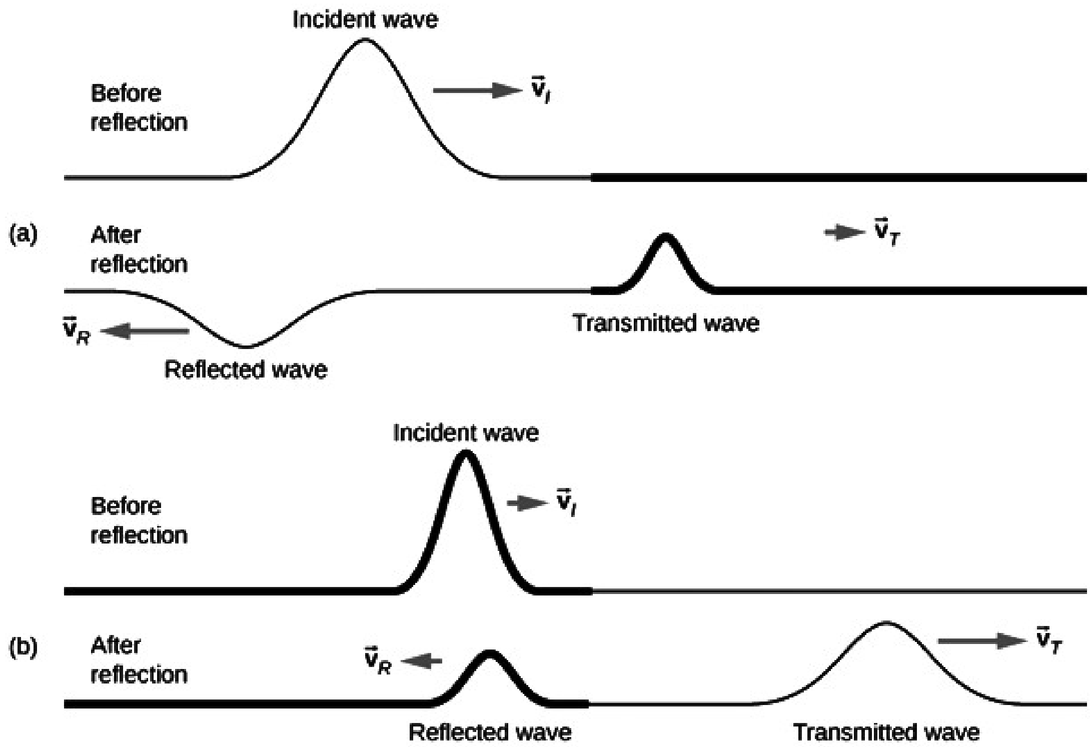


Fig. 14.5 : Reflection of wave at fixed and free end

Fig. 14.5 (a) Waves travelling along two types of strings: a thick string with a high linear density and a thin string with a low linear density. Both strings are under the same tension, so a wave moves faster on the low-density string than on the high-density string. (a) A wave moving from a low-speed to a high-speed medium results in a reflected wave that is 180° (π rad) out of phase with respect to the incident pulse (or wave) and a transmitted wave that is in phase with the incident wave Fig. 14.5 (b). When a wave moves from a low-speed medium to a high-speed medium, both the reflected and transmitted wave are in phase with respect to the incident wave.

14.5 OPEN END AND CLOSED END ORGAN PIPES

14.5.1 Closed end organ pipe

A pipe, which is closed at one end and open at other end is called closed pipe. A sound wave sent through a closed pipe gets reflected at the closed end of the pipe. Then incident and reflected waves which are having same frequency, travelling in the opposition direction are superimposed to form stationary waves. The point at which amplitude is zero is called node and the point at which amplitude is maximum is called antinode.

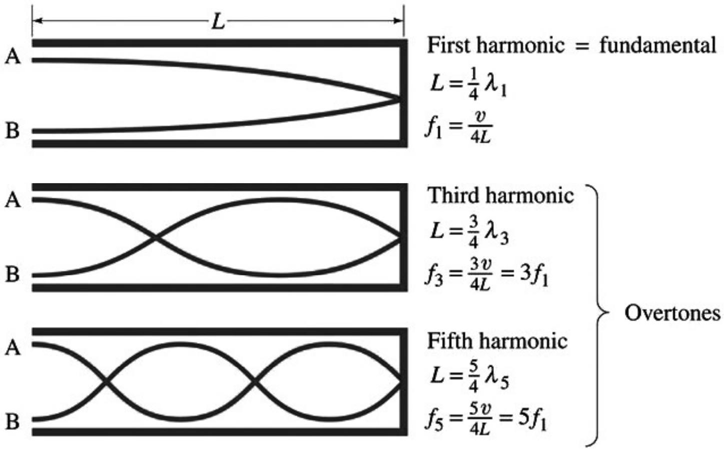


Fig. 14.6 : Normal modes of an air column open at one end

The stationary wave in closed pipe, which has atleast a node at closed end and an antinode at open end of the pipe, is known as first harmonic in closed pipe. Then length of the pipe (l) is equal to one fourth of the wave length.

$$\therefore\ l = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l$$

If 'f_1' is fundamental frequency then

$$f_1 = \frac{v}{4l} \qquad (14.2)$$

To form the next harmonic in closed pipe, two nodes and two antinodes should be formed. So that there is possible to form third harmonic in closed pipe. Since one more node and antinode should be included.

Then length of the pipe (l) is equal to $\frac{3}{4}$ of the wavelength

$\therefore\ l = \frac{3\lambda_3}{4}$ where 'λ_3' is wavelength of third harmonic

$$\lambda_3 = \frac{4l}{3}$$

If 'f_3' is third harmonic frequency (first overtone)

$$\therefore\ f_3 = \frac{v}{\lambda_3} = \frac{3v}{4l}$$

$f_3 = 3f_1$ (14.3)

Similarly the next overtone in the close pipe is only fifth harmonic it will have three nodes and 3 antinodes between the closed end and open end.

Then length of the pipe is equal to $\frac{5}{4}$ of wave length (λ_5)

$\therefore\ l = \frac{5\lambda_5}{4}$ where 'λ_5' is wavelength of fifth harmonic

$$\lambda_5 = \frac{4l}{5}$$

If 'f_5' is fifth harmonic frequency (second overtone)

$$f_5 = \frac{v}{\lambda_5} = \frac{5v}{4l}$$

$f_5 = 5f_1$ (14.4)

$\therefore$ The frequencies of higher harmonics can be determined by using the same procedure. Therefore from the above we can say that only odd harmonics are formed.

Therefore the ratio of the frequencies of harmonics in closed pipe can be written as

$f_1 : f_3 : f_5 :$ .... $= f_1 : 3f_1 : 5f_1 :$ ....

$f_1 : f_3 : f_5 :$ .... $= 1 : 3 : 5 :$ ....

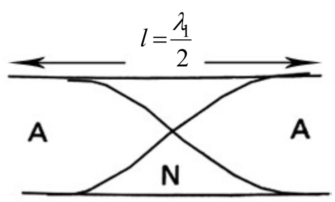
14.5.2 Open End organ pipe

A pipe, which is opened at both ends is called open pipe. A sound waves sent through an open pipe gets reflected by the earth. Then incident and reflected waves which are in same frequency and travelling in the opposite directions are super imposed to form stationary waves.

The stationery wave in open pipe should have two antinodes at two ends of the pipe with a node between them as shown in Fig. 14.7 (a)

$\therefore$ The vibrating length (l) = half of the wavelength

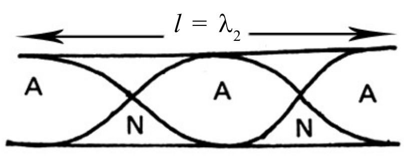
$$\left(\frac{\lambda_1}{2}\right)$$



(a) Fundamental or first harmonic

$$l = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2l$$

fundamental frequency $f_1 = \frac{v}{\lambda_1}$

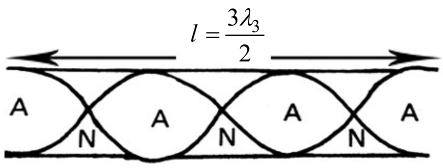


(b) Second harmonic

where, v is the velocity of sound in air

$$f_1 = \frac{v}{2l} \qquad (14.5)$$

For second harmonic (first overtone) will have one more node and antinode than the fundamental as shown in the Fig. 14.7 (b).



(c) Third harmonic

Fig. 14.7 : Standing waves in open pipe first three harmonics are shown

If λ_2 is wavelength of second harmonic $l = \frac{2\lambda_2}{2} \Rightarrow \lambda_2 = \frac{2l}{2} = l$

If 'f_2' is frequency of second harmonic, then $f_2 = \frac{v}{\lambda_2} = \frac{v \times 2}{2l} = \frac{v}{l}$

$$f_2 = 2f_1 \qquad (14.6)$$

Similarly, third harmonic (second overtone) will have three nodes and four antinodes as shown in above figure.

If λ_3 is wavelength of third harmonic $l = \frac{3\lambda_3}{2}$

If 'f_3' is frequency of third harmonic then

$$f_3 = \frac{v}{\lambda_3} = \frac{v \times 3}{2l} = 3f_1$$

$$f_3 = 3f_1 \qquad (14.7)$$

Similarly we can find the remaining or higher harmonic frequencies i.e., f_4, f_5 etc., can be determined in the same way.

Threfore, the ratio of the harmonic frequencies in the open pipe can be written as given below.

$f_1 : f_2 : f_3 : .... = 1 : 2 : 3 : ....$

14.6 ECHOES

In the context of sound waves, reflection from distant surfaces can lead to the phenomenon of echoes as shown in the Fig. 14.8.

Echoes are reflections of sound waves that arrive at the listener's ear after a brief time delay, usually due to reflections from large and distant objects.

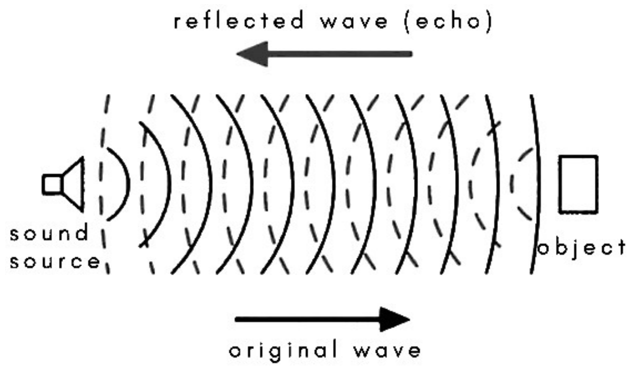


Fig. 14.8 : Formation of Echoes

Wave reflection plays a significant role in many natural and man-made processes. For example, it is crucial in the formation of standing waves, echoes, and the behaviour of electromagnetic waves in mirrors and other reflective surfaces. Understanding wave reflection is essential for various applications, including architectural acoustics, radar technology, and optics.

14.7 BEATS

We have seen that superposition of waves of same frequency propagating in the same direction produces interference. Let us now investigate what would be the outcome of superposition of waves of nearly the same frequency. First let us perform an activity.

Activity 14.2

Take two tuning forks of same frequency 512 Hz. Let us name them as A and B. Load the prong of the tuning fork B with a little wax. Now sound them together by a rubber hammer. Press their stems against a table top and note what you observe. You will observe that the intensity of sound alternately becomes maximum and minimum. These alternations of maxima and minima of intensity are called beats. One alternation of a maximum and a minimum is one beat. On loading the prong of B with a little more wax, you will find that number of beats increase. On further loading the prongs of B, no beats may be heard. The reason is that our ear is unable to hear two sounds as separate produced in an interval less than one tenths of a second. Let us now explain how beats are produced.

(a) Production of beats : Suppose we have two tuning forks A and B of frequencies N and N + n respectively; n is smaller than 10. In one second, A completes N vibrations but B completes N + n vibrations. That is, B completes n more vibrations in one second than the tuning fork A. In other words, B gains n vibrations over A in 1s and hence it gains 1 vibration in (1/n)s and half vibration over A in (1/2n)s. Suppose at t = 0, i.e. initially, both the tuning forks were vibrating in the same phase. Then after (1/2n)s, B will gain half vibration over A. Thus after $\frac{1}{2n}$s it will vibrate in oposite phase. If A sends a wave of compression then B sends a wave of rarefaction to the observer. And, the resultant intensity received by the ear would be zero. After (1/n)s, B would gain one complete vibration. If now A sends a wave of compression, B too would send a wave of compression to the observer. The intensity observed would become maximum. After (3/2n)s, the two forks again vibrate in the opposite phase and hence the intensity would again become minimum. This process would continue. The observer would hear 1 beat in (1/n)s, and hence n beats in one second. Thus, the *number of beats heard in one second equals the difference in the frequencies of the two tuning forks.* If more than 10 beats are produced in one second, the beats are not heard as separate. The beat frequency is n and beat period is $1/n$.

(b) Graphic method : Draw a 12 cm long line. Divide it into 12 equal parts of 1 cm. On this line draw 12 wavelengths each 1 cm long and height 0.5 cm. This represents

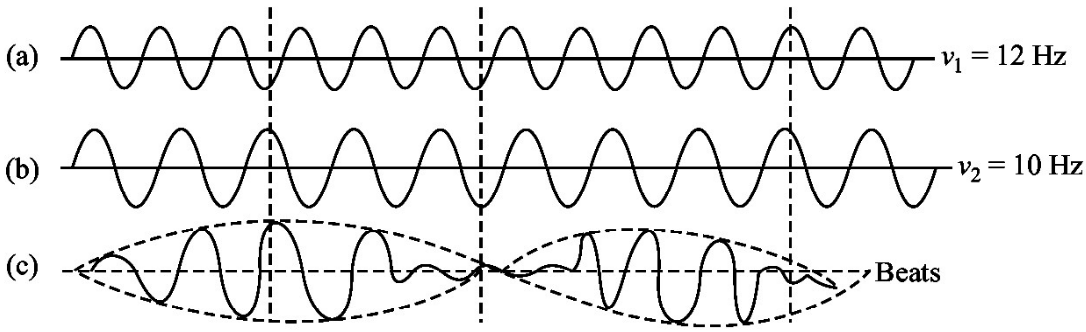


Fig. 14.9 : (a) Displacement time graph of frequency 12 Hz. (b) displacement time graph of frequency 10 Hz. Superposition of the two waves produces 2 beats per second.

a wave of frequency 12 Hz. On the line (b) draw 10 wavelengths each of length 1.2 cm and height 0.5 cm. This represents a wave of frequency 10 Hz. (c) represents the resultant wave. Fig, 14.9 is not actual waves but the displacement time graphs. Thus, the resultant intensity alternately becomes maximum and minimum. The number of beats produced in one second is Δv. Hence, the beat frequency equals the difference between the frequencies of the waves superposed.

Example 14.3

A tuning fork of unknown frequency gives 5 beats per second with another tuning of 500 Hz. Determine frequency of the unknown fork.

Solution :

$v' = v \pm n = 500 \pm 5$

$\Rightarrow$ The frequency of unknown tuning fork is either 495 Hz or 505 Hz.

Example 14.4

In an interference pattern, the ratio of maximum and minimum intensities is 9. What is the amplitude ratio of the superposing waves?

Solution :

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 \Rightarrow 9 = \left(\frac{1+r}{1-r}\right)^2, \text{ where } r = \frac{a_2}{a_1}.$$

Hence, are can write

$$\frac{1+r}{1-r} = 3$$

You can easily solve it to get $r = \frac{1}{2}$, i.e., amplitude of one wave is twice that of the other.

Doppler (1803 - 1853)

Doppler was born in Austria in 1803. Doppler started elementary education at the age of 13. After completion, he moved on to secondary education at a school in Linz. Doppler's proficiency in mathematics was discovered by Sion Stampfer, a mathematician in Salzburg. In 1829, he was chosen for an assistant position to Professor Adam Von Burg at the Polytechnic Institute of Vienna, where he continued his studies. He moved to USA in 1835. In 1842, at the age of 38, Doppler gave a lecture to the Royal Bohemian Society of Sciences and subsequently published a paper on the coloured light of the binary stars and some other stars of the heavens. In this work, Doppler postulated his principle (later named the Doppler effect) that the observed frequency of a wave depends on the relative speed of the source and the observer, and he later tried to use this concept to explain the visible colours of binary stars (this hypothesis was later proven wrong). Doppler died on 17 March 1853 at age 49.

14.8 DOPPLER EFFECT

The Doppler effect is a phenomenon observed when there is relative motion between a source of waves and an observer. It describes the apparent change in frequency or wavelength of waves as perceived by the observer due to this motion. The Doppler effect is most commonly associated with sound waves, but it applies to any type of wave, including light waves (known as the optical Doppler effect).

The theory of the Doppler effect for sound waves can be explained as follows:

14.8.1 Moving Source and Stationary Observer

If the source of sound waves is moving with a velocity (v_s) and the observer is stationary, the apparent frequency (f') perceived by the observer can be given as:

$$f' = \frac{v}{v - v_s} \cdot f$$

14.8.2 Stationary Source and Moving Observer

If the source of sound waves is stationary, and the observer is moving with a velocity (v_o),the apparent frequency (f') perceived by the observer can be given as:

$$f' = \frac{v + v_o}{v} \cdot f$$

14.8.3 Moving Source and Moving Observer

In the general case where both the source and the observer are in motion, the apparent frequency (f') perceived by the observer can be given as:

$$f' = \frac{v + v_o}{v - v_s} . f$$

Where f' is the apparent frequency perceived by the observer

f is the actual frequency emitted by the source

v is the speed of sound in the medium (constant for a given medium)

v_o is the velocity of the observer (positive if moving towards the source, negative if moving away), v_s is the velocity of the source (positive if moving away from the observer, negative if moving towards).

In these equations, if the apparent frequency (f') is greater than the actual frequency (f), it is called a "blue shift" (an increase in frequency). If the apparent frequency (f') is lower than the actual frequency (f), it is called a "redshift" (a decrease in frequency). The Doppler effect has various practical applications, including in radar systems, astronomy, and medical imaging.

Intext Questions 14.1

1. What are the basic characteristics of waves?
2. What are different types of waves?
3. Define Longitudinal and transverse waves and write the differences between them.
4. What are progressive waves? What is the speed of progressive waves?
5. A guitar string has a length of 0.75 meters. The tension in the string is 80 newtons, and the linear mass density is 0.02 kg/m. Calculate the speed of the wave travelling along the string.
6. A sound wave with a frequency of 440 Hz is produced in air. If the speed of sound in air is 343 meters per second (m/s), what is the wavelength of the sound wave?
7. Explain the principle of superposition of waves.
8. Explain the reflection of waves in different cases.
9. Explain the formation of different harmonics in (a) Closed end (b) Open end organ pipes.
10. An open-end organ pipe has a length of 2.4 meters. Calculate the fundamental frequency (first harmonic) of this pipe. Take the speed of sound in air to be approximately 343 meters per second (m/s)
11. An open-end organ pipe produces a third harmonic with a frequency of 225 Hz. Determine the length of the pipe. Take the speed of sound in air to be approximately 343 meters per second (m/s).
12. The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone (third harmonic) of an organ pipe open at both ends. What is the length of organ pipe open at both the ends.

13. How echoes are produced?
14. What are Beats?
15. Calculate the number of beats produced when two waves of frequency 10 Hz and 15 Hz overlap.
16. Two sound waves with frequencies of 320 Hz and 328 Hz are produced simultaneously. Calculate the beat frequency.
17. What is Doppler effect? Explain the Doppler effect in different cases?

WHAT YOU HAVE LEARNT

- Mechanical waves, Surface waves, Standing waves, Progressive waves are different types of waves.
- The speed of sound wave depends on density, temperature, elastic properties of the medium.
- Principle of super position of waves states that when two or more waves pass through the same medium, their displacements at any point and time add algebraically to produce a new waves
- When a wave reflects from fixed boundary and free boundary gives different results
- The ratio of the frequencies of harmonics in a closed pipe is

  $f_1 : f_3 : f_5 \ldots = 1 : 3 : 5 : \ldots$
- The ratio of the frequencies of harmonics in a open pipe is

  $f_1 : f_2 : f_3 \ldots = 1 : 2 : 3 : \ldots$
- Echoes are formed due to the reflection of sound waves from a distant object.
- Beats are produced when two waves of slightly different frequencies interfere with each other.
- When there is a relative motion between source and observer Doppler effect can be observed.
- Doppler effect is common in sound waves, but can be observed in any waves.
- The apparent frequency perceived by observer due to Doppler effect can be calculated for different cases.

TERMINAL EXERCISE

1. Mention various types of waves and give examples.
2. On what factors the speed of sound depends.
3. Explain the superposition of waves with mathematical equations.
4. Explain the reflection of waves in different cases.

5. Derive the equation for harmonics in closed and open end organ pipes.
6. How echoes are produced?
7. Explain the formation beats with diagram.
8. If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.
9. The fundamental frequency of a closed organ pipe is 400 Hz. What will be the fundamental frequency of oscillation of an open organ pipe of the same length?

ANSWERS TO INTEXT QUESTIONS

5.

Given

Length of the string (L) = 0.75 meters

Tension in the string (T) = 80 newtons

Linear mass density (m) = 0.02 kg/m

The speed of the wave on the string is given by the formula:

$$v = \sqrt{\frac{T}{\mu}}$$

Substitute the values:

$$v = \sqrt{\frac{80\text{N}}{0.02\,\text{kg/m}}}$$

$$v = \sqrt{4000\,\text{m}^2/\text{S}^2}$$

v = 63.24 m/s

6. Given

Frequency of the sound wave (f) = 440 Hz

speed of sound in air (v) = 343 m/s

The wavelength of the sound wave is given by the formula:

$$\lambda = \frac{v}{f}$$

Substitute the values:

$$\lambda = \frac{343\,\text{m/s}}{440\,\text{Hz}} \quad ; \quad \lambda = 0.7795\ \text{meters}$$

10. Given

Length of the open - end organ pipe (L) = 2.4 meters

Speed of sound in air (v) = 343 m/s

The fundamental frequency of an open-end organ pipe occurs when the length of the pipe is equal to one - fourth of the wavelength of the sound wave:

$$L = \frac{\lambda}{4}$$

We can rearrange the formula to solve for the wavelength:

$$\lambda = 4L$$

Now, we can use the formula for the speed of a wave to find the frequency.

$$\mathrm{v} = f \times \lambda$$

Substitute the values:

$$f = \frac{\mathrm{v}}{\lambda} = \frac{343\ \mathrm{m/s}}{4 \times 2.4\ \mathrm{m}}$$

$$f = \frac{343\ \mathrm{m/s}}{9.6\ \mathrm{m}}$$

$$f = 35.729\ \mathrm{Hz}$$

11. Given

Frequency of the third harmonic (f) = 225 Hz

Speed of sound in air (v) = 343 m/s.

We already know that the fundamental frequency (f_1) is given by $f_1 = \frac{\mathrm{v}}{4l}$. For the third harmonic, the frequency is three times the fundamental frequency (f_1):

$$f = 3 \times f_1 = 3 \times \frac{\mathrm{v}}{4l}$$

Now, we can solve for the length of the pipe (l):

$$l = \frac{3\mathrm{v}}{4f} = \frac{3 \times 343\ \mathrm{m/s}}{4 \times 225\ \mathrm{Hz}}$$

$$l = \frac{1029\ \mathrm{m/s}}{900\ \mathrm{Hz}}$$

l = 1.143 meters

12. $f = \frac{v}{4l}$ (for closed organ pipe) and

$f = \frac{3v}{2l}$ (second overtone for open organ pipe)

Since, both frequencies are equal $\frac{3v}{2l_o} = \frac{v}{4l_c}$

$l_o = 6\ l_c$

$l_o = 6 \times 20 = 120$ cm.

16. $f_1 = 10$ Hz, $f_2 = 15$ Hz

No. of beats = $f_2 - f_1 = 15 - 10 = 5$

17. Given:

Frequency of first sound wave (f_1) = 320 Hz

Frequency of second sound wave (f_2) = 328 Hz

The beat frequency is the absolute difference between the frequencies of the two waves:

Beat frequency = $| f_1 - f_2 |$ = | 320 Hz – 328 Hz | = 8 Hz

ANSWERS TO TERMINAL EXERCISE

8. 20 cm
9. 800 Hz